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A NOVEL APPROACH TO CHARACTERIZING THE RELATIONSHIP BETWEEN ECONOMIC GROWTH AND ENERGY CONSUMPTION

Using the tools of data-driven dynamical systems and Hamiltonian mechanics, we determine the relationship between energy costs and the distance traveled by a particular national economy in the economic space. The basis for the calculations is the time series describing the evolution of a cumulative GDP, recalculated according to the original method from monetary units to linear geometric dimensions, and energy resources consumed over a fixed period of time.

***Keywords**: macroeconomics; primary energy consumption; economic growth; GDP; data-driven dynamical systems; Hamiltonian mechanics.*

INTRODUCTION

Access to resources, notably energy, is one of the most important factors that ensure not only the growth of an economy, but also its very existence.

The study of the interdependence between two variables — the growth of gross domestic product (GDP) and the consumption of primary energy resources (PER), which include coal, oil and natural gas, electricity generated by hydro and

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nuclear power plants, as well as installations using renewable energy sources, is currently a very active area of research undergoing explosive growth. This can be attested by the high number of citations that the papers devoted to the study of this topic receive. For example, the article by J. Asafu-Adjaye that studied the relationship between energy consumption, energy prices and economic growth for a number of Asian countries [1] has been cited 1713 times as of now, while the paper by Ch. Ch. Lee about energy consumption and GDP in developed countries [2] — 1187 times.

If we consider the above-mentioned publications as belonging to the first and second generations respectively, then the paper [3], that has been cited 1567 times, can be considered as a third-generation publication, while the article [4], cited 579 times, belongs to the fourth generation. By the same token, the publication [5] (1066 citation) belongs to the fifth generation, and [6] with 151 citations — the sixth. Therefore, only within this particular chain that consists of six links, the number of papers that discuss the same topic is 6262. Nevertheless, in spite of such an impressive number of publications, the scholars working in the area have seemingly failed to reach a consensus regarding the matter [7]. Thus, having analyzed the performance of 30 countries that are members of the OECD, as well as 70 countries that do not belong to this organization, some scholars have postulated unidirectional causality from economic growth to energy consumption for the developed member countries of the OECD, at the same time it was claimed that for developing countries this relationship between economic growth and energy consumption is not common [8]. At the same time, a group of experts from the International Monetary Fund (IMF) have arrived at the opposite conclusion. Specifically, they claimed that for the OECD member countries it is possible to sustain economic growth without an increase of energy consumption (i.e., per capita consumption of PER remains nearly constant for long periods of time), while for the countries that are not members of the OECD the elasticity of energy consumption is close to one, that is for each percent of increase of GDP per capita, there is one percent increase of energy consumption per capita [9].

Such conflicting opinions have led to the realization that the discussion about the relationship between GDP and energy consumption has degenerated into a debate about econometric methods [10] and fierce disputes about the presence/absence of causality [8; 9]; see also Stern [11] and the relevant references therein. This prompted I. Ozturk, one of the most knowledgeable scientists in the field, to call for new approaches instead of the usual methods based on a set of common variables for different countries and different time intervals [3, p. 340]. Unfortunately, judging, for example, by the works [12; 13], it is safe to assume that this call has not had an effect yet and the majority of scholars working in the area, metaphorically speaking, still prefer to avoid taking the road less traveled.

That said, it must be mentioned that B. Beaudreau in a series of interesting works has exploited the fruitful idea of linking economic growth to the growth in energy consumption, which in turn is linked to fluctuations of physical quantities, such as maximum machine speed/velocity (see [14] and the relevant references therein).

The goal of this article is to take the road less traveled by establishing a relationship between GDP and energy consumption within the framework of a data-driven dynamical system approach (for example, see [15]). The dynamical system in question is defined by the pair (X, φ), where X is a two-dimensional topological space called the state space and a collection φ = {φt} of maps, called the flow. The parameter $t \in R_+$ represents time. Each map φ_t maps an open subset of X into X [16]. In this context, the space $X = R_+^2$ is determined by the variables G and E representing GDP and energy consumption for a given economy respectively. Clearly, both G and E evolve as time changes. We derive the formulas for G and E as functions of time t by making use of the corresponding time series. The relationship between G and E is then derived from the time-independent invariants of the flow φ.

METHODS

In what follows, we will interpret the evolution described by the function $G = G(t),\ t \in R_+$ as the trajectory of a particle which is subject to a constant force, $F_1 = m_1 a_1$, where $a_1 = \text{const}$ is acceleration and m represents 'mass'. The corresponding equation of motion for uniform acceleration is given by

$$\ddot{G} = a_1,$$

which yields after integration

$$G = \frac{1}{2}a_1 t^2 + b_1 t + c_1, \tag{1}$$

where a_1 is the constant acceleration; b_1 — the initial velocity at $t = 0$; c_1 — the initial 'path' traveled by a 'point' $G(t)$ at time $t = 0$.

In the above, the trajectory of a particle can be interpreted, with some assumptions, as the GDP of a national economy accumulated over the course of some period of time I_t:

$$G = \sum_{t \in I_t} GDP_{t1}, \tag{2}$$

where GDP_{t1} represents an annual GDP produced in a year $t \in I_t$, recalculated in terms of linear units, m.

Normally, the GDP of a national economy is measured in that country's currency, which makes it nearly impossible to compare GDPs of different countries in a meaningful way by employing the corresponding local currencies. Of course, in order to compare different countries' GDPs, one may convert them to a common currency, say, the US dollar. A similar problem exists in differential geometry: vectors belonging to different tangent spaces at different points on a differential manifold cannot be compared by their components. One needs a connection with connection coefficients to define parallel transport that is used to move a vector from one tangent space to another, thus making such a comparison possible (see, for example, [18]). It is worth mentioning that this geometric approach is used in gauge theories in physics to compare quantities that appear to be incomparable [19]. The notion of purchasing power parity (PPP) in economics can be viewed as the economic analogue of the notion of a connection in differential geometry. It allows economists to compare, in particular, GDP across different countries. In 1986, The Economist magazine created a survey, called the Big Mac Index, to measure PPP between different national economies [17]. It uses the prices of a McDonald's Big Mac as the benchmark under the premise that a Big Mac hamburger is roughly the same all over the world and available now almost in every country. Therefore, a real relative value of a national currency can be evaluated by comparing the local price of a Big Mac against the price of this hamburger in New York City.

In 2019, a Big Mac was selling in the USA for $5.58, in the UK it cost £3.19, while in Ukraine — 54 grn) [1]. The nominal GDP of Ukraine was 3,558.7 billion of hryvnias in 2018 [2], which implies that in terms of hamburgers the Ukrainian economy generated almost 66 billion of nominal Big Macs.

In the same year, the US GDP was estimated to be $2{,}049 \cdot 10^{13}$ in the current dollars [3], which corresponds to 3, 673 billion of Big Macs, taking into the account that the price of one Big Mac is $5.58.

The next step justifies the use of linear units. Indeed, the existence of the standard Burger Box that is used to package Big Macs leads to the following formula that allows to recalculate a GDP in terms of linear units:

$$GDP_{t1} = \frac{GDP_t}{P_{BMt}} \cdot l_{bb}, \tag{3}$$

where GDP_t represents an annual GDP produced in a year $t \in I_t$, USD; P_{BMt} — the price of a Big Mac during the same year $t \in I_t$; l_{bb} — the length of the McDonald's Burger Box.

In turn, the consumption of energy resources used by macroeconomics to generate GDP during the period under review is equal to

$$E = \sum_{t \in I_t} CE_t, \tag{4}$$

where E is the actual consumption of PER during the period I_t, J; CE_t represents the annual consumption of PER in a year $t \in I_t$.

The empirical evidence suggests (see below) that E as a function of time t can also be described by a quadratic polynomial as follows:

$$E = g(t) = \frac{1}{2} a_2 t^2 + b_2 t + c_2, \tag{5}$$

where a_2 is the acceleration of energy consumption; b_2 — the rate of consumption of energy resources at time $t = 0$; c_2 denotes the amount of energy resources used at time $t = 0$.

Next, it follows that the formulas (1) and (5) determine the flow φ of the dynamical system (X, φ) defined above that describes the evolution of a national economy in terms of the corresponding changes in its energy consumption and GDP.

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1 Индекс Биг Мака: гривня оказалась среди самых недооцененных валют. Ліга. Фінанси. 2019. 11 January. Available at: https://finance.liga.net/ekonomika/novosti/the-economist-nazvalo-grivnyu-odnoy-iz-samyh-nedootsenennyh-valyut (accessed on: 04.08.2021) [in Russian].

2 Рост ВВП Украины в 2018 году составил 3,3% — Госстат. Delo.ua. 2019. 22 March. Available at: https://delo.ua/econonomyandpoliticsinukraine/rost-vvp-ukrainy-v-2018-godu-sostavil-33-gos-351251/ (accessed on: 04.08.2021) [in Russian].

3 Соединенные Штаты Америки — ВВП, в текущих ценах (единиц нац. валюты). knoema. Available at: https://knoema.ru/atlas/Соединенные-Штаты-Америки/topics/Экономика/Национальные-счета-Валовой-внутренний-продукт/ВВП-в-текущих-ценах-единиц-нац-валюты (accessed on: 04.08.2021) [in Russian].

We note that in this context the parameters that appear in the formulas (1) and (5), namely, b_1, b_2, c_1, c_2 are strictly positive, while a_1 and a_2 can be either positive, negative, or zero.

Consider first the case when $a_1^2 + a_2^2 \neq 0$.

This assumption implies that both G and X evolve with constant acceleration, that is

$$\ddot{G} = a_1; \qquad \ddot{E} = a_2. \tag{6}$$

We can interpret either of the equations (6) as describing the motion in a uniform field.

In what follows our goal is to determine the relationships between $\dot{E}$ & E, $\dot{G}$ & G and, ultimately, $E = F(G)$, where F is a function to be determined. To achieve our goals, we will use some standard techniques from the theory of ordinary differential equations and Hamiltonian mechanics. Indeed, first let us rewrite each second-order ordinary differential equation (6) as a system of first-order ordinary differential equations by introducing new variables, namely

$$\begin{cases} \dot{E} = Y \\ \dot{Y} = a_2 \end{cases}, \tag{7}$$

and

$$\begin{cases} \dot{G} = Z \\ \dot{Z} = a_1 \end{cases}. \tag{8}$$

It follows immediately that each of the systems (7) and (8) can be viewed as a Hamiltonian system. The Hamiltonian systems in question are defined by the corresponding Hamiltonian functions:

$$H_1 = \frac{1}{2}\dot{G}^2 - a_1 G = \text{const} \tag{9}$$

and

$$H_2 = \frac{1}{2}\dot{E}^2 - a_2 G = \text{const}, \tag{10}$$

respectively w.r.t. to a standard (canonical) symplectic form.

The Hamiltonians H_1 and H_2 are invariants (i.e., functions independent of t) of the flows defined by the systems of differential equations (7) and (8) respectively. Note we can determine each value $H_1 = \text{const}$ and $H_2 = \text{const}$ from the corresponding initial conditions defined by the equations (1) and (5). This observation gives us some valuable information regarding the corresponding relationships G vs $\dot{G}$ and E vs $\dot{E}$. For example, under the assumption $\dot{E}$, a_2, $H_2 > 0$, it follows from (10) that an increase in E yields an increase in $\dot{E}$.

Let us establish now a connection between G and E. To achieve this, we will use the standard techniques of Lie group theory [20]. Indeed, consider the following system:

$$\begin{cases} \dot{G} = a_1 t + b_1, \\ \dot{E} = a_2 t + b_2, \end{cases} \tag{11}$$

or

$$\begin{cases} \dot{G}^2 = 2a_1\left(G + \dfrac{b_1^2}{2a_1} - c_1\right), \\ \dot{E}^2 = 2a_2\left(E + \dfrac{b_2^2}{2a_2} - c_2\right) \end{cases} \tag{12}$$

after completing the square in each equation, employing the equations (1) and (5). Thus, we arrive at the following system of first-order differential equations:

$$\begin{cases} \dot{G} = \pm\sqrt{2a_1\left(G + \dfrac{b_1^2}{2a_1} - c_1\right)}, \\ \dot{E} = \pm\sqrt{2a_2\left(E + \dfrac{b_2^2}{2a_2} - c_2\right)}. \end{cases} \tag{13}$$

Since the variables $G(t)$ and $E(t)$ are used here to describe the growth in both GDP and energy consumption, we can assume in what follows that $\dot{G} > 0$ and $\dot{E} > 0$ without loss of generality. Therefore, we obtain

$$\begin{cases} \dot{G} = \sqrt{2a_1\left(G + \dfrac{b_1^2}{2a_1} - c_1\right)}, \\ \dot{E} = \sqrt{2a_2\left(E + \dfrac{b_2^2}{2a_2} - c_2\right)}. \end{cases} \tag{14}$$

Now, we can view the evolution of G and E at the infinitesimal level as the dynamics generated by the vector field V_{GX} defined by (14), namely

$$V_{GE} = \sqrt{2a_1\left(G + \frac{b_1^2}{2a_1} - c_1\right)}\frac{\partial}{\partial G} + \sqrt{2a_2\left(E + \frac{b_2^2}{2a_2} - c_2\right)}\frac{\partial}{\partial E}. \tag{15}$$

Next, let us find an invariant function $W = W(G, E)$ such that $V_{GE}(W) = 0$. To find such a function, we solve this partial differential equation (PDF) by the method of characteristics. Note, the system of ordinary differential equations (14) defines the characteristics of the PDF $V_{GE}(W) = 0$. The corresponding Lagrange-Charpit equations are given by

$$\frac{dG}{\sqrt{2a_1\left(G + \dfrac{b_1^2}{2a_1} - c_1\right)}} = \frac{dE}{\sqrt{2a_2\left(E + \dfrac{b_2^2}{2a_2} - c_2\right)}} = \frac{dt}{1}. \tag{16}$$

Integrating the first two terms in (16), we arrive at the following invariant:

$$\frac{1}{a_2}\sqrt{2a_2E + b_2^2 - 2a_2c_2} = \frac{1}{a_1}\sqrt{2a_2G + b_1^2 - 2a_1c_1} + C, \tag{17}$$

where C is the constant of integration.

To determine C, we use the initial condition for $t = 0$: $G(0) = G_0 = c_1$, $E(0) = E_0 = c_2$. Substituting, we get

$$C = \frac{1}{a_2}\sqrt{2a_2E_0 + b_2^2 - 2a_2c_2} - \frac{1}{a_1}\sqrt{2a_2G_0 + b_1^2 - 2a_1c_1} = \frac{b_2}{a_2} - \frac{b_1}{a_1}, \tag{18}$$

since b_1; b_2 > 0. Substituting (18) into (17), we obtain or, multiplying through by a_1a_2, we get

$$a_1\sqrt{2a_2E+b_2^2-2a_2c_2}=a_2\sqrt{2a_1G+b_1^2-2a_1c_1}+a_1b_2-a_2b_1.$$

Finally, isolating E, we arrive at

$$E=\frac{1}{2a_2}\left(\frac{a_2}{a_1}\sqrt{2a_1G+b_1^2-2a_1c_1}+\frac{a_1b_2-a_2b_1}{a_1}\right)^2-\frac{b_2^2}{2a_2}+c_2. \quad (19)$$

The formula (19) defines the function $E = E(G)$. Note, we can express in a similar way G as a function of E. Indeed, solving for G in the above, we get

$$G=\frac{1}{2a_1}\left(\frac{a_1}{a_2}\sqrt{2a_2E+b_2^2-2a_2c_2}+\frac{a_2b_1-a_1b_2}{a_2}\right)^2-\frac{b_1^2}{2a_1}+c_1. \quad (20)$$

Next, integrating the first and the third terms in (16), we get

$$\frac{1}{a_1}\sqrt{2a_1G+b_1^2-2a_1c_1}=t+C,$$

where C is the constant of integration. To find C, we set $t = 0$ in the above to get

$$C=\frac{1}{a_1}\sqrt{2a_1G+b_1^2-2a_1c_1}.$$

Thus, we get

$$f(G)=f(G_0)+t, \quad (21)$$

where

$$f(x)=\frac{1}{a_1}\sqrt{2a_2x+b_1^2-2a_1c_1}.$$

Similarly, integrating the second and third terms in (16), we obtain

$$g(E)=g(E_0)+t, \quad (22)$$

where

$$g(x)=\frac{1}{a_2}\sqrt{2a_2x+b_2^2-2a_2c_2}.$$

It is easy to see that the functions (21) and (22) define the one-parameter Lie group transformation $\sigma: G\times R_+^2\to R_+^2$, where $G = (R, +)$, corresponding to the infinitesimal transformation given by (15). Also, we note that the formulas (21) and (22) are equivalent to the corresponding formulas given by (1) and (5) and so they also define the flow φ. Now, let us consider the case when $a_1\neq 0$, while $a_2 = 0$. The equations (11) reduce to the following system:

$$\begin{cases}\dot{G}=a_1t+b_1,\\ \dot{E}=b_2.\end{cases} \quad (23)$$

Accordingly, the system of ODEs (14) becomes

$$\begin{cases}\dot{G}=\sqrt{2a_1\left(G+\frac{b_1^2}{2a_1}-c_1\right)}.\\ \dot{E}=b_2.\end{cases} \quad (24)$$

and we have

$$V_{GE} = \sqrt{2a_1\left(G + \frac{b_1^2}{2a_1} - c_1\right)}\frac{\partial}{\partial G} + b_2\frac{\partial}{\partial E}. \quad (25)$$

The corresponding Lagrange-Charpit equations assume the following form:

$$\frac{dG}{\sqrt{2a_1\left(G + \frac{b_1^2}{2a_1} - c_1\right)}}\frac{dE}{b_2} = \frac{dt}{1}. \quad (26)$$

Repeating the same procedure as above, we integrate the first two terms to obtain

$$E = \frac{b_2}{a_1}\sqrt{2a_1G + b_1^2 - 2a_1c_1} + b_2C. \quad (27)$$

where

$$C = \frac{c_2}{b_2} - \frac{b_1}{a_1}. \quad (28)$$

Similarly, solving for *G*, we arrive at the following expression:

$$G = \frac{(a_1E - a_1b_2C)^2}{2a_1b_2^2} - \frac{b_1^2}{2a_1} + c_1. \quad (29)$$

where *C* is given by (28).

When $a_1 = 0$ and $a_2 \neq 0$, the system of differential equations (14) reduces to

$$\begin{cases} \dot{G} = b_1, \\ \dot{E} = a_2t + b_2, \end{cases} \quad (30)$$

from which we obtain

$$\begin{cases} \dot{G} = b_1, \\ \dot{E} = \sqrt{2a_2\left(E + \frac{b_2^2}{2a_2} - c_2\right)}. \end{cases} \quad (31)$$

The system of differential equations (31) corresponds to the following vector field:

$$V_{GE} = b_1\frac{\partial}{\partial G} + \sqrt{2a_2E + b_2^2 - 2a_2c_2}\frac{\partial}{\partial E}. \quad (32)$$

The corresponding Lagrange-Charpit equations are given by

$$\frac{dG}{b} = \frac{dE}{\sqrt{2a_2E + b_2^2 - 2a_2c_2}} = \frac{dt}{1}. \quad (33)$$

Integrating, we obtain from (33)

$$G = \frac{b_1}{a_2} + \sqrt{2a_2E + b_2^2 - 2a_2c_2} + b_1C, \quad (34)$$

where

$$C = \frac{c_1}{b_1} - \frac{b_2}{a_2}. \quad (35)$$

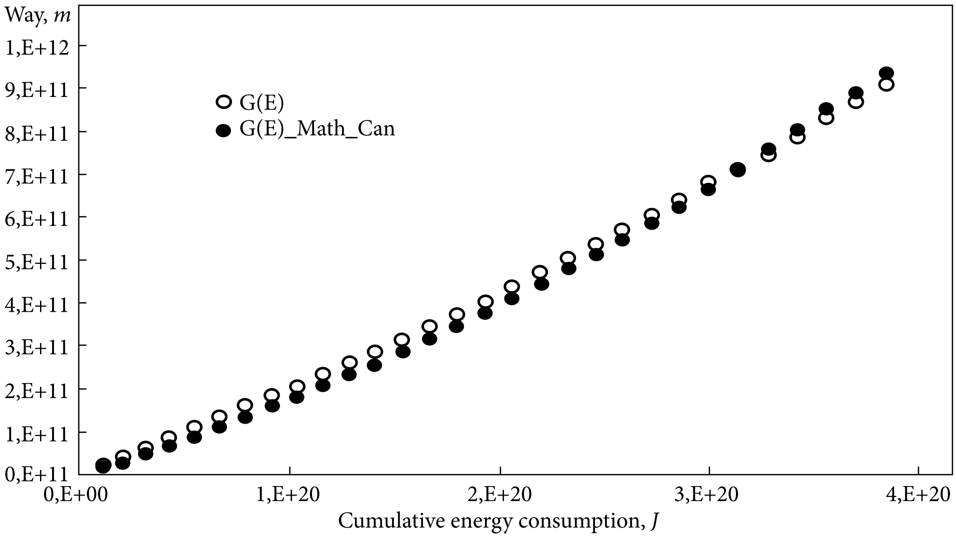

Fig. 1. The trajectory of the Canadian economy: model predictions vs empirical data
Sources: Produced by the authors.

The parameters of economic development of the four national economies in question: CAN — Canada, POL — Poland, RUS — Russia, UKR — Ukraine

Parameters	CAN	POL	RUS	UKR
a_1, 10^{-7}m/s^2	8,0	6,6	21,0	0,4
a_2, J/s^2	104,0	10,3	64,0	−120,0
b_1, m/s	594,0	239,0	1070,0	372,0
b_2, 10^{11}J/s	4,0	1,2	8,0	2,3
c_1, 10^{10}m	2,2	1,0	7,2	1,8
c_2, 10^{19}J	1,1	0,4	3,6	1,1

Sources: authors' data.

Solving for E in (34), we get

$$E = \frac{(a_2 G - a_2 b_1 C)^2}{2a_2 b_1^2} - \frac{b_2^2}{2a_2} + c_2. \quad (36)$$

Finally, consider the case when $a_1 = a_2 = 0$. This condition yields the following system:

$$\begin{cases} \dot{G} = b_1, \\ \dot{E} = b_2. \end{cases} \quad (37)$$

from which we get

$$G = \frac{b_1}{b_2} E - b_1 C \ \text{ or } \ E = \frac{b_2}{b_1} G - b_2 C, \quad (38)$$

where

$$C = \frac{c_1}{b_1} - \frac{c_2}{b_2}. \quad (39)$$

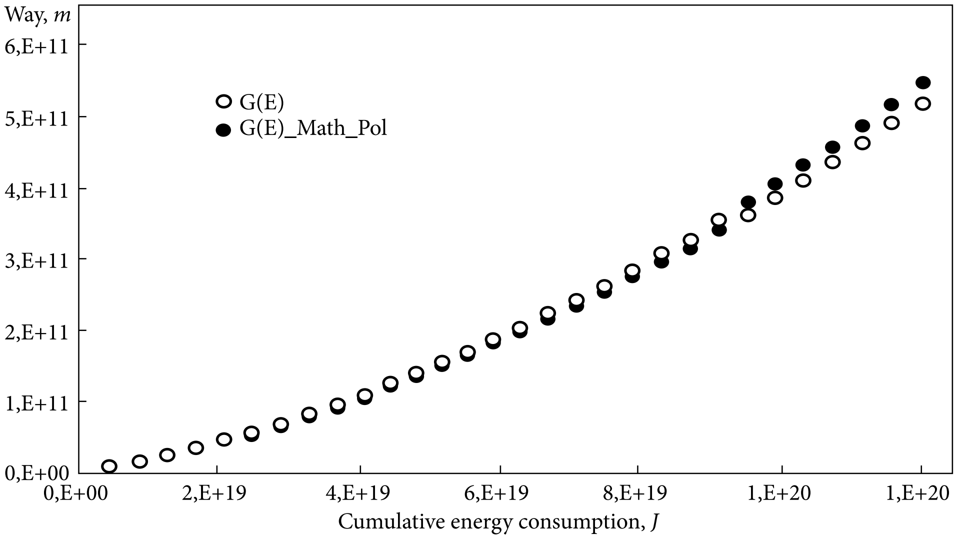

Fig. 2. The trajectory of the Polish economy: model predictions vs empirical data
Sources: Produced by the authors.

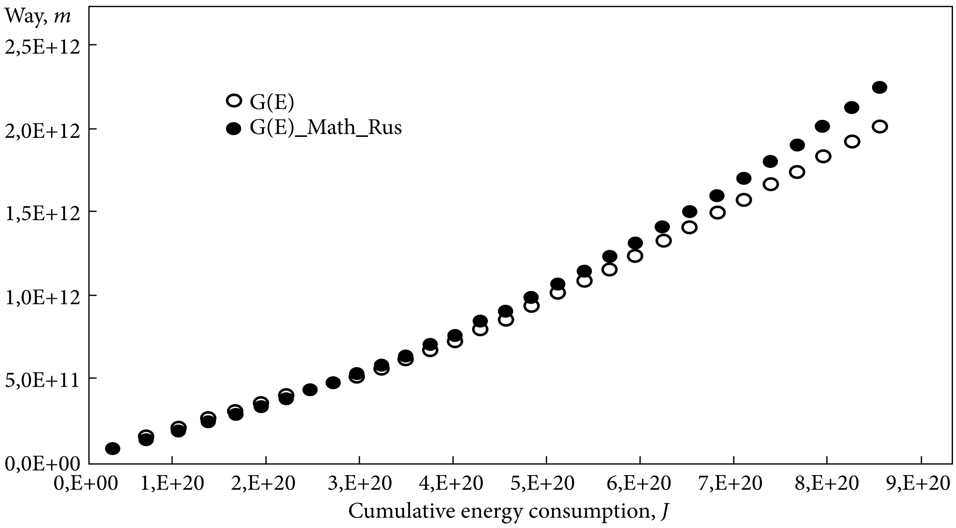

Fig. 3. The trajectory of the Russian economy: model predictions vs empirical data
Sources: Produced by the authors.

DATA

In this paper, the international dollar is used as the unit of measurement of GDP, recalculated at purchasing power parity in 2017 prices — GDP, PPP (constant 2017 International dollars [4]). The price of a Big Mac in the United States in 2017 was $5.3 per unit. The dimensions of a standard Burger Box are $0.12 \times 0.12 \times 0.065$ m. The data on the consumption of PER in various countries used in this work are derived from a survey prepared by BP [5].

[4] World Bank Open Data. Available at: https://data.worldbank.org (accessed on: 04.08.2021).
[5] Statistical Review of World Energy 2020. The 69th ed. Available at: https://www.bp.com/content/dam/bp/business-sites/en/global/corporate/pdfs/energy-economics/statistical-review/bp-stats-review-2020-full-report.pdf

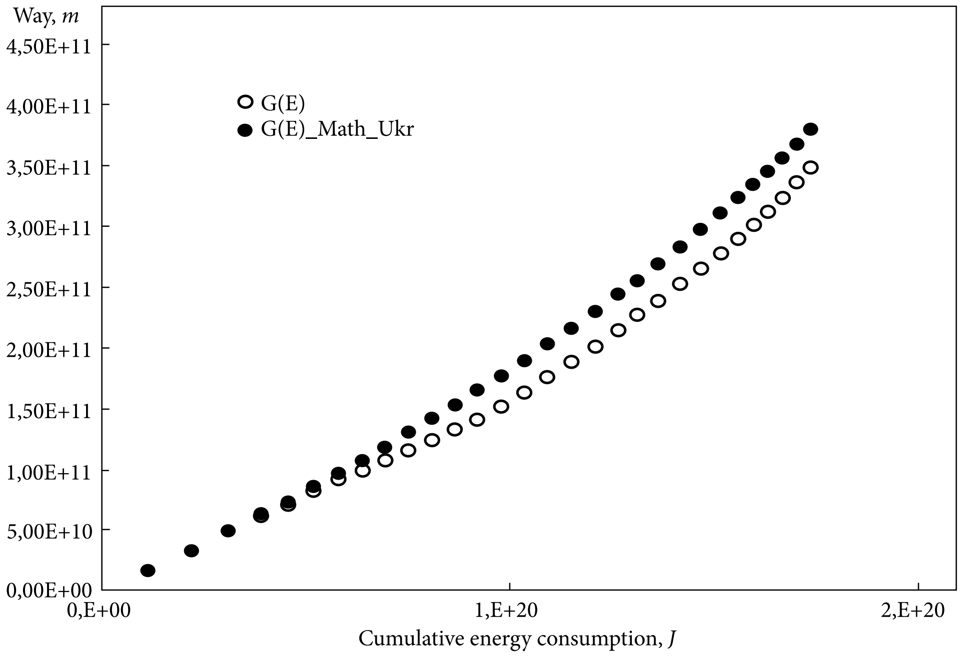

Fig. 4. The trajectory of the Ukrainian economy: model predictions vs empirical data
Sources: Produced by the authors.

Recently, instead of the previously widely used for this purpose million tons of oil equivalent (Mtoe), the energy values are represented in SI units, namely in exajoules (1 EJ=1018 J; 1 toe=41868 MJ). The macroeconomic objects of our research are the following countries: Canada, Poland, Russia, and Ukraine (see the table). The observation period spans the 30 years between 1990 and 2019.

RESULTS

The table below presents the parameters of economic development and energy consumption of the national economies of Canada, Poland, Russia, and Ukraine.

Next, we compare the path $G(E)$ as a function of the cumulative energy consumption for the four national economies in question computed according to the formula (20) — Canada, Poland, or, (29) — Russia, Ukraine, wherever applicable, against the same path determined by the actual data.

Figures 1—4 present respectively the trajectories of the Canadian, Polish, Russian, and Ukrainian economies determined by the function G(E) defined above. The light dots represent the path defining model predictions, while the dark dots — the corresponding empirical data.

CONCLUSIONS

In physics, energy is defined as the capacity to do work, or produce heat. In fact, entropy is a measure of how much energy is not available to do work. At the same time, the ability to do work is what generates GDP: the more work is done — the higher GDP per capita is. In this paper we have proposed a mathematical framework that can be used to quantify this connection. Specifically, our approach is based on the study of data-driven dynamical systems that are defined by the time series describing the growth in GDP and energy consumption. The function that determines this connection is found as an invariant of the corresponding dynamical system 'extracted' from the given data. It allows us to estimate the energy costs necessary for the national economy to pass through the economic space of the path corresponding to the GDP accumulated over a given period of time.

To test our method, we have studied the economic growth vs energy consumption of four different national economies whose growth in both GDP and energy consumption can be accurately approximated by parabolas. Specifically, we have proven the validity of our approach by comparing the calculated indicators with the actual ones. Using the example of four national economies, namely those of Canada, Poland, Russia, and Ukraine, we have demonstrated that the data produced with the aid of the function $G = G(E)$ provide us with a near perfect fit to the actual data. Importantly, the formulas (19) and (20) can be used for predicting future states of macroeconomics.

A special feature of the work is the recalculation of economic indicators into linear geometric dimensions, which makes it possible, in particular, to apply the tools of Hamiltonian mechanics to calculate the energy-economic development of macroeconomics.

The approach via approximation of economic and energy processes by parabolas proposed in the paper can be viewed as the first step in developing a new theory that will be used to analyze economic growth across a broad spectrum of countries and world regions. The work in this direction is underway.

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НОВИЙ ПІДХІД ДО ХАРАКТЕРИСТИКИ ВЗАЄМОДІЇ ЕКОНОМІЧНОГО ЗРОСТАННЯ ТА СПОЖИВАННЯ ЕНЕРГІЇ

На макроекономічному рівні споживання первинних енергоресурсів є детермінованим чинником економічного розвитку, що виправдовує актуальність і важливість розробки відповідної математичної моделі. Незважаючи на велику кількість, а саме — декілька тисяч, наукових праць з проблем взаємозалежності зростання макроекономік і витрат первинних енергетичних ресурсів, закономірності так і не виявлено, що обумовило провідних учених звернутися до наукової спільноти із закликом винайти нові підходи замість звичних методів, заснованих на наборі загальних змінних для різних країн і різних інтервалів часу.

З використанням інструментів керованих даними динамічних систем, які в міжнародній практиці відомі як Data-driven dynamical systems, і Гамільтонової механіки визначено взаємозв'язок між витратами енергії та шляхом, що проходить конкретна національна економіка в економічному просторі.

Основою для розрахунків є часові ряди, які описують еволюцію сукупного ВВП, перерахованого оригінальним способом з грошових одиниць у лінійні геометричні розміри, а також енергоресурси, які споживаються за фіксований період часу.

Актуальність отриманих математичних співвідношень підтверджується через порівняння прогнозів поведінки математичної моделі з емпіричними даними, отриманими для чотирьох країн: Канади, Польщі, Росії та України. У всіх чотирьох випадках модель продемонструвала високоточну відповідність фактичним даним.

Запропонований підхід апроксимації еволюції економічних показників і споживання енергії параболами є основою для розробки загальної теорії, яка може бути використана для аналізу економічних показників широкого спектра національних економік і регіонів світу.

Ключові слова: *макроекономічний рівень; споживання первинної енергії; економічний розвиток; керовані даними динамічні системи; Гамільтонова механіка; витрати енергії; пройдений в економічному просторі шлях.*