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TWO-LEVEL BALANCE MODEL OF PRODUCTS DISTRIBUTION BASED ON MARKOV CHAINS

Abstract. On the basis of one of approaching for models of exchange the two-level balance model of products distribution is created at the use of mathematical theory of probabilistic processes. The participants of nearby overhead and lower channels of commodities distribution in the chains of deliveries can be situated on these levels: producers-consumers, mediators of distribution. A model on the basis of Markov chains allows to determine self-congruent equilibrium relative distributions of finances and commodities monetary at every level accordingly. The relations of amount of the got commodities at bottom level will be saved if volume of deliveries changes at top level proportionally to the equilibrium distribution. In this case every recipient of bottom level knows the part from the distributed products, that assists more effective organization of trade process. The offered methodology is interesting for creation of multilevel channels models of commodities distribution.

Keywords: exchange model, balance, Markov chain.
JEL Classification: C25, C35
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ДВОРІВНЕВА БАЛАНСОВА МОДЕЛЬ РОЗПОДІЛУ ПРОДУКЦІЇ НА ОСНОВІ ЛАНЦЮГІВ МАРКОВА

Анотація. На основі одного з підходів до моделей обміну при використанні математичної теорії випадкових процесів створена дворівнева балансова модель розподілу продукції. На цих рівнях можуть розташовуватися учасники сусідніх по вертикалі каналів розподілу товарів в ланцюгах постачань: виробники-споживачі, посередники розподілу. Модель на основі ланцюгів Маркова дозволяє визначати самоузгоджені рівноважні відносні розподіли фінансів і товарів в грошовому вираженні на кожному рівні відповідно. При зміні об'єму поставок пропорційно рівноважному розподілу на верхньому рівні відношення кількості отриманих товарів на нижньому рівні зберігатимуться. В цьому випадку кожен одержувач нижнього рівня знає свою долю від продукції, яка розподіляється, що сприяє ефективнішій організації торговельного процесу. Використовувана методика представляє інтерес для створення багаторівневих моделей каналів розподілу товарів.

Ключові слова: модель обміну, баланс, ланцюги Маркова.

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ДВУХУРОВНЕВАЯ БАЛАНСОВАЯ МОДЕЛЬ РАСПРЕДЕЛЕНИЯ ПРОДУКЦИИ НА ОСНОВЕ ЦЕПЕЙ МАРКОВА

Аннотация. На основе одного из подходов к моделям обмена при использовании математической теории случайных процессов создана двухуровневая балансовая модель распределения продукции. На этих уровнях могут располагаться участники соседних по вертикали каналов распределения товаров в цепях поставок: производители-потребители, посредники распределения. Модель на основе цепей Маркова позволяет определять самосогласованные равновесные относительные распределения финансов и товаров в денежном выражении на каждом уровне соответственно. При изменении объёма поставок пропорционально равновесному распределению на верхнем уровне отношения количества полученных товаров на нижнем уровне будут сохраняться. В этом случае каждый получатель нижнего уровня знает свою долю от распределяемой продукции, что способствует более эффективной организации торгового процесса. Используемая методика представляет интерес для создания многоуровневых моделей каналов распределения товаров.

Ключевые слова: модель обмена, баланс, цепи Маркова.

Формул: 7; рис.: 0; табл.: 2; библ.: 21.

Introduction. Activity of man is constantly related to the processes of exchange by different values Trade became the result of development of exchange processes, that presently is the important constituent of socio-economic development of countries, plays a considerable role their integrations in an international association. The researches of Blyde, Faggioni, [1] can be confirmation of it. Description of difficult dynamic processes supposes the use of mathematical models. Quality of prediction depends on them. In this connection the models of exchange in an economy present substantial interest for planning, and their development and creation of new models are actual.

Basic questions in the models of exchange are determination of the state of equilibrium of process and study of dynamics that results in him.

Analysis of the last researches and publications. One of approaches for the study of the states of equilibrium in the economic systems that describe an exchange blessing between the subjects of process, on the basis of the recurrent systems of equalizations, differential equalizations, and also determination of own vectors of matrices was worked out and develops scientists of Valras

[2], Arrow, Debreu [3,4], McKenzie [5], Sonnenschein [6], Makarov et al. [7], Makarov [8], Brodsky [9], Andreev [10].

Other approach for the study of exchange processes was worked out on the basis of mathematical theory of probabilistic processes-chains of Markov (look the monographs of Howard [11], Kemeny, Snell [12], Zhluktenko et al. [13], Sokolov, Chistyakov [14], Kuznichenko et al. [15]). Note that at first deficit-free and deficit-having models both without a management and with an external management developed and used only for discrete processes. In deficit-having model unlike deficit-free the processes of exchange («demand» and «supply») are investigated separately and on leaving on the stationary mode their participants have a not zero balance from a transmission and receipt of blessing.

This approach gives an opportunity not only to determine the equilibrium states of the systems but also to study the dynamics of moving from the initial state to equilibrium one.

A further step was development of continuous deficit-free and deficit-having models both without a management and with an external management [16-18]). In the built models of exchange for continuous processes results continuous and discrete models coincide in moments of time, multiple to the step of discrete model. It is necessary to mark the stochastic models for inflation, investments and exchange rates which worked out by Wilkie [19]. There is of interest another stochastic approach to the exchange models. Different aspects and applications of kinetic exchange model from statistical physics to the economic and sociological processes are considered by Goswami, Chakraboti [20].

Goal. The aim of this work is creation of two-level balance model of distribution of products from producers to the consumers at determination of the consistent equilibrium states of every level on the basis of stochastic approach-Markov chains.

Originality of the research. Balance method and created on it basis balance models, that is included in the models of exchange, serve as the basic instrument of maintenance of proportions in a national economy. Balance models are built as numerical matrices, therefore they are attributed to the type of economics-mathematic matrix models. Complication of processes in a market economy is required by maintenances in her equilibrium in a macroeconomic scale: the concerted balancing of allocation of limit resources and requirements in them.

The most well-known and until now used balance model is a model of linear inter-branch balance of Leontief [21]. To the balance models it is possible to take deficit-having models [15–18]. There are «salespeople» and «customers» the same in these models, and the processes of sales and purchases divide. The found steady-states of these processes for every participant are given by possibility to define trade balance – equilibrium distribution of money and commodity streams.

Both the processes of sales and purchases and «salespeople» and «customers» (their amount can be anything) are divided in this article. A model on the basis of Markov chains allows to determine the concerted equilibrium distributions of finances and commodities at every level accordingly. Under co-ordination we will understand maintenance of proportional distribution of exchange products at every level at the change of their volume.

Research results. The direct chain of deliveries (simple channel of distribution of commodities): «salespeople (producers)»-»customers (consumers)» is considered. Let there is P_j^0 ($j=\overline{1,m_0}$) an amount of «salespeople» at zero overhead level, and on the first – P_i^1 ($i=\overline{1,m_1}$) amount of «customers». Every «salesman» distributes a commodity between «customers» according to agreements that take into account possibility of change of volumes of delivery at maintenance of the proportion between them. «Customers» pay commodities, carrying out a money stream to the «salespeople». The matrices of pair comparisons of volumes of commodities monetary for «customers» in relation to every «salesman» are presented in a table. 1.

Table 1

Matrices of pair comparisons for the participants of 1 level in relation to every participant of 0 level

P_j^0	P_1^1	P_2^1	P_3^1	…	$P_{m_1}^1$	*Sum*	$V(P_j^0, P^1)$
P_1^1	1	x_{1j}/x_{2j}	x_{1j}/x_{3j}	…	x_{1j}/x_{m_1j}	$x_{1j} * t_j$	x_{1j}/X_j^0
P_2^1	x_{2j}/x_{1j}	1	x_{2j}/x_{3j}	…	x_{2j}/x_{m_1j}	$x_{2j} * t_j$	x_{2j}/X_j^0
P_3^1	x_{3j}/x_{1j}	x_{3j}/x_{2j}	1	…	x_{3j}/x_{m_1j}	$x_{3j} * t_j$	x_{3j}/X_j^0
…	…	…	…	…	…	…	…
$P_{m_1}^1$	x_{m_1j}/x_{1j}	x_{m_1j}/x_{2j}	x_{m_1j}/x_{3j}	…	1	$x_{m_1j} * t_j$	x_{m_1j}/X_j^0
$t_j = \frac{1}{x_{1j}} + \frac{1}{x_{2j}} + \frac{1}{x_{3j}} + ... + \frac{1}{x_{m_1j}}$ $X_j^0 = x_{1j} + x_{2j} + x_{3j} + ... x_{m_1j}$					*Sum*	$X_j^0 * t_j$	1

In the table 1 x_{ij} – amount of money, that the i «customer» passes to the j «salesman» or vice versa (j «salesman» passes to the i «customer» commodities monetary), $V(P_j^0, P^1)$ is eigenvector of matrices P_j^0. For all tables (matrices) of exchange distribution between participants 0 and 1 levels the next equalities are executed:

$$\sum_{i=1}^{m_1} x_{ij} = X_j^0, \sum_{j=1}^{m_0} x_{ij} = X_i^1, \sum_{j=1}^{m_0} X_j^0 = \sum_{i=1}^{m_1} X_i^1 = D \quad (1)$$

The matrices of pair comparisons of the got money between «salespeople» in relation to every «customer» are presented in a table 2.

Table 2.

Matrices of pair comparisons for participants of 0 level in relation to every participant of 1 level

P_i^1	P_1^0	P_2^0	P_3^0	…	$P_{m_0}^0$	*Sum*	$V(P_i^1, P^0)$
P_1^0	1	x_{i1}/x_{i2}	x_{i1}/x_{i3}	…	x_{i1}/x_{im_0}	$x_{i1} * t_i$	x_{i1}/X_i^1
P_2^0	x_{i2}/x_{i1}	1	x_{i2}/x_{i3}	…	x_{i2}/x_{im_0}	$x_{i2} * t_i$	x_{i2}/X_i^1
P_3^0	x_{i3}/x_{i1}	x_{i3}/x_{i2}	1	…	x_{i3}/x_{im_0}	$x_{i3} * t_i$	x_{i3}/X_i^1
…	…	…	…	…	…	…	…
$P_{m_0}^0$	x_{im_0}/x_{i1}	x_{im_0}/x_{i2}	x_{im_0}/x_{i3}	…	1	$x_{im_0} * t_i$	x_{im_0}/X_i^1
$t_i = \frac{1}{x_{i1}} + \frac{1}{x_{i2}} + \frac{1}{x_{i3}} + ... + \frac{1}{x_{im_0}}$ $X_i^1 = x_{i1} + x_{i2} + x_{i3} + ... + x_{im_0}$					*Sum*	$X_i^1 * t_i$	1

Here $V(P_i^1, P^0)$ – eigenvector of matrices P_i^1. On the found eigenvectors $V(P_j^0, P^1)$ $(j = \overline{1, m_0})$ and $V(P_i^1, P^0)$ $(i = \overline{1, m_1})$ matrices $V(P^0, P^1)$ and $V(P^1, P^0)$ are constructed:

$$V(P^0, P^1) = \begin{pmatrix} V^T(P_1^0, P^1) \\ V^T(P_2^0, P^1) \\ V^T(P_3^0, P^1) \\ \dots \\ V^T(P_{m_0}^0, P^1) \end{pmatrix} \quad (2)$$

$$V(P^1, P^0) = \begin{pmatrix} V^T(P_1^1, P^0) \\ V^T(P_2^1, P^0) \\ V^T(P_3^1, P^0) \\ \dots \\ V^T(P_{m_1}^1, P^0) \end{pmatrix} \quad (3)$$

The stochastic matrix $C(P^0)$ of size $m_0 * m_0$ is determined on a formula:

$$C(P^0) = V(P^0, P^1) * V(P^1, P^0) \quad (4)$$

The stochastic matrix $C(P^1)$ of size $m_1 * m_1$ is determined on a formula:

$$C(P^1) = V(P^1, P^0) * V(P^0, P^1) \quad (5)$$

The vector of global priorities $\overline{W}(P^0)$ is found from the system of equalizations:

$$\begin{cases} \overline{W}(P^0) = \overline{W}(P^0) * C(P^0) \\ \sum_{j=1}^{m_0} w_j(P^0) = 1 \\ \overline{W}(P^0) = \left(w_1(P^0), \quad w_2(P^0), \quad w_3(P^0), \quad \dots \quad, w_{m_0}(P^0)\right) \end{cases} \quad (6)$$

The vector of global priorities $\overline{W}(P^1)$ is found from the system of equalizations:

$$\begin{cases} \overline{W}(P^1) = \overline{W}(P^1) * C(P^1) \\ \sum_{j=1}^{m_1} w_j(P^1) = 1 \\ \overline{W}(P^1) = \left(w_1(P^1), \quad w_2(P^1), \quad w_3(P^1), \quad \dots \quad, w_{m_1}(P^1)\right) \end{cases} \quad (7)$$

Vectors of global priorities $\overline{W}(P^0)$, $\overline{W}(P^1)$ determine the equilibrium distribution of products system monetary in a two-level balance model. If a force change of relations of vectors components $\overline{W}(P^0)$, $\overline{W}(P^1)$ takes place, the used method of Markov chains allows to define the amount of steps for arrival of the system in the equilibrium state. In this case changed vectors play role of initial vectors of the systems states and the matrices $C(P^0)$, $C(P^1)$ are in a role of transitional matrices in the chains of Markov. At the certain number of steps n the system will get around a steady-state with large exactness.

Conclusion. The two-level balance model of distribution of products monetary, that determines the equilibrium states of every level in the process of exchange, on the basis of stochastic approach is worked out in this work. Both producers and consumers and participants of middle channels of distribution of commodities in the chains of deliveries can be on these levels. Not only the processes of sales and purchases but also their participants divide unlike the deficit-having models of exchange, here their amount can be anything. A model on the basis of Markov chains allows to determine equilibrium relative distributions of finances and commodities monetary at every level accordingly. Thus the found proportions of distribution at bottom level will be saved at the change of volume of deliveries to the equilibrium distribution at top level. It facilitates planning and prognostication of supplying with products substantially. Note that stationary distributions can be found not only at the decision of the systems of equalizations (6) and (7), and at successive erection of matrices (4) and (5) in a degree. The index of degree of matrices depends on exactness of approaching of matrix rows to equilibrium distribution. Further researches suppose development of model on the multilevel systems of channels of commodities distribution.

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