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INTERPOLATED SCALES OF APPROXIMATION SPACES FOR REGULAR ELLIPTIC OPERATORS ON COMPACT MANIFOLDS

We define the interpolated scales of approximation spaces, generated by regular elliptic operators on compact manifolds. The appropriate Bernstein-Jackson inequalities and application to spectral approximations of regular elliptic operators are considered.

Key words and phrases: approximation spaces, Bernstein-Jackson-type inequalities, regular elliptic operators, compact manifolds.

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INTRODUCTION AND PRELIMINARIES

One of the problems in the approximation theory is to characterize the set of functions which have a prescribed order of an approximation by a given method of an approximation [3], [7], [8], [11]. The classical results in this subject are the Jackson and Bernstein inequalities that express a relation between smoothness modules of functions and properties of their best approximations by polynomials.

From direct and inverse theorems it follows that certain classical function spaces can be viewed as special approximation spaces and this is one of the best mathematical expression of equivalences between the degree of smoothness of functions and the behaviour of their best approximation errors [1]. In many instances the approximation spaces can be identified with the interpolation spaces obtained by the real method of interpolation [9].

Approximation spaces and appropriate Bernstein-Jackson inequalities, generated by an unbounded linear operator A in a Banach space, are considered in [4]. Such inequalities are applied to a best approximation problem by invariant subspaces of exponential type entire vectors of A and to spectral approximations of an operator with the point spectrum.

The aim of the paper is to investigate a best approximation problem by subspaces of exponential type vectors of the regular elliptic operators on compact manifolds. Approximation spaces associated with such operators coincide with the Besov-type spaces (Theorem 1). The Bernstein-Jackson-type inequalities, estimating the minimal distance from a given element to a subspace of exponential type vectors and application to spectral approximations of regular elliptic operators are shown in Theorem 2.

Let a compact manifold has the form of the infinitely smooth boundary $\partial\Omega$ of an open bounded domain $\Omega \subset \mathbb{R}^n$ (see [12, Definition 3.2.1/2]). On $\partial\Omega$ we consider a regular elliptic operator A, that in the local coordinates $y^{(j)} = (y_1^{(j)}, \dots, y_{n-1}^{(j)})$ $(j = 1, \dots, N)$ has the form

$$Au = \sum_{|s| \le 2m} a_s^{(j)} D^s_{y^{(j)}} u \,, \quad a_s^{(j)} \in \mathbb{C},$$

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where $|s| = s_1 + \ldots + s_{n-1}$, $s_i \in \mathbb{Z}_+$ $(i = 1, \ldots, n-1)$.

We assume that $a_s^{(j)} \in \mathbb{R}$ for $|s| = 2m$ and there exists a constant $c > 0$ such that for all $\xi \in \mathbb{R}^{n-1}$ we have $(-1)^m \sum_{|s|=2m} a_s^{(j)} \xi^s \geq c\,|\xi|^{2m}$ (see [12, Definition 5.2.1/4]).

Let A with the domain $\mathcal{C}^1(A) = C^\infty(\partial\Omega)$ is the symmetrical operator in the complex space $L_2(\partial\Omega)$. Then by [12, Theorem 7.6.1]) the operator $\overline{A}$ has a point spectrum and $\mathcal{C}^k(\overline{A}) = W_2^{2mk}(\partial\Omega)$ for $k = 1, 2, \ldots$, $\mathcal{C}^\infty(\overline{A}) = \bigcap_{k=1}^\infty \mathcal{C}^k(\overline{A}) = C^\infty(\partial\Omega)$.

Let $U_j \subset \mathbb{R}^{n-1}$ $(j = 1, \ldots, N)$ be open balls that covered $\partial\Omega$ and $\chi_j \in C^\infty(\partial\Omega)$ is the partition of unity that corresponds to the covering $\{U_j\}$: $\sum_{j=1}^N \chi_j = 1$ on $\partial\Omega$, $\chi_j \in C_0^\infty(U_j)$, $0 \leq \chi_j \leq 1$. For $0 < \alpha < \infty$ and $1 \leq \tau \leq \infty$ we consider the spaces $B_{2,\tau}^\alpha(\partial\Omega) = \big\{u \in L_2(\partial\Omega)\colon (\chi_j u)(\psi^{(j)-1}(y)) \in B_{2,\tau}^\alpha(\mathbb{R}^{n-1}), j = 1, \ldots, N\big\}$ with the norm

$$\|u\|_{B_{2,\tau}^\alpha(\partial\Omega)} = \sum_{j=1}^N \|(\chi_j u)(\psi^{(j)-1}(y))\|_{B_{2,\tau}^\alpha(\mathbb{R}^{n-1})}$$

(see [12, Definition 3.6.1]).

We use the real method of an interpolation (see [2, Section 3.11]). Let $(X, |\cdot|_X)$ and $(Y, |\cdot|_Y)$ be quasi-normed complex spaces and $\{0 < \vartheta < 1,\ 1 \leq q \leq \infty\}$ or $\{0 < \vartheta \leq 1,\ q = \infty\}$. The interpolation space can be defined as the set $(X, Y)_{\vartheta,p} = \big\{u \in X + Y\colon |u|_{(X,Y)_{\vartheta,p}} < \infty\big\}$ endowed with the quasi-norm

$$|u|_{(X,Y)_{\vartheta,q}} = \begin{cases} \left(\int_0^\infty \left[\tau^{-\vartheta} K(\tau, u; X, Y)\right]^q \frac{d\tau}{\tau}\right)^{1/q}, & q < \infty, \\ \sup_{0<\tau<\infty} \tau^{-\vartheta} K(\tau, u; X, Y), & q = \infty, \end{cases}$$

where $K(\tau, u; X, Y) = \inf\limits_{u=x+y} \left(|x|_X + \tau\,|y|_Y\right)$.

MAIN RESULTS

For any $t > 0$ we define the normed space

$$\mathcal{E}_2^t(\overline{A}) = \left\{u \in \mathcal{C}^\infty(\overline{A})\colon \|u\|_{\mathcal{E}_2^t(\overline{A})} < \infty\right\},$$

where $\|u\|_{\mathcal{E}_2^t(\overline{A})} = \left(\sum_{k\in\mathbb{Z}_+} \left\|\left(\overline{A}/t\right)^k u\right\|_{L_2(\partial\Omega)}^2\right)^{1/2}$. The elements of $\mathcal{E}_2^t(\overline{A})$ can be called the exponential type vectors of $\overline{A}$ (see [10]).

Lemma 1. (i) *The following embeddings $\mathcal{E}_2^t(\overline{A}) \subset \mathcal{E}_2^\tau(\overline{A}) \subset L_2(\partial\Omega)$ with $\tau > t$ hold.*

(ii) *Each space $\mathcal{E}_2^t(\overline{A})$ is $\overline{A}$-invariant and the restriction $\overline{A}|_{\mathcal{E}_2^t(\overline{A})}$ is a bounded operator over $\mathcal{E}_2^t(\overline{A})$ with the norm $\|\overline{A}|_{\mathcal{E}_2^t(\overline{A})}\|_{\mathcal{E}_2^t(\overline{A})} \leq t$.*

Proof. (i) The inequalities $\|u\|_{L_2(\partial\Omega)} \leq \|u\|_{\mathcal{E}_2^t(\overline{A})}$ and $\|u\|_{\mathcal{E}_2^\tau(\overline{A})} \leq \|u\|_{\mathcal{E}_2^t(\overline{A})}$ with $\tau > t$ yield the embeddings $\mathcal{E}_2^t(\overline{A}) \subset L_2(\partial\Omega)$ and $\mathcal{E}_2^t(\overline{A}) \subset \mathcal{E}_2^\tau(\overline{A})$, respectively.

(ii) Using $\overline{A}(\overline{A}/t)^k u = t(\overline{A}/t)^{k+1} u$, we obtain $\|\overline{A}u\|_{\mathcal{E}_2^t(\overline{A})} \leq t\|u\|_{\mathcal{E}_2^t(\overline{A})}$. □

On the subspace $\mathcal{E}(\overline{A}) = \bigcup_{t>0} \mathcal{E}_2^t(\overline{A})$ we define the function

$$|u|_{\mathcal{E}(\overline{A})} = \|u\|_{L_2(\partial\Omega)} + \inf\left\{t > 0\colon u \in \mathcal{E}_2^t(\overline{A})\right\}. \tag{1}$$

Lemma 2. *The function* (1) *is a quasi-norm satisfying the inequality*

$$|u+v|_{\mathcal{E}(\overline{A})} \leq |u|_{\mathcal{E}(\overline{A})} + |v|_{\mathcal{E}(\overline{A})}$$

for all $u, v \in \mathcal{E}(\overline{A})$ *and the embedding* $\mathcal{E}(\overline{A}) \subset L_2(\partial\Omega)$ *holds.*

Proof. Let $r(u) = \inf\{t > 0 \colon u \in \mathcal{E}_2^t(\overline{A})\}$. The values $\|u\|_{\mathcal{E}_2^{r(u)+\varepsilon}(\overline{A})}$, $\|v\|_{\mathcal{E}_2^{r(v)+\varepsilon}(\overline{A})}$ are finite for each $u, v \in \mathcal{E}(\overline{A})$ and $\varepsilon > 0$ and the inequalities $\|u+v\|_{\mathcal{E}_2^{r+\varepsilon}(\overline{A})} \leq \|u\|_{\mathcal{E}_2^{r(u)+\varepsilon}(\overline{A})} + \|v\|_{\mathcal{E}_2^{r(v)+\varepsilon}(\overline{A})}$, with $r = \max\{r(u), r(v)\}$ hold. It follows that $r(u+v) \leq r + \varepsilon \leq r(u) + r(v) + \varepsilon$. Since ε is arbitrary, $r(u+v) \leq r(u) + r(v)$ for all $u, v \in \mathcal{E}(\overline{A})$. Evidently, $r(u) = r(-u)$ for all $u \in \mathcal{E}(\overline{A})$. So, (1) is a quasi-norm. The embedding $\mathcal{E}(\overline{A}) \subset L_2(\partial\Omega)$ follows from the inequality $\|u\|_{L_2(\partial\Omega)} \leq |u|_{\mathcal{E}(\overline{A})}$ for all $u \in \mathcal{E}(\overline{A})$. □

Given a pair of numbers $\{0 < \alpha < \infty,\ 0 < \tau \leq \infty\}$ and $\{0 \leq \alpha < \infty,\ \tau = \infty\}$ we consider the scale of spaces $\mathcal{B}^{\alpha}_{2,\tau}(\overline{A}) = \left\{u \in L_2(\partial\Omega) :\ |u|_{\mathcal{B}^{\alpha}_{2,\tau}(\overline{A})} < \infty\right\}$,

$$|u|_{\mathcal{B}^{\alpha}_{2,\tau}(\overline{A})} = \begin{cases} \left(\int_0^\infty \left[t^\alpha E(t,u)\right]^\tau \frac{dt}{t}\right)^{1/\tau}, & 0 < \tau < \infty, \\ \sup_{t>0} t^\alpha E(t,u), & \tau = \infty, \end{cases}$$

where $E(t,u) = \inf\left\{\|u - u^0\|_{L_2(\partial\Omega)} : u^0 \in \mathcal{E}(\overline{A}),\ |u^0|_{\mathcal{E}(\overline{A})} < t\right\}$, $u \in L_2(\partial\Omega)$.

The space $\mathcal{B}^{\alpha}_{2,\tau}(\overline{A})$ can be identified with the interpolation space. If $[\mathcal{B}^{\alpha}_{2,\tau}(\overline{A})]^\vartheta$ is the space $\mathcal{B}^{\alpha}_{2,\tau}(\overline{A})$ with the quasi-norm $|u|^\vartheta_{\mathcal{B}^{\alpha}_{2,\tau}(\overline{A})}$ then by [2, Theorem 7.1.7] the following equality

$$[\mathcal{B}^{\alpha}_{2,\tau}(\overline{A})]^\vartheta = \left(\mathcal{E}(\overline{A}), L_2(\partial\Omega)\right)_{\vartheta,g}, \quad \vartheta = 1/(\alpha+1), \quad \tau = g\vartheta,$$

holds with equivalent quasi-norms.

Theorem 1. *If* $\alpha > 1/2$, $1 \leq \tau \leq \infty$, *then the following isomorphism holds*

$$\mathcal{B}^{\alpha}_{2,\tau}(\overline{A}) = B^{\alpha-1/2}_{2,\tau}(\partial\Omega). \tag{2}$$

Proof. Consider the space

$$\mathcal{E}_2^t(D) = \left\{ u \in C^\infty(\partial\Omega) : D^s u \in L_2(\partial\Omega), |s| = k \in \mathbb{Z}_+ \right\}$$

endowed with the norm

$$\|u\|_{\mathcal{E}_2^t(D)} = \Big(\sum_{k \in \mathbb{Z}_+} \sum_{|s|=k} t^{-2k} \| D^s u\|^2_{L_2(\partial\Omega)} \Big)^{1/2}.$$

The union $\mathcal{E}(D) = \bigcup_{t>0} \mathcal{E}_2^t(D)$ we endow with the quasi-norm

$$|u|_{\mathcal{E}(D)} = \|u\|_{L_2(\partial\Omega)} + \inf\{t > 0 \colon u \in \mathcal{E}_2^t(D)\}.$$

If $l > (n-1)/2$ and $u \in \mathcal{E}_2^t(D)$ then the Sobolev embedding theorem yields

$$\sum_{j=1}^N \sup_{y \in \mathbb{R}^{n-1}} |D^s(\chi_j u)(\psi^{(j)-1}(y))| \leq c \max\{1, t, \ldots, t^l\} t^k \|u\|_{\mathcal{E}_q^t(D)} \leq c_0 t^k. \tag{3}$$

It follows that

$$\sum_{j=1}^{N} \left|(\chi_j u)(\psi^{(j)-1}(y+i\eta))\right| \le \sum_{k\in\mathbb{Z}_+}\sum_{|s|=k}\sum_{j=1}^{N}\left|D^s(\chi_j u)(\psi^{(j)-1}(y))\right|\frac{|\eta|^k}{k!} \le c_1 e^{t|\eta|} \tag{4}$$

for all $\eta \in \mathbb{R}^{n-1}$, where the constant c_1 is independent of $k \in \mathbb{Z}_+$.

Let a function u satisfy (4). Then the inequality $\sum_{|s|=k} \| D^s u\|_{L_2(\partial\Omega)} \le c_2(2(n-1)^2 t)^k$ holds and we have

$$\sum_{k\in\mathbb{Z}_+}\sum_{|s|=k}(4(n-1)^2 t)^{-2k}\| D^s u\|^2_{L_2(\partial\Omega)} \le \frac{4}{3}\sup_{k\in\mathbb{Z}_+}\frac{\sum\limits_{|s|=k}\| D^s u\|^2_{L_2(\partial\Omega)}}{(2(n-1)^2 t)^{2k}}. \tag{5}$$

It follows that $u \in \mathcal{E}_2^{4(n-1)^2 t}(D)$ and consequently $u \in \mathcal{E}(D)$.

Using the inequality (3), (5) and the Paley-Wiener theorem, we obtain the quasi-norm equivalence

$$|u|_{\mathcal{E}(D)} \sim \inf_{v|_{\partial\Omega}=u,\, v\in L_2(\mathbb{R}^{n-1})}\Big\{\|v\|_{L_2(\mathbb{R}^{n-1})} + \sup_{\zeta\in \operatorname{supp} Fv}|\zeta|\Big\},$$

where supp Fv denotes the support of the Fourier-image Fv of a function $v \in L_2(\mathbb{R}^{n-1})$.

Applying [12, Theorems 3.6.1, 4.2.2], [2, Theorem 7.1.7] and Bernstein-Jackson inequalities from [2, Section 7.2] for $l \in \mathbb{N}$, we obtain

$$\|u\|^{1/(l+1)}_{W_2^{l-1/2}(\partial\Omega)} \le c_l\, |u|^{1-1/(l+1)}_{\mathcal{E}(D)}\, \|u\|^{1/(l+1)}_{L_2(\partial\Omega)}, \quad u \in \mathcal{E}(D), \tag{6}$$

$$K\big(t,u;\mathcal{E}(D),L_2(\partial\Omega)\big) \le c_l\, t^{1/(l+1)}\|u\|^{1/(l+1)}_{W_2^{l-1/2}(\partial\Omega)}, \quad u \in W_2^{l-1/2}(\partial\Omega). \tag{7}$$

We define the space

$$\mathcal{B}^\alpha_{2,\tau}(D) = \Big\{u \in L_2(\partial\Omega) : |u|_{\mathcal{B}^\alpha_{2,\tau}(D)} = \Big(\int_0^\infty \big(t^\alpha E(t,u)\big)^\tau \frac{dt}{t}\Big)^{1/\tau} < \infty\Big\},$$

where $E(t,u) = \inf\Big\{\|u-u^0\|_{L_2(\partial\Omega)} \colon u^0 \in \mathcal{E}(D),\ |u^0|_{\mathcal{E}(D)} < t\Big\}$. Using [2, Theorems 3.11.5, 3.11.6, 7.1.7], [12, Theorems 2.4.2/2, 3.6.1, 3.6.3] and the inequalities (6), (7), we obtain

$$\begin{aligned}\mathcal{B}^\alpha_{2,\tau}(D) &= \Big(\big(\mathcal{E}(D),L_2(\partial\Omega)\big)_{1/(\alpha+1),\tau(\alpha+1)}\Big)^{\alpha+1}\\ &= \big(L_2(\partial\Omega), W_2^{l-1/2}(\partial\Omega)\big)_{\alpha/l,\tau} = B^{\alpha-1/2}_{2,\tau}(\partial\Omega).\end{aligned} \tag{8}$$

By [12, Theorems 5.4.3, 7.6.1] for any $k \in \mathbb{N}$ there exist positive numbers c and C such that

$$c^k\|u\|_{W_2^{2mk}(\partial\Omega)} \le \|\overline{A}^k u\|_{L_2(\partial\Omega)} \le C^k\|u\|_{W_2^{2mk}(\partial\Omega)}, \quad u \in \mathcal{C}^k(\overline{A}).$$

It follows that we have the inequality

$$\sum_{k\in\mathbb{Z}_+}(C((n-1)t)^{2m})^{-2k}\|\overline{A}^k u\|^2_{L_2(\partial\Omega)} \le C_1\sum_{k\in\mathbb{Z}_+}\sum_{|s|=2mk} t^{-4mk}\| D^s u\|^2_{L_2(\partial\Omega)}.$$

Thus, the embedding $\mathcal{E}_2^t(D) \subset \mathcal{E}_2^\tau(\overline{A})$ with $\tau = C((n-1)t)^{2m}$ holds. Conversely, let $u \in \mathcal{E}_2^t(\overline{A})$. Then

$$\sum_{k\in\mathbb{Z}_+} t^{-2k}\|\overline{A}^k u\|^2_{L_2(\partial\Omega)} \ge \sum_{k\in\mathbb{Z}_+}\sum_{|s|=k}(c^{-1}t)^{-2k}\| D^s u\|^2_{L_2(\partial\Omega)}.$$

It follows that $\mathcal{E}_2^t(\overline{A}) \subset \mathcal{E}_2^{c^{-1}t}(D)$. So, we have the equality $\mathcal{E}(\overline{A}) = \mathcal{E}(D)$. Using (8), we obtain the required equality (2). ∎

The distance between $u \in L_2(\partial\Omega)$ and $\mathcal{E}_2^t(\overline{A})$ we denote by

$$d(t,u) = \inf\left\{\|u - u^0\|_{L_2(\partial\Omega)} \colon u^0 \in \mathcal{E}_2^t(\overline{A})\right\}, \quad u \in L_2(\partial\Omega).$$

Let $\mathcal{R}^t$ be the complex linear span of all $\{\mathcal{R}(\lambda_n) \colon |\lambda_n| < t\}$, where $\mathcal{R}(\lambda_n)$ is the root subspace of $\overline{A}$ corresponding to λ_n.

Theorem 2. *Let $\alpha > 1/2$ and $1 \leq \tau \leq \infty$. There are constants c_1 and c_2 such that the following inequalities*

$$\|u\|_{B^{\alpha-1/2}_{2,\tau}(\partial\Omega)} \leq c_1 |u|^{\alpha}_{\mathcal{E}(D)} \|u\|_{L_2(\partial\Omega)}, \quad u \in \mathcal{E}(D), \tag{9}$$

$$d(t,u) \leq c_2\, t^{-\alpha} \|u\|_{B^{\alpha-1/2}_{2,\tau}(\partial\Omega)}, \quad u \in B^{\alpha-1/2}_{2,\tau}(\partial\Omega), \tag{10}$$

hold. In particular, there is a constant c such that

$$\inf\left\{\|u - u^0\|_{L_2(\partial\Omega)} \colon u^0 \in \mathcal{R}^t\right\} \leq c\, t^{-\alpha} \|u\|_{B^{\alpha-1/2}_{2,\tau}(\partial\Omega)}, \quad u \in B^{\alpha-1/2}_{2,\tau}(\partial\Omega). \tag{11}$$

Proof. By [4, Theorem 5] for some constants c_1 and c_2 we have

$$|u|_{\mathcal{B}^{\alpha}_{2,\tau}(\overline{A})} \leq c_1 |u|^{\alpha}_{\mathcal{E}(\overline{A})} \|u\|_{L_2(\partial\Omega)}, \quad u \in \mathcal{E}(\overline{A}), \tag{12}$$

$$d(t,u) \leq c_2\, t^{-\alpha} |u|_{\mathcal{B}^{\alpha}_{2,\tau}(\overline{A})}, \quad u \in \mathcal{B}^{\alpha}_{2,\tau}(\overline{A}). \tag{13}$$

The inequalities (12), (13) and the isomorphism (2) imply that the inequalities (9), (10) hold.

Using [6, Theorem 2.2] and [5, Proposition 2], we obtain the equality $\mathcal{E}_2^t(\overline{A}) = \mathcal{R}^t$. Hence, the inequality (10) directly implies the estimation (11). □

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Дмитришин М.I. *Інтерполяційні шкали апроксимаційних просторів для регулярних еліптичних операторів на компактних многовидах* // Карпатські матем. публ. — 2014. — Т.6, №1. — С. 26–31.

Визначено інтерполяційні шкали апроксимаційних просторів, асоційованих з регулярними еліптичними операторами на компактних многовидах. Встановлено відповідні нерівності типу Бернштейна і Джексона та показано їх застосування до спектральних апроксимацій регулярних еліптичних операторів.

Ключові слова і фрази: апроксимаційні простори, нерівності типу Бернштейна і Джексона, регулярні еліптичні оператори, компактні многовиди.

Дмитришин М.И. *Интерполяционные шкалы аппроксимационных пространств для регулярных эллиптических операторов на компактных многообразиях* // Карпатские матем. публ. — 2014. — Т.6, №1. — С. 26–31.

Определены интерполяционные шкалы аппроксимационных пространств, ассоциированных с регулярными эллиптическими операторами на компактных многообразиях. Установлены соответствующие неравенства типа Бернштейна и Джексона и показано их применение к спектральным аппроксимациям регулярных эллиптических операторов.

Ключевые слова и фразы: аппроксимационные пространства, неравенства типа Бернштейна и Джексона, регулярные эллиптические операторы, компактные многообразия.