DOI: https://doi.org/10.15407/kvt192.02.044

UDC 681.5

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ADAPTIVE STABILIZATION OF SOME MULTIVARIABLE SYSTEMS WITH NONSQUARE GAIN MATRICES OF FULL RANK

Introduction. *The paper states and solves a new problem concerning the adaptive stabilization of a specific class of linear multivariable discrete-time memoryless systems with nonsquare gain matrices at their equilibrium states. This class includes the multivariable systems in which the number of outputs exceeds the number of control inputs. It is assumed that the unknown gain matrices have full rank.*

The purpose *of this paper is to answer the question of how the pseudoinverse model-based adaptive approach might be utilized to deal with the uncertain multivariable memoryless system if the number of control inputs is less than the number of outputs.*

Results. *It is shown that the parameter estimates generated by the standard adaptive projection recursive procedure converge always to some finite values for any initial values of system's parameters. Based on these ultimate features, it is proved that the adaptive pseudoinverse model-based control law makes it possible to achieve the equilibrium state of the nonsquare system to be controlled. The asymptotical properties of the adaptive feedback control system derived theoretically are substantiated by a simulation experiment.*

Conclusion. *It is established that the ultimate behavior of the closed-loop control system utilizing the adaptive pseudoinverse model-based concept is satisfactory.*

Keywords: *adaptive control, multivariable system, discrete time, feedback, pseudoinversion, stability, uncertainty.*

INTRODUCTION

A long-standing problem of the optimal controller design for multivariable systems has been solved by using different approaches including the l_1 optimal control approach [1, 2]. It remains the important problem in the modern control theory [3–5].

Based on the well-known internal model principle, multivariable control problem was first approached in the paper [6]. Within the framework of this principle, the so-called inverse model approach seems to be perspective to deal with improving MIMO (multi-input multi-output) feedback controls. Since the pioneering work [7], the problem of inversion of linear time-invariant MIMO systems has attracted an attention of several researches; see, e.g. [8–10]. Last time, a significant progress in this scientific area has been achieved in [11–14].

The inverse model approach to ensuring perfect steady-state regulation of linear discrete-time memoryless multivariable systems was first advanced in [15]. Similar approach has also been discussed in [10] dealing with the problem of minimal inversion. However, the inverse model approach is quite unacceptable if the MIMO systems to be controlled are nonsquare.

It turned out that the so-called pseudoinverse (generalized inverse) model approach first proposed in the paper [9] can be exploited to cope with the non-inevitability of nonsquare system. Recently, this approach was extended in [16–19] for controlling a wide class of discrete-time memoryless multivariable systems. In particular, in [16] it was first established that pseudoinverse model-based controller for the steady-state regulation of the MIMO systems having singular or nonsquare gain matrices is indeed optimal. But such controller can be implemented if system parameters are known *a priori*. In the case of the parameter uncertainty, the nonadaptive controller employing a fixed linear pseudoinverse model can be shown to be acceptable to ensure the robust stability of multivariable closed-loop systems containing uncertain linear and some nonlinear memoryless plants [17–19]. Nevertheless, this controller may not be suitable if parameter uncertainty is great enough.

An adaptation of control law is known as some universal concept to deal with uncertain systems. Results obtained within the framework of adaptive controls were summarized in many books [20–27], etc. A key question in these controls concerns the stability of resulting systems, i.e. the boundedness of the control input and output signals [20, 21]. In order to resolve this important question, two different tools were independently advanced in above two books. Namely, the so-called Frequency Theorem was exploited in [20, Theorem 4.17.3] to establish the ultimate boundedness properties of linear adaptive control systems including multivariable plants with square gain matrices, whereas the so-called Key Technical Lemma of [21, item 6.2] was used to derive such properties in MIMO case where the number of outputs does not exceed the number of control inputs, see [21, subitem 6.3.6]. Unfortunately, these tools seems to be not admissible to an adaptive nonsquare case. To the best of author's knowledge, there are no theoretical results concerning adaptive controls of these MIMO systems while they may appear in practice [21, p.141].

The purpose of this paper is to answer the question of how the pseudoinverse model-based adaptive approach might be utilized to deal with the uncertain multivariable memoryless system in which the number of its output exceeds the number of control inputs.

PROBLEM STATEMENT

Let

$$y_n = Bu_{n-1} \tag{1}$$

be the difference equation of an static (memoryless) plant that is some MIMO discrete-time system to be stabilized. In this equation, $y_n = [y_n^{(1)},...,y_n^{(m)}]^T$ and $u_n = [u_n^{(1)},...,u_n^{(r)}]^T$ are its m-dimensional output and r-dimensional control input vectors, respectively, at the nth time instant $(n = 1, 2, \ldots)$, and

$$B = \begin{pmatrix} b^{(11)} & \ldots & b^{(1r)} \\ \cdots & \cdots & \cdots \\ b^{(m1)} & \ldots & b^{(mr)} \end{pmatrix} \tag{2}$$

denotes the time-invariant $m \times r$ gain matrix.

Consider a nonsquare system, where

$$r < m, \tag{3}$$

i.e., where the number of output variables $y_n^{(i)}$ $(1 \le i \le m)$ exceeds the number of control variables $u_n^{(j)}$ $(1 \le j \le r)$.

Suppose that B is some unknown matrix of full rank meaning that $\operatorname{rank} B = \min\{r, m\}$. Due to (3) we have

$$\operatorname{rank} B = r. \tag{4}$$

Introducing the vector $y^0 = [y^{0(1)},..., y^{0(m)}]^T$ whose components are the desired output variables (the given set-points for outputs), define the current ith output error $e_n^{(i)}$ as

$$e_n^{(i)} = y^{0(i)} - y_n^{(i)}, \quad i = 1, \ldots, m. \tag{5}$$

Then the output error vector will be given by

$$e_n = y^0 - y_n. \tag{6}$$

It is assumed that the elements $b^{(ij)}$ $(i = 1, \ldots, m,\ j = 1, \ldots, r)$ of B in (2) are unknown *a priori*. Moreover, the bounds on these elements are assumed to be unknown (contrary to [18, 19]) and it is essential.

The problem stated below is as follows. Based on the available observations of $e_n, e_{n-1}, \ldots, e_0$ given by (6), devise an adaptive controller of a general form

$$u_n = U_n(e_n, e_{n-1}, \ldots, e_0), \tag{7}$$

such that the closed-loop control system containing the uncertain plant (1) and the feedback (7) will be stable. More specifically, we require the sequences

$\{u_n\} := u_0, u_1, \dots$ and $\{y_n\} := y_0, y_1, \dots$ to be bounded uniformly $(\{u_n\} \in \ell_\infty, \{y_n\} \in \ell_\infty)$ and for any initial conditions to achieve

$$u_n \xrightarrow[n\to\infty]{} u^{\mathrm{e}}, \qquad y_n \xrightarrow[n\to\infty]{} y^{\mathrm{e}}, \tag{8}$$

where the pair $(u^{\mathrm{e}}, y^{\mathrm{e}})$ with $y^{\mathrm{e}} = Bu^{\mathrm{e}}$ defines the equilibrium state of the feedback control system (1), (6), (7).

Remark 1. Note that it is not required for the errors $e_n^{(1)}, \dots, e_n^{(m)}$ given by (5) to be asymptotically equal to zero. In fact, m zero errors cannot be achieved simultaneously except a unique case when $y^0 \in \Re(B)$, where $\Re(B)$ denotes the so-called range of B (the definition of $\Re(\cdot)$ can be found in [28, Exercise 2.8.6]). Thus without less of generality we assume that $y^0 \notin \Re(B)$.

ADAPTIVE CONTROLLER DESIGN

Suppose that B is known. Then the pseudoinverse model-based control law of the form

$$u_n = u_{n-1} + B^+ e_n \tag{9}$$

advanced in [16] can here be chosen. In this equation, B^+ specifies the so-called pseudoinverse matrix given by [28, Theorem 3.4]

$$B^+ = \lim_{\delta\to 0} (B^T B + \delta I_r)^{-1} B^T, \tag{10}$$

where I_q denotes the identity $q \times q$ matrix.

Note that under conditions (3), (4) on B, instead of (10), a very simple formula

$$B^+ = (B^T B)^{-1} B^T \tag{11}$$

may be employed to calculate B^+ for given B; see [28, Exercise 3.5.3].

Following to the standard identification approach, we will design an adaptive control by replacing the unknown matrix B in (9) by its suitable estimate B_n updated at the nth time instant. Then the control law takes the form

$$u_n = u_{n-1} + B_n^+ e_n, \tag{12}$$

where

$$B_n = \begin{pmatrix} b_n^{(11)} & \dots & b_n^{(1r)} \\ \cdots & \cdots & \cdots \\ b_n^{(m1)} & \dots & b_n^{(mr)} \end{pmatrix}. \tag{13}$$

To derive the estimation algorithm for updating B_n, we first define the *i*th current estimation error $\tilde{e}_n^{(i)}$ given as follows:

$$\tilde{e}_n^{(i)} = e_n^{(i)} - e_{n-1}^{(i)} + b_{n-1}^{(i)T} \nabla u_{n-1} \quad (i = 1, \ldots, m). \tag{14}$$

In these expressions, $b_n^{(i)T} = [b_n^{(i1)}, \ldots, b_n^{(ir)}]$ is the *i*th row of B_n selected from (13), and the notation $\nabla u_n := u_n - u_{n-1}$ is used.

Remark 2. By virtue of (1) together with (8), (5), it follows that if $b_n^{(i)} = b^{(i)}$, where $b^{(i)T} = [b^{(i1)}, \ldots, b^{(ir)}]$ denotes the *i*th row of *B* then

$$\tilde{e}^{(i)} = 0 \tag{15}$$

will be ensured. Based on (15) define a set Γ_n of possible $b^{(i)}$s under which the estimation errors are equal to zero for given observable $e_n^{(i)}, e_{n-1}^{(i)}, \nabla u_{n-1}$. Obviously, Γ_n represents the hyperplane

$$\Gamma_n = \{ b^{(i)} : \; e_n^{(i)} - e_{n-1}^{(i)} + b_{n-1}^{(i)T} \nabla u_{n-1} = 0 \} \subset \mathbf{R}^r \tag{16}$$

belonging to the *r*-dimensional Euclidean space $\mathbf{R}^r$. It is not hard to see that $b^{(i)} \in \Gamma_n$ for all $n = 1, 2, \ldots$. □

Now, similar to [21, sect. 3.3], we will choose the adaptation algorithm as the recursive estimation procedure

$$b_n^{(i)} = b_{n-1}^{(i)} + \gamma_n^{(i)} \frac{\tilde{e}_n^{(i)}}{c_0 + \| \nabla u_{n-1} \|^2} \nabla u_{n-1} \quad (i = 1, \ldots, m), \tag{17}$$

where c_0 is an arbitrary sufficiently small positive constant $(c_0 << 1)$ needed to avoid the possibility of division by zero, and $\gamma_n^{(i)}$ is a scalar possibly time-varying multiplier (in contrast to Equation (3.3.19) of [21]) satisfying

$$0 < \underline{\gamma}^{(i)} \le \gamma_n^{(i)} \le \overline{\gamma}^{(i)} < 2. \tag{18}$$

This procedure describes the so-called projection algorithm which is also known in the literature as the normalized least-mean-squares algorithm [21, p. 52].

There is a simple geometrical interpretation of (17), (18) (in terms of orthogonal projection of vector $b_{n-1}^{(i)}$ onto the hyperplane Γ_n represented by (16) if $\gamma_n^{(i)} = 1$ and $c_0 = 0$). It is given in Fig. 1, where admissible $b_n^{(i)}$s for $\gamma_n^{(i)} \in (0, 2)$ are also shown.

Remark 3. In order to do not deal with the possible division by zero, instead of (17), other estimation algorithm

$$b_n^{(i)} = \begin{cases} b_{n-1}^{(i)} & \text{if } \| \nabla u_{n-1} \| = 0 \\ b_{n-1}^{(i)} + \gamma_n^{(i)} \dfrac{\tilde{e}_n^{(i)}}{\| \nabla u_{n-1} \|^2} \nabla u_{n-1} & \text{otherwise} \end{cases}, \tag{19}$$

can also be proposed as the adaptive estimation procedure. This algorithm represents the slightly modified well known Kaczmarz's algorithm who proposed it in 1937 for solving a set of linear equations (a translation of his original work can be found in the recent paper [29]). □

Thus, the algorithm (17) (or (19)) together with (16), (18) leads to forming the estimate matrix B_n given by (13). It turns out that it is possible to ensure

$$\operatorname{rank} B_n = r \quad \forall n = 1, 2, \dots \tag{20}$$

by suitable choice of $\gamma_n^{(i)}$s from $[\underline{\gamma}^{(i)}, \overline{\gamma}^{(i)}]$ with arbitrary numbers $i = i_1, \dots, i_r$ such that $1 \le i_1 < \dots < i_r \le m$. To substantiate this fact, consider the so-called $r \times r$ submatrix $B_n[i_1, \dots, i_r \,|\, 1, \dots, r]$ of B_n consisting of its r rows with the numbers $i_1, \dots, i_r$ and of all columns (the definition of some submatrix of an arbitrary P and its symbol notation $P[\cdot\,|\,\cdot]$ have been taken from [30, part 1, item 2.2]).

Following to [20, item 4.2.2] it can be shown that if $\gamma_n^{(i)} \in [\gamma', \gamma'']$ then the requirement $\operatorname{rank} B_n[i_1, \dots, i_r \,|\, 1, \dots, r] = r$ can always be satisfied because

$$\det B_n[i_1, \dots, i_r \,|\, 1, \dots, r] \ne 0$$

may take place at some isolated $\gamma_n^{(i)}$s. Thereby, the condition (20) can be met. This makes it possible to calculate B_n^+ by

$$B_n^+ = (B_n^T B_n)^{-1} B_n^T \tag{21}$$

similarly to (11).

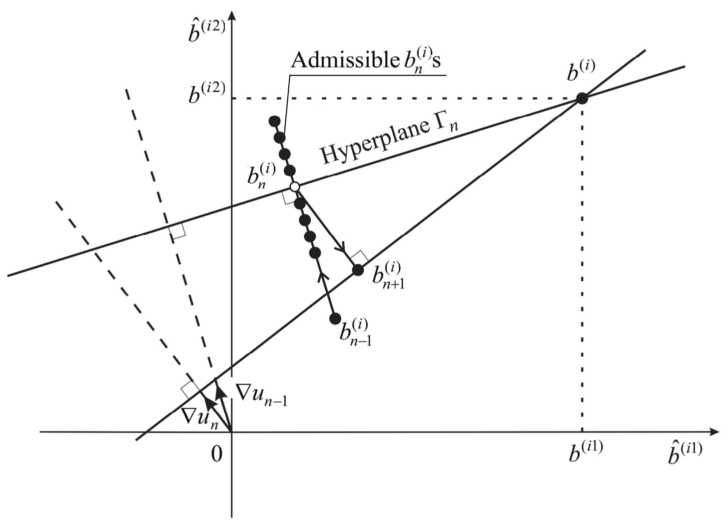

Fig. 1. The algorithm (17), (18) as an orthogonal projection process for the two-dimensional case ($r = 2$)

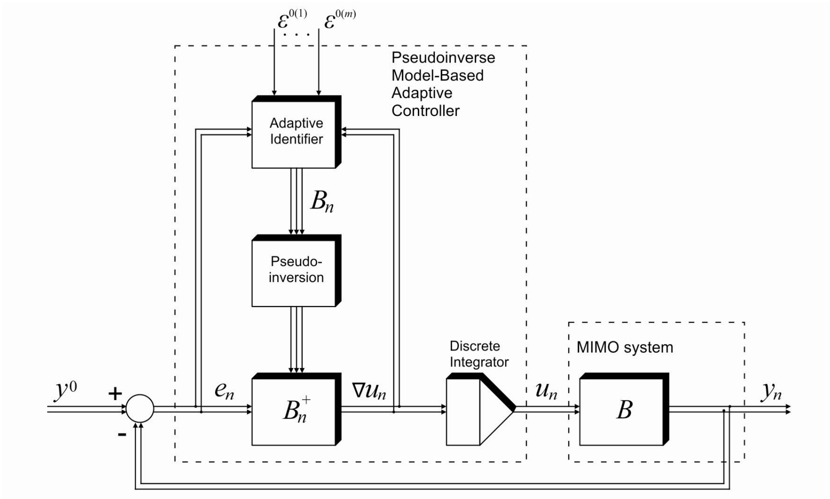

Fig. 2. Configuration of adaptive control system

The choice of $\gamma_n^{(i)}$ completes the synthesis of the adaptive control algorithm determined in the expressions (12), (17) together with (6), (13), (14), (18) and (21) in full detail. To implement this algorithm, the adaptive pseudoinverse model-based control system is designed as shown in Fig. 2.

As it is seen, the controller of this system contains the discrete integrator summing the increments

$$\nabla u_n = B_n^+ e_n \tag{22}$$

from 0 to n at each nth time instant and giving

$$u_n = \sum_{k=0}^{n} \nabla u_k$$

(in accordance with (12)).

ASYMPTOTIC BEHAVIOR OF ADAPTIVE FEEDBACK CONTROL SYSTEM

To study the ultimate behavior of the adaptive control algorithm (12), (17) together with (6), (14), (21), the preliminary results formulated in [21, Lemma 3.3.2] are needed. From these results we can derive the following asymptotic properties:

(i) the scalar variables $V_n^{(i)} := \| b^{(i)} - b_n^{(i)} \|$ are the Lyapunov function of the algorithm (17) meaning

$$V_n^{(i)} \le V_{n-1}^{(i)} \quad \forall i = 1, \ldots, m;$$

(ii) the sequences $\{b_n^{(i)}\}$ satisfy

$$\| b_n^{(i)} - b_{n-1}^{(i)} \| \to 0 \quad \text{as} \quad n \to \infty \quad \forall i = 1, \ldots, m;$$

(iii) there exist the limits

$$\lim_{n\to\infty}\frac{\tilde{e}_n^{(i)}}{(c_0+\|\nabla u_{n-1}\|^2)^{1/2}}=0 \qquad \forall i=1,\dots,m. \tag{23}$$

With the foregoing properties (i) – (iii), the following lemma can be shown to be valid.

Lemma. *For the algorithm* (17) *together with* (14) *and subject to* (18), *it follows that:*

(a) *the current estimate* B_n *of unknown* B *remains always upper bounded implying*

$$\| B_n \| < \infty \quad \forall n;$$

(b) *the matrix sequence* $\{B_n\}$ *satisfies*

$$\| B_n - B_{n-1} \| \xrightarrow[n\to\infty]{} 0;$$

(c) *the zero limit*

$$\lim_{n\to\infty}\frac{\|\tilde{e}_n\|}{c_1+\|\nabla u_{n-1}\|}=0 \tag{24}$$

is achieved, where $\tilde{e}_n$ *represents the estimation error vector defined as*

$$\tilde{e}_n=[\tilde{e}_n^{(1)},\dots,\tilde{e}_n^{(m)}]^T, \tag{25}$$

and $c_1=c_0^{1/2}$.

Proof. Part (a) follows from the property (i) and the definition (13) of B_n. Part (b) holds due to the property (ii).

To prove part (c) we can write

$$\lim_{n\to\infty}\frac{\|\tilde{e}_n\|}{(c_0+\|\nabla u_{n-1}\|^2)^{1/2}}=0, \tag{26}$$

using (23) and the definition (25) of $\tilde{e}_n$. Since $(h_1+h_2)^{1/2}\le h_1^{1/2}+h_2^{1/2}$ for any numbers $h_1, h_2 \ge 0$, the inequality

$$\frac{\|\tilde{e}_n\|}{(c_0+\|\nabla u_{n-1}\|^2)^{1/2}} \ge \frac{\|\tilde{e}_n\|}{c_1+\|\nabla u_{n-1}\|}$$

with $c_1=c_0^{1/2}$ is valid. Taking this inequality into account, due to (26) we immediately obtain (24). □

Now, we are able to present some basic result as

Theorem. *The adaptation procedure* (17), (14) *has the following ultimate property:*

$$\lim_{n\to\infty}\|\tilde{e}_n\|=0. \tag{27}$$

Proof. First, recalling that $e_n = [e_n^{(1)}, \ldots, e_n^{(m)}]^T$, due to (13), (14) we obtain

$$\widetilde{e}_n = e_n - e_{n-1} + B_{n-1}\nabla u_{n-1}. \tag{28}$$

Substituting (28) into (24) gives

$$\lim_{n\to\infty} \frac{\| e_n - e_{n-1} + B_{n-1}\nabla u_{n-1} \|}{c_1 + \| \nabla u_{n-1} \|} = 0. \tag{29}$$

By virtue of (22), the expression (29) can then be rewritten as follows:

$$\lim_{n\to\infty} \frac{\| e_n - (I_m - B_{n-1}B_{n-1}^{+})e_{n-1} \|}{c_1 + \| \nabla u_{n-1} \|} = 0. \tag{30}$$

Consider the equation

$$e_n - e_{n-1} = -B\nabla u_{n-1} \tag{31}$$

produced by (1) together with (6). Using this equation, represent (28) and (29) in the form

$$\widetilde{e}_n = (B_{n-1} - B)\nabla u_{n-1}, \tag{32}$$

$$\lim_{n\to\infty} \frac{\| (B_{n-1} - B)\nabla u_{n-1} \|}{c_1 + \| \nabla u_{n-1} \|} = 0. \tag{33}$$

It is clear that if $\| \nabla u_{n-1} \|$ tends to 0 as n goes to infinity, then (33) is always satisfied. Assume that $\lim\,\sup_{n\to\infty} \| \nabla u_{n-1} \| = \infty.$ To study this case, write

$$\lim_{n\to\infty} \frac{\| (B_{n-1} - B)\nabla \bar{u}_{n-1} \|}{1 + c_1 / \| \nabla u_{n-1} \|} = 0, \tag{34}$$

dividing the numerator and the denominator of (33) by $\| \nabla u_{n-1} \|$. In this expression, $\nabla \bar{u}_{n-1} := \nabla u_{n-1} / \| \nabla u_{n-1} \|$ denotes the unit vector of the same direction as ∇u_{n-1}.

It is not hard to establish that when $B_{n-1} \neq B$ and $\sup_{n\in[0,\infty)} \| \nabla u_n \|$ tends to ∞ then zero limit (34) will be satisfied if and only if

$$\nabla \bar{u}_n \xrightarrow[n\to\infty]{} \aleph(B_{n-1} - B), \tag{35}$$

where the notation $\aleph(P)$ of the null-space of an arbitrary matrix P taken from [28, Exercise 2.8.6] has been used.

By definition of $\aleph(\cdot)$, it follows that (35) implies also that

$$\nabla u_n \xrightarrow[n\to\infty]{} \aleph(B_{n-1} - B) \tag{36}$$

becomes the necessary and sufficient condition to achieve the limit (33) for any $\{\nabla u_n\}$. Taking (34) in to account, due to (32) which may be rewritten as

$$\lim_{n\to\infty} \| (B_{n-1} - B)\nabla u_{n-1} \| = 0,$$

result (27) follows. □

Further, the following proposition is advanced.

Proposition. If the adaptation algorithm (17) together with (14) and with $\gamma_n^{(i)}$ chosen as in (18) is applied to the MIMO system (1), then there exist a finite limit

$$\lim_{n\to\infty} B_n = B_\infty \quad (\| B_\infty \| < \infty). \tag{37}$$

This proposition is based on the observation that each $\{b_n^{(i)}\}$ is the so-called Fejer's sequence because $b_n^{(i)}$ is pointwise closer than $b_{n-1}^{(i)}$ to the intersection $\bigcap_{\nu=n}^{\infty} \Gamma_{\nu+1}$ of all the hyperplanes Γ_νs defined in (16) (since they contain the point $b^{(i)}$, this intersection is non-empty set).

Notice that, in this proposition nothing has been said about the convergence $\{B_n\}$ to true B, and it not necessary, in principle.

By virtue of (27) and (37), from the definition (28) of $\tilde{e}_n$, it follows that our time-varying control system becomes asymptotical close to a time-invariant system described by

$$e_n - (I_m - B_\infty B_\infty^+) e_{n-1} = 0_m \tag{38}$$

as $n \to \infty$, where $0_r := [\underbrace{0, \ldots, 0}_{r}]^T$ denotes the r-dimensional zero vector. Since (38) produces

$$e_n = e_{n-1} \quad \text{with } \| e_n \| < \infty$$

for any integer positive n and for any finite $\| e_0 \| < \infty$ [16, 18], according to [31], it can be concluded that the adaptive control system given in equations (1), (6), (12) has the following main ultimate properties:

$$1) \quad \lim_{n\to\infty} \| e_n - e_{n-1} \| = 0; \tag{39}$$

there is a finite limit

$$2) \quad \lim_{n\to\infty} e_n = e_\infty \quad (\| e_\infty \| < \infty). \tag{40}$$

Using the fact that B is the matrix of full rank (see (4)), from (31) we derive

$$\nabla u_{n-1} = -B^+ (e_n - e_{n-1})$$

to establish

$$\| \nabla u_{n-1} \| \le \| B^+ \| \| e_n - e_{n-1} \|. \tag{41}$$

Due to (39) from (41) it follows that

$$\| \nabla u_{n-1} \| \xrightarrow[n\to\infty]{} 0. \tag{42}$$

By (40) and (42) we conclude that $\{y_n\}$ and $\{u_n\}$ will go to the equilibrium state $(u^{\mathrm{e}}, y^{\mathrm{e}})$ with $y^{\mathrm{e}} = y^0 - e_\infty$ and $u^{\mathrm{e}} = B^+ y^{\mathrm{e}}$ as n tends to infinity.

Hence, the problem (8) stated in this paper will be solved.

Comment. It can understand that u^{e} may be specified by solving the vector equation

$$B_{\infty}^{+}Bu^{\mathrm{e}} = B_{\infty}^{+}y^{0} \qquad (43)$$

yielding by the condition

$$B_{n}^{+}\underbrace{(y^{0} - Bu_{n-1})}_{e_n} \xrightarrow[n\to\infty]{} 0_{r}$$

under which the equilibrium state should asymptotically be achieved.

A SIMULATION EXAMPLE

To illustrate how the adaptive pseudoinverse model-based control algorithm performs, a simulation of the closed-loop system consisting of the nonsquare memoryless MIMO system (1) (the plant) and of the adaptive controller described in equation (6), (12), (17) together with (13), (14), (21) was conducted. The system to be stabilized at an equilibrium state was given by

$$B = \begin{pmatrix} 0.2 & 1.4 \\ 0.8 & 2.4 \\ 1.1 & 0.5 \end{pmatrix}.$$

with the matrix B of the full rank ($\operatorname{rank} B = 2$).

The desired output vector y^{0} was taken as $y^{0} = [2, 7, 3]^{T}$ to ensure $y^{0} \notin \Re(B)$. The duration of the simulation experiment was chosen as long as adaptation of the controller parameters continues.

Table 1 sets out the true system parameters and their initial estimates.

Results of the simulation experiment are presented in Figs. 3 to 5. Fig. 3 shows how the estimate vectors $b_{n}^{(i)}$ $(i = 1, 2, 3)$ move to their final $b_{\infty}^{(i)}$. We can see that they differ from $b^{(i)}$. These final estimates given in Table 1 yield

$$B_{\infty} \cong \begin{pmatrix} 5.83 & 9.63 \\ 4.68 & 8.07 \\ 2.24 & 2.17 \end{pmatrix}.$$

Table 1. **System parameters**

Parameters	$b^{(11)}$	$b^{(12)}$	$b^{(21)}$	$b^{(22)}$	$b^{(31)}$	$b^{(32)}$
True value	0.2	1.4	0.8	2.4	1.1	0.5
Initial value	50	20	30	40	10	10
Final value	$\cong 5.83$	$\cong 9.63$	$\cong 4.68$	$\cong 8.07$	$\cong 2.24$	$\cong 2.17$

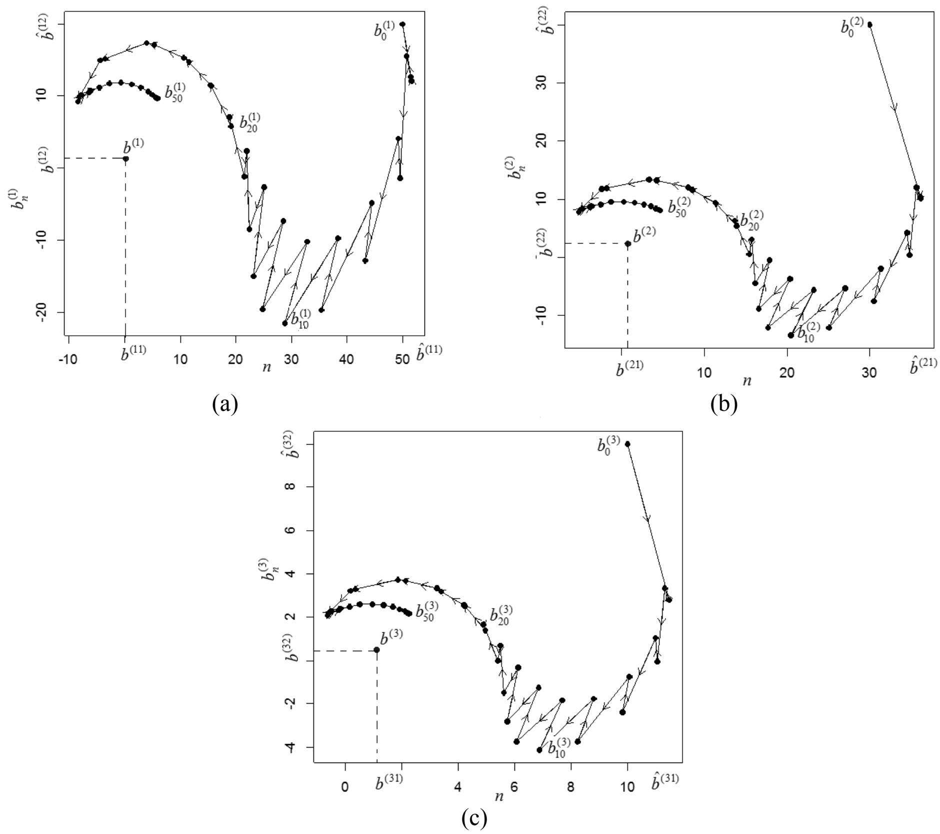

Fig. 3. Trajectories of adaptive estimation processes: (a) Vectors $b_n^{(1)}$; (b) Vectors $b_n^{(2)}$; (c) Vectors $b_n^{(3)}$;

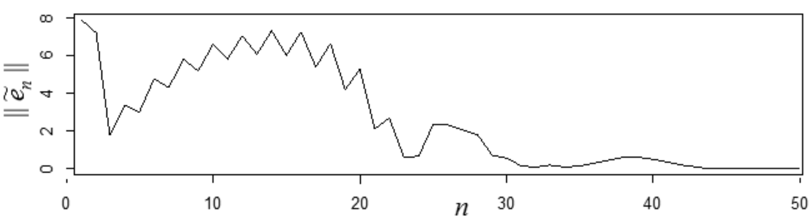

Fig. 4. Norm of estimation error vector

From Fig. 4 it is seen that the norm of the estimation error $\tilde{e}_n$ converges to 0 as $n \to \infty$ as predicted by the theorem above established. Fig. 5 shows the input control variables $u_n^{(i)}$ $(i = 1, 2)$ and the output variables $y_n^{(i)}$ $(i = 1, 2, 3)$. It is seen from Fig. 5b that the performance of the adaptive controller is satisfactory because this controller is able to stabilize the output vector y_n at some ultimate y_∞ given by

$$y_\infty \cong [2.99, 5.88, 2.75]^T$$

as n tends to infinity. Namely, this vector specifies the equilibrium state of the adaptive control system: $y^{\mathrm{e}} = y_\infty$.

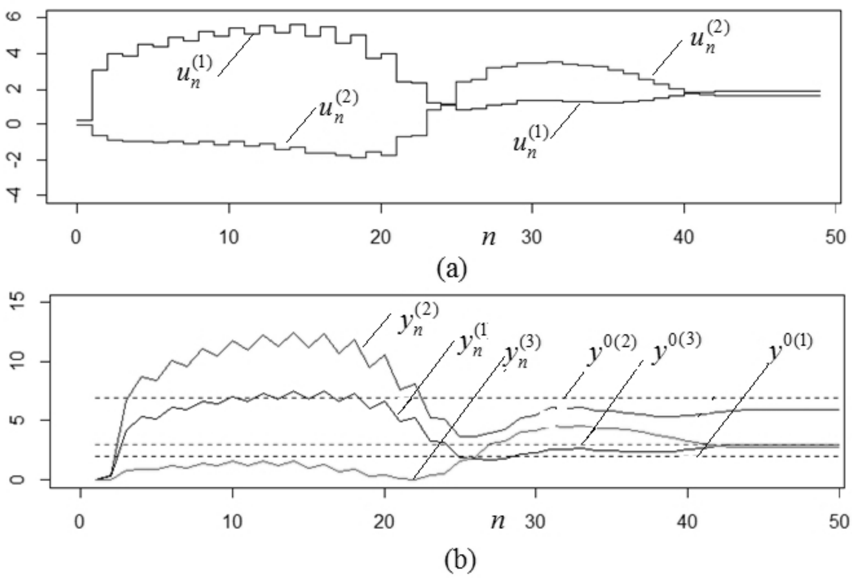

Fig. 5. Systems variables: (a) Control inputs $u_n^{(j)}$; (b) Outputs $y_n^{(i)}$ (solid lines) and desired value $y^0(i)$ (dashed lines)

By solving (43) we numerically determine

$$u^{\mathrm{e}} \cong [1.634, 1.905]^T$$

that is the same as observed in the simulation example (see Fig.5a).

We also see that although the equilibrium state is asymptotically achieved, there is a difference between the components $y^{0(1)}, y^{0(2)}, y^{0(3)}$ of desired output vector y^0 and their ultimate values $y_\infty^{(1)}, y_\infty^{(2)}, y_\infty^{(3)}$, respectively, if $y^0 \notin \Re(B)$.

CONCLUSION

This paper shed light on the adaptive pseudoinverse model-based approach to deal with the stabilization of uncertain nonsquare memoryless MIMO systems in which the number of the outputs exceeds the number of their control inputs. It ensure the asymptotical stabilization of such systems at equilibrium states for any initial estimates of system's parameters. A simulation experiment has demonstrated a good performance of these systems.

REFERENCES

1. Dahleh M.A., Pearson J.B. l_1 optimal-feedback controllers for MIMO discrete-time systems. *IEEE Trans. Autom. Contr.*, 1987, vol. 32, no. 4, pp. 314–322.
2. McDonald J.S., Pearson J.B. l_1 optimal control of multivariable systems with output norm constraints. *Automatica*, 1991, vol. 27, no. 2, pp. 317–329.
3. Maciejowski J. M. *Multivariable Feedback Design*. Wokinghan: Addison-Wesley, 1989.
4. Skogestad S., Postlethwaite I. *Multivariable Feedback Control*. UK, Chichester: Wiley, 1996.

5. Albertos P., Sala A. *Multivariable Control Systems: an Engineering Approach*. London: Springer , 2006.
6. Francis B., Wonham W. The internal model principle of control theory. *Automatica*, 1976, vol. 12, no. 5, pp. 457–465.
7. Brockett R. W. The invertibility of dynamic systems with application to control. *Ph. D. Dissertation*, Case Inst. of Technology, Cleveland, Ohio, 1963.
8. Silverman L. M. Inversion of multivariable linear systems. *IEEE Trans. Autom. Contr.*, 1969, vol. AC-14, no. 3, pp. 270–276..
9. Lovass-Nagy V., Miller J. R., Powers L. D. On the application of matrix generalized inversion to the construction of inverse systems. *Int. J. Control*, 1976, vol. 24, no. 5, pp. 733–739.
10. Seraji H. Minimal inversion, command tracking and disturbance decoupling in multivariable systems. *Int. J. Control*, 1989, vol. 49, no. 6, pp. 2093–2191.
11. Marro G., Prattichizzo D., Zattoni E. Convolution profiles for right-inversion of multivariable non-minimum phase discrete-time systems. *Automatica*, 2002, vol. 38, no. 10, pp. 1695–1703.
12. Liu C., Peng H. Inverse-dynamics based state and disturbance observers for linear time-invariant systems. *ASME J. Dyn Syst., Meas. and Control*, 2002, vol. 124, no. 5, pp. 376–381.
13. Lyubchyk L. M. Disturbance rejection in linear discrete multivariable systems: inverse model approach. *Prep. 18th IFAC World Congress*, Milano, Italy, 2011, pp. 7921–7926.
14. Pushkov S. G. Inversion of linear systems on the basis of state space realization. *Journal of Computer and Systems Sciences International*, 2018, vol. 57, vo. 1, pp. 7–17.
15. Pukhov G. E., Zhuk K. D. *Synthesis of Interconnected Control Systems via Inverse Operator Method*. Kiev: Nauk. dumka, 1966 (in Russian).
16. Skurikhin V. I., Gritsenko V. I., Zhiteckii L. S., Solovchuk K. Yu. Generalized inverse operator method in the problem of optimal controlling linear interconnected static plants. *Dopovidi NAN Ukrainy*, no. 8, pp. 57–66, 2014 (in Russian).
17. Zhiteckii L. S., Azarskov V. N., Solovchuk K. Yu., Sushchenko O. A. Discrete-time robust steady-state control of nonlinear multivariable systems: a unified approach. *Proc. 19th IFAC World Congress*, Cape Town, South Africa, 2014, pp. 8140–8145.
18. Zhitetskii L. S., Skurikhin V. I., Solovchuk K. Yu. Stabilization of a nonlinear multivariable discrete-time time-invariant plant with uncertainty on a linear pseudoinverse model. *Journal of Computer and Systems Sciences International*, 2017, vol. 56, no. 5, pp. 759–773.
19. Zhiteckii L. S., Solovchuk K. Yu. Pseudoinversion in the problems of robust stabilizing multivariable discrete-time control systems of linear and nonlinear static objects under bounded disturbances. *Journal of Automation and Information Sciences*, 2017, vol. 49, no. 5, pp. 35–48.
20. Fomin V. N., Fradkov A. L., Yakubovich V. A. *Adaptive Control of Dynamic Plants*. Moscow: Nauka, 1981 (in Russian).
21. Goodwin G.C., Sin K.S. *Adaptive Filtering, Prediction and Control*. Engewood Cliffs, NJ.: Prentice-Hall, 1984.
22. Landau I. D., Lozano R., M'Saad M. *Adaptive Control*. London: Springer, 1997.
23. Kuntsevich V. M. *Control under Uncertainty: Guaranteed Results in Control and Identification Problems*. Kiev: Nauk. dumka, 2006 (in Russian).
24. Zhiteckii L. S., Skurikhin V. I. *Adaptive Control Systems with Parametric and Nonparametric Uncertainties*. Kiev: Nauk. dumka, 2010 (in Russian).
25. Narendra K. S., Annaswamy A. M. *Stable Adaptive Systems*. NY: Dover Publications, 2012.
26. Ioannou P., Sun J. *Robust Adaptive Control*. NY: Dover Publications, 2013.
27. Aström K. J., Wittenmark B. *Adaptive Control: 2nd Edition*. NY: Dover Publications, 2014.
28. Albert A. *Regression and the Moore-Penrose Pseudoinverse*. New York: Academic Press, 1972.

29. Kaczmarz S. Approximate solution of systems of linear equations. *Internat. J. Control*, 1993. vol. 57, no. 6. pp. 1269–1271.
30. Marcus M., Minc H. *A Survey of Matrix Theory and Matrix Inequalities*. Boston: Aliyn and Bacon, 1964.
31. Desoer C.A., Vidyasagar M. *Feedback Systems: Input–Output Properties*. New York: Elsevier, 1975.

Received 29.03.2018

ЛИТЕРАТУРА

1. Dahleh M.A., Pearson J.B. l_1 optimal-feedback controllers for MIMO discrete-time systems. *IEEE Trans. Autom. Contr.*, 1987, vol. 32, no. 4, pp. 314–322.
2. McDonald J.S., Pearson J.B. l_1 optimal control of multivariable systems with output norm constraints. *Automatica*, 1991, vol. 27, no. 2, pp. 317–329.
3. Maciejowski J. M. *Multivariable Feedback Design*. Wokinghan: Addison-Wesley, 1989.
4. Skogestad S., Postlethwaite I. *Multivariable Feedback Control*. UK, Chichester: Wiley, 1996.
5. Albertos P., Sala A. *Multivariable Control Systems: an Engineering Approach*. London: Springer , 2006.
6. Francis B., Wonham W. The internal model principle of control theory. *Automatica*, 1976, vol. 12, no. 5, pp. 457–465.
7. Brockett R. W. The invertibility of dynamic systems with application to control. *Ph. D. Dissertation*, Case Inst. of Technology, Cleveland, Ohio, 1963.
8. Silverman L. M. Inversion of multivariable linear systems. *IEEE Trans. Autom. Contr*., 1969, vol. AC-14, no. 3, pp. 270–276..
9. Lovass-Nagy V., Miller J. R., Powers L. D. On the application of matrix generalized inversion to the construction of inverse systems. *Int. J. Control*, 1976, vol. 24, no. 5, pp. 733–739.
10. Seraji H. Minimal inversion, command tracking and disturbance decoupling in multivariable systems. *Int. J. Control*, 1989, vol. 49, no. 6, pp. 2093–2191.
11. Marro G., Prattichizzo D., Zattoni E. Convolution profiles for right-inversion of multivariable non-minimum phase discrete-time systems. *Automatica*, 2002, vol. 38, no. 10, pp. 1695–1703.
12. Liu C., Peng H. Inverse-dynamics based state and disturbance observers for linear time-invariant systems. *ASME J. Dyn Syst., Meas. and Control*, 2002, vol. 124, no. 5, pp. 376–381.
13. Lyubchyk L. M. Disturbance rejection in linear discrete multivariable systems: inverse model approach. *Prep. 18th IFAC World Congress*, Milano, Italy, 2011, pp. 7921–7926.
14. Pushkov S. G. Inversion of linear systems on the basis of state space realization. *Journal of Computer and Systems Sciences International*, 2018, vol. 57, vo. 1, pp. 7–17.
15. Пухов Г.Е., Жук К.Д. *Синтез многосвязных систем управления по методу обратных операторов*, Киев: Наук. думка, 218 с., 1966.
16. Скурихин В.И., Гриценко В.И., Житецкий Л.С., Соловчук К.Ю. Метод обобщенного обратного оператора в задаче оптимального управления линейными многосвязными статическими объектами. *Доклады НАН Украины*, 2014, №8, С. 57–66.
17. Zhiteckii L. S., Azarskov V. N., Solovchuk K. Yu., Sushchenko O. A. Discrete-time robust steady-state control of nonlinear multivariable systems: a unified approach. *Proc. 19th IFAC World Congress*, Cape Town, South Africa, 2014, pp. 8140–8145.
18. Zhitetskii L. S., Skurikhin V. I., Solovchuk K. Yu. Stabilization of a nonlinear multivariable discrete-time time-invariant plant with uncertainty on a linear pseudoinverse model. *Journal of Computer and Systems Sciences International*, 2017, vol. 56, no. 5, pp. 759–773.
19. Zhiteckii L. S., Solovchuk K. Yu. Pseudoinversion in the problems of robust stabilizing multivariable discrete-time control systems of linear and nonlinear static objects under bounded disturbances. *Journal of Automation and Information Sciences*, 2017, vol. 49, no. 5, pp. 35–48.
20. Фомин В.Н., Фрадков А. Л., Якубович В. А. *Адаптивное управление динамическими объектами*. М.: Наук, 448 с., 1981.

21. Goodwin G.C., Sin K.S. *Adaptive Filtering, Prediction and Control*. Engewood Cliffs, NJ.: Prentice-Hall, 1984.
22. Landau I. D., Lozano R., M'Saad M. *Adaptive Control*. London: Springer, 1997.
23. Кунцевич В. М. *Управление в условиях неопределенности: гарантированные результаты в задачах управления и идентификации*. Киев: Наук. думка, 264с., 2006.
24. Житецкий Л.С., Скурихин В.И. *Адаптивные системы управления с параметрическими и непараметрическими неопределенностями*. Киев: Наук. думка, 301 с., 2010.
25. Narendra K. S., Annaswamy A. M. *Stable Adaptive Systems*. NY: Dover Publications, 2012.
26. Ioannou P., Sun J. *Robust Adaptive Control*. NY: Dover Publications, 2013.
27. Aström K. J., Wittenmark B. *Adaptive Control: 2nd Edition*. NY: Dover Publications, 2014.
28. Albert A. *Regression and the Moore-Penrose Pseudoinverse*. New York: Academic Press, 1972.
29. Kaczmarz S. Approximate solution of systems of linear equations. *Internat. J. Control*, 1993. vol. 57, no. 6. pp. 1269–1271.
30. Marcus M., Minc H. *A Survey of Matrix Theory and Matrix Inequalities*. Boston: Aliyn and Bacon, 1964.
31. Desoer C.A., Vidyasagar M. *Feedback Systems: Input–Output Properties*. New York: Elsevier, 1975.

Получено 29.03.2018

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АДАПТИВНА СТАБІЛІЗАЦІЯ ДЕЯКИХ БАГАТОВИМІРНИХ СИСТЕМ З ПРЯМОКУТНИМИ МАТРИЦЯМИ КОЕФІЦІЄНТІВ ПІДСИЛЕННЯ ПОВНОГО РАНГУ

Вступ. У статті поставлено та розв'язано одну нову задачу, яка стосується адаптивної стабілізації положення рівноваги певного класу лінійних багатовимірних дискретних систем без пам'яті з прямокутними матрицями коефіцієнтів підсилення. Цей клас включає багатовимірні системи, у яких кількість виходів перевищує кількість входів керування. Введено припущення, що невідомі матриці коефіцієнтів підсилення мають повний ранг.

Метою даного дослідження є відповідь на питання про те, чи можна реалізувати адаптивний підхід на основі псевдооберненої моделі для керування невизначеною багатомірною системою без пам'яті, в якій кількість входів керування є менша за кількість вихідних змінних.

Результати. Показано, що оцінки параметрів, які формуються стандартною адаптивною рекурентною процедурою проекційного типу, завжди збігаються до деяких скінченних значень за будь-яких початкових оцінок параметрів системи. Доведено, що адаптивний закон керування на основі псевдооберненої моделі дозволяє досягти положення рівноваги системи, яка підлягає керуванню. Асимптотичні властивості системи керування з адаптивним зворотним зв'язком, встановлені теоретично, підтверджуються модельним експериментом.

Висновки. Встановлено, що гранична поведінка замкненої системи керування з використанням адаптивної концепції, основаної на псевдообернені, є задовільною.

Ключові слова: адаптивне керування, багатовимірна система, дискретний час, зворотний зв'язок, псевдообернення, стійкість, невизначеність.

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АДАПТИВНАЯ СТАБИЛИЗАЦИЯ НЕКОТОРЫХ
МНОГОМЕРНЫХ СИСТЕМ С ПРЯМОУГОЛЬНЫМИ МАТРИЦАМИ
КОЭФФИЦИЕНТОВ УСИЛЕНИЯ ПОЛНОГО РАНГА

Введение. В настоящей статье ставится и решается одна новая задача, касающаяся адаптивной стабилизации положения равновесия определенного класса линейных многомерных дискретных систем без памяти с прямоугольными матрицами коэффициентов усиления. Этот класс включает многомерные системы, у которых число выходов превышает число управляющих входов. Предполагается, что неизвестные матрицы коэффициентов усиления имеют полный ранг.

Цель этой статьи — ответить на вопрос, можно ли реализовать адаптивный подход на основе псевдообратной модели для управления неопределенной многомерной системой без памяти, в которой число управляющих входов меньше числа выходных переменных.

Результаты. Показано, что оценки параметров, генерируемые стандартной адаптивной рекуррентной процедурой проекционного типа, всегда сходятся к некоторым конечным значениям для любых начальных оценок параметров системы. Доказано, что адаптивный псевдообратный закон управления позволяет достичь положения равновесия управляемой системы. Асимптотические свойства адаптивной системы управления с обратной связью, полученные теоретически, подтверждены модельным экспериментом.

Выводы. Установлено, что предельное поведение замкнутой системы управления, построенной на основе адаптивного псевдообращения, является удовлетворительным.

***Ключевые слова*:** адаптивное управление, многомерная система, дискретное время, обратная связь, псевдообращение, устойчивость, неопределенность.