УДК 539.3

Mathematical model for interaction of normally incident elliptically polarized light with a stressed dielectric layer

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A model for interaction of polarized light with strained non-absorbing dielectric layer is considered in the paper. The effects of stresses, applied to layer, and polarization of normally incident light beam on the reflectance and transmittivity of the strained layer placed between two homogeneous isotropic dielectric half-spaces were studied. Possible application of the studied effects for determination of non-destructive stress-strained state of solids is discussed.

Keywords: polarized light, strained dielectrics, photoelasticity effect, polarized light reflectance, polarized light transmittivity.

Introduction. Isotropic dielectric elastic mediums acquire the properties of optical anisotropy under mechanical stresses. This effect is caused by the dependence of permittivity of the material on the strain. Polarized light, propagating in the deformed dielectric medium, changes its parameters due to strain components distribution along the ray. This phenomenon is known as the effect of photoelasticity [1]. It is widely used in non-destructive optical methods for the stress-strain state of solids determination [2]. Ellipticity of light and the angle of orientation of the polarization ellipse are used as informative parameters in photoelasticity methods [3]. However the changes of dielectric permeability, induced by the stresses, can impact on reflection and refraction of light on the boundaries dividing different dielectric mediums. Therefore, the coefficients of reflection and transmission of light beam, passing through the stressed medium, also can be used as informative parameters for non-destructive determination of the stress-strain state. To confirm the possibility of practical application of this phenomenon is necessary to evaluate the sensitivity of a stressed medium’s reflectance and transmissivity on the level of applied stresses.

Analytical formulas, that determinate effect of thermal stresses, caused by short optical pulses, on the reflectance and transmittance of light normally incident on a multilayer thin-film structure, have been obtained in the paper [4]. The results can be useful for analysis of photoacoustic and photothermal experiments based on optical probing.

In this paper we consider the problem of interaction of polarized light with an optically homogeneous non-absorbing dielectric layer, being in stress-strained state, which is characterized by a diagonal strain tensor. In assumption of linear constitutive relationships between the components of permittivity and strain tensors, considering the light beam as normally incident plane electromagnetic wave, the analytical solution of the appropriate electrodynamics problem for the strained layer that divide two isotropic semi-spaces, was obtained. The formulas for the reflectance and transmittance of light normally incident on the stressed layer were derived with use of obtained solution. On this basis the effect of stresses, applied to layer, and incident beam polarization on the coefficients of light reflection by the layer and transmission through it were studied. On this basis the sensitivity of these photometric parameters to strain and a possibility of their application as informative parameters for non-destructive methods of stress-strained state determination were evaluated.

1. Formulation of the problem

Consider a non-absorptive dielectric layer $S=\left(0<x<h_0\right)$, that separates two homogeneous isotropic non-absorptive dielectric half-spaces $S^{(1)}=\left(-\infty<x<0\right)$ and $S^{(2)}=$ $=\left(h_0<x<+\infty\right)$. Let denote as $\varepsilon^{(\alpha)}$, $\mu^{(\alpha)}$ and $n^{(\alpha)}=\sqrt{\mu^{(\alpha)}\varepsilon^{(\alpha)}}$ ($\alpha=1,2$) the relative dielectric permittivity, magnetic permeability and refractive index of the half-spaces $S^{(1)}$ and $S^{(2)}$. The medium of layer S in it's initial unstressed state is considered as homogeneous and isotropic one with the dielectric permittivity ε, magnetic permeability μ and refraction index $n=\sqrt{\mu\varepsilon}$.

Impact of mechanical stress on the layer S is accounted by the dependence of the dielectric properties of the medium on the of elastic strain tensor $\hat{\mathbf{e}}=\left(e_{ij}\right)$. The case of a homogeneous stress-strain state, determined by the diagonal strain tensor $\left(e_{ij}\right)=$ $=\operatorname{diag}\left(e_{11},e_{22},e_{33}\right)$ is considered. In this case the dielectric tensor $\hat{\boldsymbol{\varepsilon}}=\left(\varepsilon_{ij}\right)$ is also diagonal one: $\left(\varepsilon_{ij}\right)=\operatorname{diag}\left(\varepsilon_{11},\varepsilon_{22},\varepsilon_{33}\right)$, and it's principal components can be expressed via the components of strain tensor $\hat{\mathbf{e}}$ by the linear constitutive relationship [1]

$$\varepsilon_{11}=\varepsilon+\left(2p+p_0\right)e_{11}+p_0\left(e_{22}+e_{33}\right),$$
$$\varepsilon_{22}=\varepsilon+\left(2p+p_0\right)e_{22}+p_0\left(e_{11}+e_{33}\right),$$
$$\varepsilon_{33}=\varepsilon+\left(2p+p_0\right)e_{33}+p_0\left(e_{11}+e_{22}\right).$$

Let an elliptically polarized monochromatic light beam falls normally from the half-space $S^{(1)}$ on the surface $x=0$ of the layer S. The beam, interacting with the layer, splits on reflected and refracted beams on the surface $x=0$. The refracted beam, in turn, partially reflects back to the layer and refracts into the half-space $S^{(2)}$ on the opposite layer's surface $x=h_0$. We will consider the light beams, interacting with the layer, as electromagnetic wave of fixed cyclic frequency ω. The vectors of the electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields, electrical $\mathbf{D}$ and magnetic $\mathbf{B}$ inductions are functions of spatial coordinates x,y,z satisfy within the domains $S^{(1)}$, S and $S^{(2)}$ the equations

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t},\quad \nabla\cdot\mathbf{D}=0,\quad \nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t},\quad \nabla\cdot\mathbf{B}=0. \tag{1}$$

where as on the discontinuity boundaries, dividing the mediums with different dielectric properties, these vector fields satisfy the jump conditions

$$[\mathbf{E}_\tau]\big|_{x=0,h_0}=0,\quad [\mathbf{D}_n]\big|_{x=0,h_0}=0,\quad [\mathbf{H}_\tau]\big|_{x=0,h_0}=0,\quad [\mathbf{B}_n]\big|_{x=0,h_0}=0. \tag{2}$$

Here the subscripts τ and n are used to denote the tangential and normal components of the corresponding vector; the brackets $[\ldots]\big|_{x=0}$ and $[\ldots]\big|_{x=h_0}$ denote the jumps of the corresponding physical parameter on the boundaries $x=0$ and $x=h_0$ respectively.

We will use the linear physical relationships for pairs of parameters **D**, **E** and **B**, **H**

$$\mathbf{D}^{(i)}=\varepsilon_0\varepsilon_1\mathbf{E}^{(i)},\quad \mathbf{B}^{(i)}=\mu_0\mu_1\mathbf{H}^{(i)},\quad \mathbf{D}^{(r)}=\varepsilon_0\varepsilon_1\mathbf{E}^{(r)},\quad \mathbf{B}^{(r)}=\mu_0\mu_1\mathbf{H}^{(r)}; \tag{3}$$

$$\mathbf{D}=\varepsilon_0\hat{\boldsymbol{\varepsilon}}\cdot\mathbf{E},\quad \mathbf{B}=\mu_0\mu\mathbf{H},\quad \mathbf{D}^{(t)}=\varepsilon_0\varepsilon_2\mathbf{E}^{(t)},\quad \mathbf{B}^{(t)}=\mu_0\mu_2\mathbf{H}^{(t)}. \tag{4}$$

The field parameters of different waves are denoted here by superscripts enclosed in the parenthesis: the denotations (*i*) and (*r*) are used for incident and reflected waves correspondently, the superscript (*t*) is used for to denote the field parameters of the wave, transmitted through the layer into the half-space $S^{(2)}$, the free of indices symbols **D**, **E**, **B** and **H** denote the wave field parameters in the layer *S*.

Since the optical anisotropy of the layer is induced by strain, the resulting electromagnetic field, caused by interaction of the incident wave with the piecewise-homogeneous dielectric medium $S^{(1)}\cup S\cup S^{(2)}$, becomes dependent on strain. Due to that we formulate the problem as: for given incident wave's parameters and strain tensor $\hat{\mathbf{e}}$ it is necessary to determinate the wave field parameters in the domains $S^{(1)}$, S and $S^{(2)}$, which satisfy the equations (1), (3), (4) in these domains and the conditions (2) on their boundaries.

2. The solution of the problem

We consider the light beam as the plane electromagnetic wave, that normally falls from the half-space $S^{(1)}$ on the interface $x=0$ between the domains $S^{(1)}$ and *S*. Due to that the electric fields vector of the incident wave is defined as $\mathbf{E}^{(i)}(x,t)=$ $=\dot{\mathbf{E}}^{(i)}\exp\left[i\left(k^{(i)}x-\varpi t\right)\right]$ ($l=y,z$), where $\dot{\mathbf{E}}^{(i)}=\left(0,\dot{E}_y^{(i)},\dot{E}_z^{(i)}\right)$ is the vector of complex amplitude, $\dot{E}_l^{(i)}$ ($l=y,z$) are the components of the complex vector $\dot{\mathbf{E}}^{(i)}$ in the Cartesian coordinate system (x,y,z), $k^{(i)}=-\omega c_0^{-1}n^{(1)}=-2\pi n^{(1)}/\lambda_0$, $c_0\equiv 1/\sqrt{\mu_0\varepsilon_0}$ and λ_0 stands for the phase velocity and wavelength of the wave in vacuum, μ_0,ε_0 are magnetic and dielectric constants.

The incident wave $\dot{\mathbf{E}}^{(i)}(x,t)$ is elliptically polarized. Its polarization is determined by the complex scalar $\dot{\chi}^{(i)} = \dot{E}_z^{(i)} / \dot{E}_y^{(i)}$. The polarimetric parameters — polarization azimuth $\psi^{(i)}$ and light ellipticity $a^{(i)}/b^{(i)} \equiv \tan\left(\eta^{(i)}\right)$, where $a^{(i)}$ and $b^{(i)}$ are the axes of the polarization ellipse, can be expressed via $\dot{\chi}^{(i)}$ by the relationships [2]

$$\tan\left(2\psi^{(i)}\right) = \left(\tan\left(2\alpha^{(i)}\right)\right)\cos\left(\delta^{(i)}\right), \quad \sin\left(2\eta^{(i)}\right) = \left(\sin\left(2\alpha^{(i)}\right)\right)\sin\left(\delta^{(i)}\right),$$

where $\alpha^{(i)} = \arctan\left(\chi^{(i)}\right)$, $\chi^{(i)} = \mathrm{mod}\left(\dot{\chi}^{(i)}\right)$, $\delta^{(i)} = \arg\left(\dot{\chi}^{(i)}\right)$.

In the considered case of plane incident wave the wave the field arising in the domain $S^{(1)} \cup S \cup S^{(2)}$ is depending only on the spatial coordinate x, normal to the layer. So the reflected $\dot{\mathbf{E}}^{(r)}(x,t)$ and transmitted $\dot{\mathbf{E}}^{(t)}\left(x,t\right)$ waves are also plane ones [5]

$$\dot{\mathbf{E}}^{(r)}(x,t) = \dot{\mathbf{E}}^{(r)} \exp\left[i\left(k^{(r)}x - \omega t\right)\right], \quad \dot{\mathbf{E}}^{(t)}(x,t) = \dot{\mathbf{E}}^{(t)} \exp\left[i\left(-k^{(t)}x - \omega t\right)\right] \qquad (5)$$

with wave numbers:

$$k^{(r)} = 2\pi n^{(1)}/\lambda_0, \qquad k^{(t)} = 2\pi n^{(2)}/\lambda_0 .$$

Here $\dot{\mathbf{E}}^{(r)} = \left(0, \dot{E}_y^{(r)}, \dot{E}_z^{(r)}\right)$ and $\dot{\mathbf{E}}^{(t)} = \left(0, \dot{E}_y^{(t)}, \dot{E}_z^{(t)}\right)$ are the complex amplitudes of the reflected and transmitted waves.

The Cartesian components $\dot{E}_l^{(r)}$ and $\dot{E}_l^{(t)}$ ($l = y,z$) of complex vectors $\dot{\mathbf{E}}^{(r)}$ and $\dot{\mathbf{E}}^{(t)}$ can be presented as $\dot{E}_l^{(r)} = E_l^{(r)} \exp\left(-i\varphi_l^{(r)}\right)$, $\dot{E}_l^{(t)} = E_l^{(t)} \exp\left(-i\varphi_l^{(t)}\right)$, where $E_l^{(r)}$, $E_l^{(t)}$, $\varphi_l^{(r)}$ and $\varphi_l^{(t)}$ are real constant scalars.

The wave $\dot{\mathbf{E}}(x,t)$ in the layer S is also a plane transversal wave. But, because of optical anisotropy, induced by strain, the wave numbers k_y and k_z of its Cartesian components $\dot{E}_y(x,t)$ and $\dot{E}_z(x,t)$ are different [6]

$$k_y = 2\pi n_{22}/\lambda_0, \qquad k_z = 2\pi n_{33}/\lambda_0,$$

where $n_{33} = \sqrt{\mu\varepsilon_{33}}$, $n_{22} = \sqrt{\mu\varepsilon_{22}}$.

This wave can be presented as superposition of the direct $\mathbf{E}^{(d)}(x,t)$ and backward $\mathbf{E}^{(b)}(x,t)$ waves: $\mathbf{E}(x,t) = \mathbf{E}^{(d)}(x,t) + \mathbf{E}^{(b)}(x,t)$. In the Cartesian components the wave $\mathbf{E}(x,t)$ is written as

$$E_l(x,t) = \mathrm{Re}\left\{\dot{E}_l^{(d)} \exp\left[i\left(-k_l x - \omega t\right)\right] + \dot{E}_l^{(b)} \exp\left[i\left(k_l x - \omega t\right)\right]\right\}, \quad l = y,z, \qquad (6)$$

where $\dot{E}_l^{(d)} = E_l^{(d)} \exp\left(-i\varphi_l^{(d)}\right)$ and $\dot{E}_l^{(b)} = E_l^{(b)} \exp\left(-i\varphi_l^{(b)}\right)$ are the Cartesian components of complex amplitudes of the direct and backward waves in the layer, $E_l^{(d)}$, $E_l^{(b)}$, $\varphi_l^{(d)}$ and $\varphi_l^{(b)}$ are real constant scalars.

Substitution the representation (5) and (6) into relationships (2) with accounting the equations (1) and material relationships (3), (4) yields

$$E_y^{(r)} = E_y^{(i)} L_{22}^{-1} \sqrt{\left(N_y^{(r)}\right)^2 + \left(M_y^{(r)}\right)^2}, \quad E_z^{(r)} = E_z^{(i)} L_{33}^{-1} \sqrt{\left(N_z^{(r)}\right)^2 + \left(M_z^{(r)}\right)^2},$$

$$E_y^{(d)} = -E_y^{(i)} 2n^{(1)} \left(\Sigma_{22}^{(1)} \sqrt{L_{22}}\right)^{-1}, \quad E_z^{(d)} = -E_z^{(i)} 2n^{(1)} \left(\Sigma_{33}^{(1)} \sqrt{L_{33}}\right)^{-1},$$

$$E_y^{(b)} = E_y^{(i)} \frac{-2n^{(1)} \Delta_{22}^{(2)}}{\Sigma_{22}^{(1)} \Sigma_{22}^{(2)} \sqrt{L_{22}}}, \quad E_z^{(b)} = E_z^{(i)} \frac{-2n^{(1)} \Delta_{33}^{(2)}}{\Sigma_{33}^{(1)} \Sigma_{33}^{(2)} \sqrt{L_{33}}},$$

$$E_y^{(t)} = -4E_y^{(i)} n^{(1)} n_{22} \left(\Sigma_{22}^{(1)} \Sigma_{22}^{(2)} L_{22}\right)^{-1} \sqrt{\left(N_y^{(t)}\right)^2 + \left(M_y^{(t)}\right)^2},$$

$$E_z^{(t)} = -4E_z^{(i)} n^{(1)} n_{33} \left(\Sigma_{33}^{(1)} \Sigma_{33}^{(2)} L_{33}\right)^{-1} \sqrt{\left(N_z^{(t)}\right)^2 + \left(M_z^{(t)}\right)^2},$$

$$\varphi_y^{(r)} = \arctan\left(M_{22}^{(r)} / N_{22}^{(r)}\right) + \varphi_y^{(i)}, \quad \varphi_z^{(r)} = \arctan\left(M_{33}^{(r)} / N_{33}^{(r)}\right) + \varphi_z^{(i)},$$

$$\varphi_y^{(t)} = \arctan\left(M_{22}^{(t)} / N_{22}^{(t)}\right) + \varphi_y^{(i)}, \quad \varphi_z^{(t)} = \arctan\left(M_{33}^{(t)} / N_{33}^{(t)}\right) + \varphi_z^{(i)},$$

$$\varphi_y^{(d)} = \arctan \frac{\sin\left(4\pi n_{22} h\right)}{\Delta_{22}^{(1)} \Delta_{22}^{(2)} / \Sigma_{22}^{(1)} \Sigma_{22}^{(2)} - \cos\left(4\pi n_{22} h\right)} + \varphi_y^{(i)},$$

$$\varphi_z^{(d)} = \arctan \frac{\sin\left(4\pi n_{33} h\right)}{\Delta_{33}^{(1)} \Delta_{33}^{(2)} / \Sigma_{33}^{(1)} \Sigma_{33}^{(2)} - \cos\left(4\pi n_{33} h\right)} + \varphi_z^{(i)},$$

$$\varphi_y^{(b)} = \arctan \frac{\Delta_{22}^{(1)} \Delta_{22}^{(2)} \sin\left(4\pi n_{22} h\right)}{\Delta_{22}^{(1)} \Delta_{22}^{(2)} \cos\left(4\pi n_{22} h\right) - 1} + \varphi_y^{(i)},$$

$$\varphi_z^{(b)} = \arctan \frac{\Delta_{33}^{(1)} \Delta_{33}^{(2)} \sin\left(4\pi n_{33} h\right)}{\Delta_{33}^{(1)} \Delta_{33}^{(2)} \cos\left(4\pi n_{33} h\right) - 1} + \varphi_z^{(i)}.$$

Here the denotations are used:

$$\Sigma_{22}^{(\alpha)} \equiv n_{22} + n^{(\alpha)}, \ \Delta_{22}^{(\alpha)} \equiv n_{22} - n^{(\alpha)}, \ \Sigma_{33}^{(\alpha)} \equiv n_{33} + n^{(\alpha)}, \ \Delta_{33}^{(\alpha)} \equiv n_{33} - n^{(\alpha)} \ \left(\alpha = 1, 2\right);$$

$$N_y^{(r)} = \frac{\Delta_{22}^{(2)}}{\Sigma_{22}^{(2)}} \left[\left(\frac{\Delta_{22}^{(1)}}{\Sigma_{22}^{(1)}}\right)^2 + 1\right] \cos\left(4\pi n_{22} h\right) - \frac{\Delta_{22}^{(1)}}{\Sigma_{22}^{(1)}} \left[\left(\frac{\Delta_{22}^{(2)}}{\Sigma_{22}^{(2)}}\right)^2 + 1\right],$$

$$N_z^{(r)} = \frac{\Delta_{33}^{(2)}}{\Sigma_{33}^{(2)}} \left[\left(\frac{\Delta_{33}^{(1)}}{\Sigma_{33}^{(1)}}\right)^2 + 1\right] \cos\left(4\pi n_{33} h\right) - \frac{\Delta_{33}^{(1)}}{\Sigma_{33}^{(1)}} \left[\left(\frac{\Delta_{33}^{(2)}}{\Sigma_{33}^{(2)}}\right)^2 + 1\right],$$

$$M_y^{(r)} = \left(\Delta_{22}^{(2)}/\Sigma_{22}^{(2)}\right)\left[\left(\Delta_{22}^{(1)}/\Sigma_{22}^{(1)}\right)^2 - 1\right]\sin\left(4\pi n_{22}h\right),$$

$$M_z^{(r)} = \left(\Delta_{33}^{(2)}/\Sigma_{33}^{(2)}\right)\left[\left(\Delta_{33}^{(1)}/\Sigma_{33}^{(1)}\right)^2 - 1\right]\sin\left(4\pi n_{33}h\right),$$

$$L_{22} = \left(\Delta_{22}^{(1)}\Delta_{22}^{(2)}/\Sigma_{22}^{(1)}\Sigma_{22}^{(2)}\right)^2 - 2\left(\Delta_{22}^{(1)}\Delta_{22}^{(2)}/\Sigma_{22}^{(1)}\Sigma_{22}^{(2)}\right)\cos\left(4\pi n_{22}h\right) + 1,$$

$$L_{33} = \left(\Delta_{33}^{(1)}\Delta_{33}^{(2)}/\Sigma_{33}^{(1)}\Sigma_{33}^{(2)}\right)^2 - 2\left(\Delta_{33}^{(1)}\Delta_{33}^{(2)}/\Sigma_{33}^{(1)}\Sigma_{33}^{(2)}\right)\cos\left(4\pi n_{33}h\right) + 1,$$

$$N_y^{(t)} = \left(\Delta_{22}^{(1)}\Delta_{22}^{(2)}/\Sigma_{22}^{(1)}\Sigma_{22}^{(2)}\right)\cos\left(2\pi\Sigma_{22}^{(2)}h\right) - \cos\left(2\pi\Delta_{22}^{(2)}h\right),$$

$$N_z^{(t)} = \left(\Delta_{33}^{(1)}\Delta_{33}^{(2)}/\Sigma_{33}^{(1)}\Sigma_{33}^{(2)}\right)\cos\left(2\pi\Sigma_{33}^{(2)}h\right) - \cos\left(2\pi\Delta_{33}^{(2)}h\right),$$

$$M_y^{(t)} = \left(\Delta_{22}^{(1)}\Delta_{22}^{(2)}/\Sigma_{22}^{(1)}\Sigma_{22}^{(2)}\right)\sin\left(2\pi\Sigma_{22}^{(2)}h\right) + \sin\left(2\pi\Delta_{22}^{(2)}h\right),$$

$$M_z^{(t)} = \left(\Delta_{33}^{(1)}\Delta_{33}^{(2)}/\Sigma_{33}^{(1)}\Sigma_{33}^{(2)}\right)\sin\left(2\pi\Sigma_{33}^{(2)}h\right) + \sin\left(2\pi\Delta_{33}^{(2)}h\right),$$

h is the dimensionless thickness of the layer $h \equiv h_0/\lambda_0$.

3. The effect of stresses on light reflection and transmission

Using the obtained solution we can calculate the coefficients of reflection *R* and transmission *T* of incident polarized wave for the stained layer

$$R = \frac{\left(E_y^{(r)}\right)^2 + \left(E_z^{(r)}\right)^2}{\left(E_y^{(i)}\right)^2 + \left(E_z^{(i)}\right)^2}, \qquad T = \left(\frac{n^{(2)}}{n^{(1)}}\right)\frac{\left(E_y^{(t)}\right)^2 + \left(E_z^{(t)}\right)^2}{\left(E_y^{(i)}\right)^2 + \left(E_z^{(i)}\right)^2}.$$

The influence of mechanical stress on the parameters *R* and *T* was studied for three cases of stress state of the layer: 1) uniaxial stretch/compression in the direction of *x*-axis, when $\sigma_1 = \sigma \neq 0,\ \sigma_2 = \sigma_3 = 0$; 2) isotropic stretch/compression in the plane yOz, when $\sigma_2 = \sigma_3 \equiv \sigma \neq 0,\ \sigma_1 = 0$; 3) uniaxial stretch/compression in the direction of *y*-axis, when $\sigma_2 = \sigma \neq 0, \sigma_1 = \sigma_3 = 0$. The strain components for this cases are defined as

$$e_1 = \sigma_1/E, \qquad e_2 = e_3 = -\nu\sigma_1/E, \tag{7}$$

$$e_1 = -2\nu\sigma/E, \qquad e_2 = e_3 = (1-\nu)\sigma/E, \tag{8}$$

$$e_2 = \sigma_2/E, \qquad e_1 = e_3 = -\nu\sigma_2/E, \tag{9}$$

where *E* and ν stand for Young modulus and Poisson ratio of the layer respectively.

Fig. 1 shows the dependences of reflection *R* (solid curves) and transmission *T* (dashed curves) coefficients on stress value for different stress-strain state (7)-(9) and different polarization of incident light. The fig. 1*a* and 1*b* represent the cases (7) (curve 1-4) and (8) (curve 1'-4'). The fig. 1*c* and 1*d* represent case (9) (curve 1-4). The plots on fig. 1*a*, 1*c* are obtained for linear polarization of the incident light with

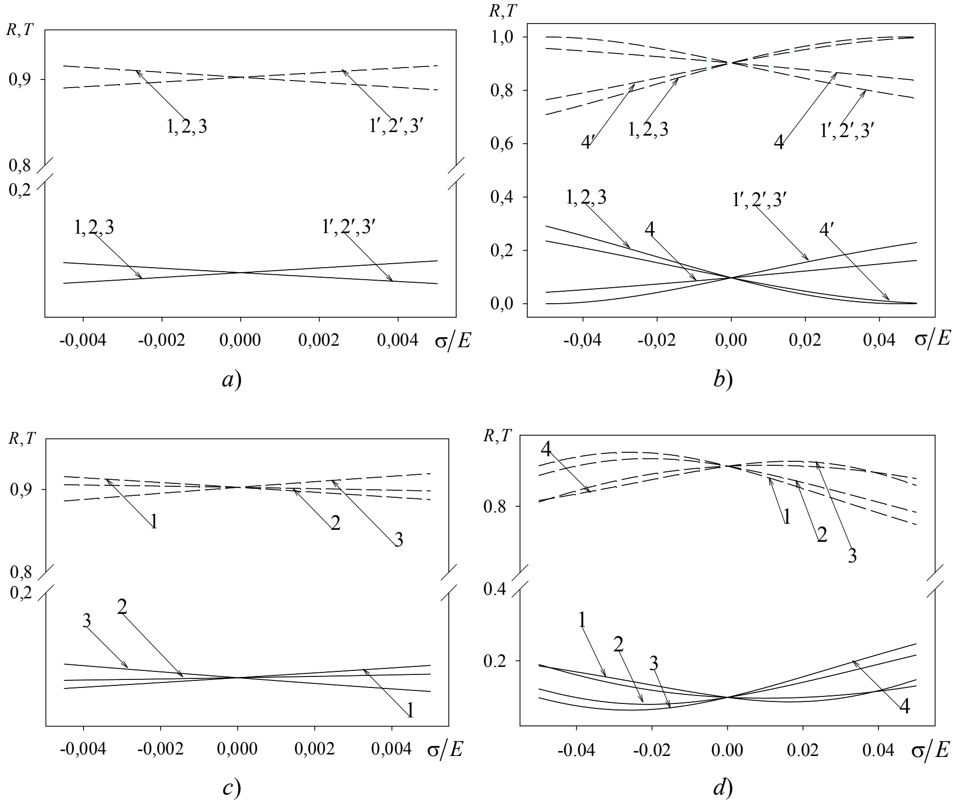

Fig. 1 Effect of stresses on the reflectance and transmittivity for light of different polarization

azimuth's values: $\psi = 0, \pi/6, \pi/3$. The plots on fig. 1*b*, 1*d* are obtained for three values of elliptic polarization (curves 1-3) and circular polarization (curve 4), defined by parameters $(\delta,\chi)=(0{,}187\pi;0{,}629)$, $(0{,}352\pi;1{,}292)$, $(0{,}648\pi;0{,}773)$ and $(0{,}5\pi;1)$.

As it can be seen from the plots, the coefficients of wave reflection *R* and transmission *T* are substantially dependent on the applied stresses. For the third type of stress-strained state a significant dependence of *R* and *T* coefficients on stress value and incident wave polarization state is observed. Whereas strain tensor for the first and second types of stress-strained state is isotropic one, the coefficients *R* and *T* are not dependent on incident wave polarization state in these cases (see fig. 1*a* and 1*b*).

Conclusions. The exact analytical solution for problem of monochromatic polarized light reflection and refraction in piece-wise homogeneous medium, consisting of two dielectric half-space, separated by elastically strained plane layer, has been obtained within the wave theory of light. The problem was formulated for the case of homogeneous stress-strained state of the layer with a diagonal strain tensor and condition when the incident light beam falls from a half-space normally to its boundary with the strained layer. Effect of stress state and incident light polarization on the layer's reflectance

and transmittivity was numerically studied with use of obtained solution. Three kinds of stress-strained state were considered — uniaxial compression/tension stress, applied in direction of light propagation, uniaxial compression/tension stress, applied normally to the direction of light propagation, and isotropic compression/tension in the plane, normal to the light beam. The obtained numerical results revealed a significant sensitivity of the coefficients of light reflection by the layer and transmission through it to the kind of stressed state, the stress value and to polarization parameters of the incident light beam (the polarization azimuth and light ellipticity). On this basis the reasonable conclusion about possibility to use data of polarimetric measuring of the layer's reflectance and transmittivity as informative parameters for non-destructive systems for stress-strained state determination can be made.

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Математична модель взаємодії еліптично поляризованого світла нормального падіння з напруженим діелектричним шаром

Василь Чекурін, Наталія Васьо

У рамках хвильової теорії отримано точний розв'язок задачі відбивання та заломлення монохроматичного поляризованого світла в кусково-однорідному середовищі, яке утворене двома діелектричними півпросторами та пружно деформованим діелектричним плоским шаром, який їх розділяє. Розглянуто випадок однорідного напружено-деформованого стану шару з діагональним тензором деформації за умови нормального падіння пучка світла з півпростору на поверхню його розділу з напруженим шаром. Із використанням отриманого розв'язку досліджено вплив напруженого стану шару та поляризації падаючого світла на коефіцієнти його відбивання шаром і проходження крізь шар. Розглядали випадки одновісних напружень стиску-розтягу, прикладених у напрямках поширення світла та нормальному до нього, а також ізотропного стиску-розтягу шару в площині нормальній до напрямку світлового променя. Проведені числові дослідження виявили істотну чутливість коефіцієнтів відбивання та проходження світла від виду напруженого стану і величини напружень у шарі, а також від параметрів поляризації світла, що падає (азимуту поляризації та еліптичності). На цій підставі можна зробити обґрунтований висновок про те, що дані фотометричних вимірювань відбивальної та пропускної здатності напруженого шару можна використати як інформативні параметри для створення систем неруйнівного визначення напружено-деформованого стану діелектричних тіл.

Математическая модель взаимодействие нормально падающего эллиптически поляризованного света с напряженным диэлектрическим слоем

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В рамках волновой теории получено точное решение задачи отражения и преломления монохроматического поляризованного света в кусочно-однородной среде, которое образовано двумя диэлектрическими полупространствами и упругодеформированным диэлектрическим плоским слоем, который их разделяет. Рассмотрен случай однородного напряженно-деформированного состояния слоя с диагональным тензором деформации при условии нормального падения пучка света с полупространства на поверхность его раздела с напряженным слоем. С использованием полученного решения исследовано влияние напряженного состояния слоя и поляризации падающего света на коэффициенты его отражения слоем и прохождения через слой. Рассматривались случаи одноосных напряжений сжатия-растяжения, приложенных в направлениях распространения света и нормальном к нему, а также изотропного сжатия-растяжения слоя в плоскости нормальной к направлению светового луча. Проведенные численные исследования обнаружили существенную чувствительность коэффициентов отражения и прохождения света от вида напряженного состояния и величины напряжений в слое, а также от параметров поляризации падающего света (азимуту поляризации и эллиптичности). На этом основании можно сделать обоснованный вывод о том, что данные фотометрических измерений отражательной и пропускной способности напряженного слоя можно использовать для как информативные параметры для создания систем неразрушающего определения напряженно-деформированного состояния диэлектрических тел.

Отримано 17.11.12