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COUNTABLE HYPERBOLIC SYSTEMS IN THE THEORY OF NONLINEAR OSCILLATIONS

In this article a model example of a mixed problem for a fourth-order differential equation is reduced to initial-boundary value problem for countable hyperbolic system of first order coherent differential equations.

Key words and phrases: countable hyperbolic system, initial-boundary value problem.

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INTRODUCTION

Many problems from Elasticity Theory, Gas dynamics, Theory of plates and shells reduced to partial higher order differential equations [1, 2, 3] using Fourier method [3] or the method of Principal coordinates [1]. As a result we get a infinite system of ordinary differential equations. The Theory of countable ordinary differential systems is described in the monograph [4]. However, in many cases, particularly in the famous Hadamard's example [5, p.112] about correct solvability of initial problem for Cauchy-Riemann equation, if interpret partial solutions like $u_n = I_n(t)\cos nx$, $v_n = J_n(t)\sin nx$, we get a countable system of partial first order differential equations. Similar systems occur in determining of the generalized solution for hyperbolic first order equations [5, p.132], in the investigation of mathematical models of self-excited oscillator with distributed parameters [6], in many periodic solutions of quasi-linear hyperbolic systems [7] and others. Some questions about the correct solvability of initial-boundary value problems for countable hyperbolic systems of first order differential equations are considered in [8, 9, 10, 13].

1 STATEMENT OF PROBLEM

In the domain $Q = \{(t,x,y) : t \in (0,T), x \in (0,l_1), y \in (0,l_2)\}$ we consider fourth order partial differential equation

$$u_{tt} + B(t,x)(u_{tx} + u_{xyy}) + C(t,x)u_{xx} + u_{yyyy} + 2u_{tyy} = f\Big(t,x,y,u,u_t,u_x,u_{yy}\Big) \tag{1}$$

with initial

$$\begin{aligned} u|_{t=0} &= \varphi(x,y), \\ u_t|_{t=0} &= \psi(x,y), \quad 0 \le x \le l_1,\ 0 \le y \le l_2, \end{aligned} \tag{2}$$

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and boundary conditions

$$\begin{aligned} &u|_{y=0} = u|_{y=l_2} = 0, \\ &\frac{\partial^2 u}{\partial y^2}\Big|_{y=0} = \frac{\partial^2 u}{\partial y^2}\Big|_{y=l_2} = 0, \quad 0 \le x \le l_1,\ 0 \le t \le T, \\ &u|_{x=0} = \mu(t,y),\ u|_{x=l_1} = \nu(t,y), \quad 0 \le y \le l_2,\ 0 \le t \le T, \end{aligned} \tag{3}$$

where

$$\begin{aligned} &\mu(0,y) = \varphi(0,y), \quad \nu(0,y) = \varphi(l_1,y), \quad \mu'_t(0,y) = \psi(0,y), \quad \nu'_t(0,y) = \psi(l_1,y), \\ &\varphi(x,0) = \varphi(x,l_2) = 0, \quad \psi(x,0) = \psi(x,l_2) = 0, \\ &\varphi''_{yy}(x,0) = \varphi''_{yy}(x,l_2) = 0, \quad \psi''_{yy}(x,0) = \psi''_{yy}(x,l_2) = 0. \end{aligned}$$

2 THE REDUCTION EQUATION (1) TO A COUNTABLE SYSTEM OF SECOND ORDER DIFFERENTIAL EQUATIONS

We will search solution of the problem (1)–(3) using separation of variables method, namely in the form of a series

$$u(t,x,y) = v_0(t,x) + \sum_{n=1}^{\infty} \big(v_n(t,x)\cos\alpha_n y + w_n(t,x)\sin\alpha_n y\big), \tag{4}$$

where $\alpha_n = \frac{2\pi n}{l_2}$ (see [12, 13]). Substituting (4) in boundary conditions (3), we obtain $\sum\limits_{n=0}^{\infty} v_n(t,x) = 0$ and $\sum\limits_{n=1}^{\infty} \alpha_n^2 v_n(t,x) = 0$. Suppose, that $v_n(t,x) \equiv 0$ for all $n \in \mathbb{N}$ and $(t,x) \in \Pi^{t,x} = (0,T) \times (0,l_1)$.

Assume that the initial data of the problem (1)–(3) are sufficiently smooth. Let compatibility conditions are fulfilled and the initial data are unambiguous decomposed in a series

$$f\Big(t,x,y,u,\frac{\partial u}{\partial t},\frac{\partial u}{\partial x},\frac{\partial^2 u}{\partial y^2}\Big) = \sum_{n=1}^{\infty} f_n\Big(t,x,w_1,w_2,\dots,\frac{\partial w_1}{\partial t},\frac{\partial w_2}{\partial t},\dots,\frac{\partial w_1}{\partial x},\frac{\partial w_2}{\partial x},\dots\Big)\sin\alpha_n y, \tag{5}$$

$$\varphi(x,y) = \sum_{n=1}^{\infty} \varphi_n(x)\sin\alpha_n y, \quad \psi(x,y) = \sum_{n=1}^{\infty} \psi_n(x)\sin\alpha_n y, \tag{6}$$

$$\mu(t,y) = \sum_{n=1}^{\infty} \mu_n(t)\sin\alpha_n y, \quad \nu(t,y) = \sum_{n=1}^{\infty} \nu_n(t)\sin\alpha_n y. \tag{7}$$

Let $\omega_n = \big(\frac{2\pi n}{l_2}\big)^2$. Substitute equality (4) in equation (1) and conditions (2) and (3). After multiplying received equalities by $\sin\alpha_m y$, $(m = 1,2,\dots)$ and integrating in the interval from 0 to l_2, with considering conditions (5)–(7), we obtain the countable system of second-order differential equations

$$\begin{aligned} &\frac{\partial^2 w_n}{\partial t^2} + B(t,x)\Big(\frac{\partial^2 w_n}{\partial t \partial x} - \omega_n \frac{\partial w_n}{\partial x}\Big) + C(t,x)\frac{\partial^2 w_n}{\partial x^2} + \omega_n^2 w_n - 2\omega_n \frac{\partial w_n}{\partial t} \\ &\quad = f_n\Big(t,x,w_1,w_2,\dots,\frac{\partial w_1}{\partial t},\frac{\partial w_2}{\partial t},\dots,\frac{\partial w_1}{\partial x},\frac{\partial w_2}{\partial x},\dots\Big), \quad n \in \mathbb{N}, \end{aligned} \tag{8}$$

with initial and boundary conditions

$$\begin{aligned} &w_n|_{t=0} = \varphi_n(x),\ \frac{\partial w_n}{\partial t}\Big|_{t=0} = \psi_n(x), \quad 0 \le x \le l_1, \\ &w_n|_{x=0} = \mu_n(t),\ w_n|_{x=l_1} = \nu_n(t), \quad 0 \le t \le T. \end{aligned}$$

Propose a change of variables $w_n = v_n e^{\omega_n t}$. Then all derivatives will be rewritten in a form

$$\begin{aligned}
&\frac{\partial w_n}{\partial t} = \Big(\frac{\partial v_n}{\partial t} + \omega_n v_n\Big)e^{\omega_n t}, \quad \frac{\partial w_n}{\partial x} = \frac{\partial v_n}{\partial x}e^{\omega_n t},\\
&\frac{\partial^2 w_n}{\partial t^2} = \Big(\frac{\partial^2 v_n}{\partial t^2} + 2\omega_n\frac{\partial v_n}{\partial t} + \omega_n^2 v_n\Big)e^{\omega_n t},\\
&\frac{\partial^2 w_n}{\partial t\partial x} = \Big(\frac{\partial^2 v_n}{\partial t\partial x} + \omega_n\frac{\partial v_n}{\partial x}\Big)e^{\omega_n t}, \quad \frac{\partial^2 w_n}{\partial x^2} = \frac{\partial^2 v_n}{\partial x^2}e^{\omega_n t}.
\end{aligned}$$

As a result, we obtain the countable system of second order differential equations

$$\begin{aligned}
&\frac{\partial^2 v_n}{\partial t^2} + B(t,x)\frac{\partial^2 v_n}{\partial t\partial x} + C(t,x)\frac{\partial^2 v_n}{\partial x^2}\\
&\quad = \tilde{f}_n\Big(t, x, v_1, v_2, \ldots, \frac{\partial v_1}{\partial t}, \frac{\partial v_2}{\partial t}, \ldots, \frac{\partial v_1}{\partial x}, \frac{\partial v_2}{\partial x}, \ldots\Big), \quad n \in \mathbb{N},
\end{aligned}$$

where

$$\begin{aligned}
\tilde{f}_n = e^{-\omega_n t} f_n\Big(&t, x, v_1 e^{\omega_n t}, v_2 e^{\omega_n t}, \ldots,\\
&\frac{\partial v_1}{\partial t}e^{\omega_n t} + \omega_n v_1 e^{\omega_n t}, \frac{\partial v_2}{\partial t}e^{\omega_n t} + \omega_n v_2 e^{\omega_n t}, \ldots, \frac{\partial v_1}{\partial x}e^{\omega_n t}, \frac{\partial v_2}{\partial x}e^{\omega_n t}, \ldots\Big).
\end{aligned}$$

Initial and boundary conditions will be rewritten in a form

$$\begin{aligned}
&v_n|_{t=0} = \varphi_n(x), \quad \frac{\partial v_n}{\partial t}\Big|_{t=0} = \tilde{\psi}_n(x), \qquad 0 \le x \le l_1,\\
&v_n|_{x=0} = \tilde{\mu}_n(t), \quad v_n|_{x=l_1} = \tilde{\nu}_n(t), \qquad 0 \le t \le T,
\end{aligned}$$

where $\tilde{\mu}_n(t) = \mu_n(t)e^{-\omega_n t}$, $\tilde{\nu}_n(t) = \nu_n(t)e^{-\omega_n t}$, $\tilde{\psi}_n(x) = \psi_n(x) - \omega_n\varphi_n(x)$.

3 THE REDUCTION TO COUNTABLE SYSTEM OF FIRST ORDER DIFFERENTIAL EQUATIONS

Suppose that $\Delta(t,x) = B^2(t,x) - 4C(t,x) > 0$, for all $(t,x) \in \Pi^{t,x}$, so each equation of the system (8) has hyperbolic type. We denote

$$\begin{aligned}
&\lambda_i(t,x) = \frac{B(t,x) + (-1)^i\sqrt{\Delta(t,x)}}{2},\\
&v_{i,n} = \frac{\partial v_n}{\partial t} + \lambda_i\frac{\partial v_n}{\partial x}, \quad i = 1,2.
\end{aligned}$$

Then

$$\begin{aligned}
&\frac{\partial v_n}{\partial x} = \frac{v_{2,n} - v_{1,n}}{\sqrt{\Delta}},\\
&\frac{\partial v_n}{\partial t} = v_{2,n} - (B + \sqrt{\Delta})\frac{v_{2,n} - v_{1,n}}{2\sqrt{\Delta}}.
\end{aligned}$$

Due to variables changes, each equation of the system (8) would be equivalent to the system of equations [5, 11]

$$\begin{aligned}
&\frac{\partial v_{i,n}}{\partial t} + \lambda_{3-i}\frac{\partial v_{i,n}}{\partial x} = \frac{1}{\sqrt{\Delta}}\Big(\frac{\partial \lambda_i}{\partial t} + \lambda_{3-i}\frac{\partial \lambda_i}{\partial x}\Big)(v_{2,n} - v_{1,n})\\
&\qquad + \tilde{f}_n\Big(t, x, v_1, \ldots, v_{2,1} - (B + \sqrt{\Delta})\frac{v_{2,1} - v_{1,1}}{2\sqrt{\Delta}}, \ldots, \frac{v_{2,1} - v_{1,1}}{\sqrt{\Delta}}, \ldots\Big), \qquad (9)\\
&\frac{\partial v_n}{\partial t} = v_{2,n} - (B + \sqrt{\Delta})\frac{v_{2,n} - v_{1,n}}{2\sqrt{\Delta}}, \quad i = 1,2, \quad n \in \mathbb{N}.
\end{aligned}$$

Suppose, that $\lambda_1(t,x) \geq 0$, $\lambda_2(t,x) \leq 0$ $\big($a sufficient condition is execution the inequality $|B(t,x)| \leq \sqrt{\Delta(t,x)}\big)$. Conduct characteristic $L_1(0,0)$ through the point $(0,0)$ and characteristic $L_2(0,l_1)$ through the point $(0,l_1)$, which are the solutions of Cauchy problems

$$\frac{dx}{dt} = \lambda_1(t,x),\ x(0)=0,\ \frac{dx}{dt} = \lambda_2(t,x),\ x(0)=l_1.$$

Thus, rectangle $\Pi^{t,x}$ is divided into three parts (see Figure 1).

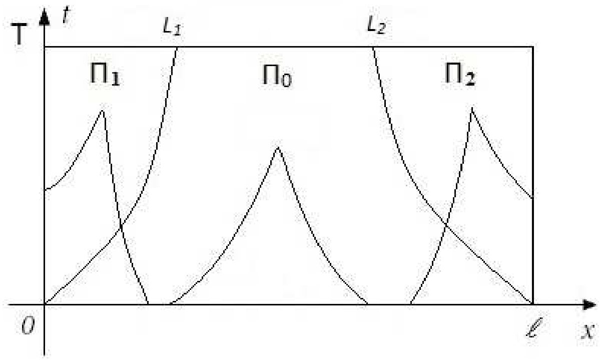

Figure 1: Partition of domain by characteristics with slope $\lambda_1 \geq 0, \lambda_2 \leq 0$.

In subdomain Π_0 for system (9) define the initial conditions

$$v_n|_{t=0} = \varphi_n(x), v_{i,n}|_{t=0} = \tilde{\psi}_n(x) + \lambda_i|_{t=0}\frac{d\varphi_n}{dx}(x), \quad i=1,2.$$

In Π_1 for v_n and $v_{2,n}$ define the initial conditions, and for $v_{1,n}$ define the boundary conditions on the left side

$$v_n|_{t=0} = \varphi_n(x), \quad v_{2,n}|_{t=0} = \tilde{\psi}_n(x) + \lambda_2|_{t=0}\frac{d\varphi_n}{dx}(x),$$
$$v_{1,n}|_{x=0} = \frac{2\sqrt{\Delta}}{B+\sqrt{\Delta}}\Big|_{x=0}\frac{d\tilde{\mu}_n}{dt}(t) + \Big(1-\frac{2\sqrt{\Delta}}{B+\sqrt{\Delta}}\Big)\Big|_{x=0} v_{2,n}|_{x=0}.$$

In subdomain Π_2 for v_n and $v_{1,n}$ define the initial conditions, and for $v_{2,n}$ define the boundary conditions on the right side

$$v_n|_{t=0} = \varphi_n(x), \quad v_{1,n}|_{t=0} = \tilde{\psi}_n(x) + \lambda_1|_{t=0}\frac{d\varphi_n}{dx}(x),$$
$$v_{2,n}|_{x=l_1} = \frac{2\sqrt{\Delta}}{\sqrt{\Delta}-B}\Big|_{x=l_1}\frac{d\tilde{\mu}_n}{dt}(t) + \frac{B+\sqrt{\Delta}}{B-\sqrt{\Delta}}\Big|_{x=l_1} v_{1,n}|_{x=l_1}.$$

Remark 3.1. *If the following condition is not fulfilled $\lambda_1 \geq 0, \lambda_2 \leq 0$, there is possible to get such cases:*

i) $\lambda_1 \geq \lambda_2 \geq 0, \lambda_1^2 + \lambda_2^2 \neq 0;$

ii) $\lambda_1 \leq \lambda_2 \leq 0, \lambda_1^2 + \lambda_2^2 \neq 0.$

In the first case, for system (1) *it is necessary to define the boundary conditions in the next form*

$$u|_{x=0} = \mu(t,y), u_x|_{x=0} = \nu(t,y), \quad 0 \leq y \leq l_2,\ 0 \leq t \leq T.$$

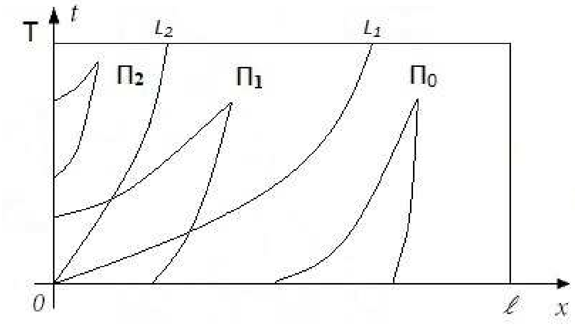

Figure 2: Partition of domain by characteristics with slope $\lambda_1, \lambda_2 > 0$.

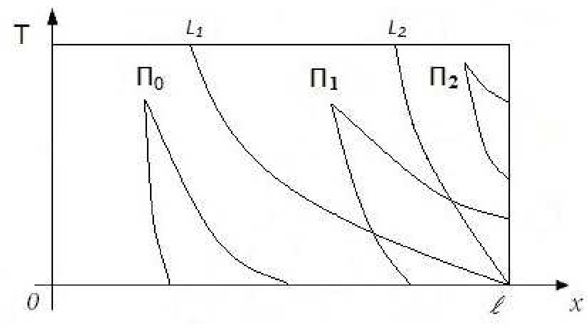

Figure 3: Partition of domain by characteristics with slope $\lambda_1, \lambda_2 < 0$.

Conduct characteristics $L_1(0,0)$ and $L_2(0,0)$ through the point $(0,0)$, which are the solutions of Cauchy problems

$$\frac{dx}{dt} = \lambda_i, \ x(0) = 0, \quad i = 1,2.$$

Thus, rectangle $\Pi^{t,x}$ is devided into three parts (see Figure 2).

In subdomain Π_0 define the initial conditions

$$v_n|_{t=0} = \varphi_n(x), \ v_{i,n}|_{t=0} = \tilde{\psi}_n(x) + \lambda_i|_{t=0}\frac{d\varphi_n}{dx}(x), \quad i = 1,2.$$

In Π_1 for v_n and $v_{2,n}$ define the initial conditions, and for $v_{1,n}$ define the boundary conditions on the left side

$$v_n|_{t=0} = \varphi_n(x), \quad v_{2,n}|_{t=0} = \tilde{\psi}_n(x) + \lambda_2|_{t=0}\frac{d\varphi_n}{dx}(x),$$
$$v_{1,n}|_{x=0} = \frac{d\tilde{\mu}_n}{dt}(t) + \lambda_1|_{x=0}\tilde{v}_n(t).$$

In subdomain Π_2 for v_n define the initial conditions, and for $v_{1,n}$ and $v_{2,n}$ define the boundary conditions on the left side

$$v_n|_{t=0} = \varphi_n(x), \quad v_{i,n}|_{x=0} = \frac{d\tilde{\mu}_n}{dt}(t) + \lambda_i|_{x=0}\nu_n(t).$$

Similarly, the initial and boundary conditions would be defined in case, when $\lambda_1 \leq \lambda_2 \leq 0$, $\lambda_1^2 + \lambda_2^2 > 0$ (see Figure 3). In this case for the system (1) *we have to set the boundary conditions in the following form*

$$u|_{x=l_1} = \mu(t,y), \ u_x|_{x=l_1} = \nu(t,y), \quad 0 \leq y \leq l_2, \ 0 \leq t \leq T.$$

4 EXAMPLE

For example, consider a differential equation

$$u_{tt} - x^2 u_{xx} + u_{yyyy} + 2u_{tyy} = -xu_x + \Big(-\frac{\pi}{6} + \pi y - y^2 \Big) u + f(t,x,y), \tag{10}$$

where $f(t,x,y)$ — some polynomial of (t,x,y), with initial conditions

$$u|_{t=0}=0,\ u_t|_{t=0}=\Big(y^5-\frac{5}{2}\pi y^4+\frac{5}{3}\pi^2y^3-\frac{1}{6}\pi^4 y\Big)(\pi x-x^2),\quad 0\le x\le\pi,\ 0\le y\le\pi,$$

and homogeneous boundary conditions

$$\begin{aligned}&\frac{\partial^2 u}{\partial y^2}\Big|_{y=0}=\frac{\partial^2 u}{\partial y^2}\Big|_{y=\pi}=0,\\ &u|_{y=0}=u|_{y=\pi}=0,\ u|_{x=0}=u|_{x=\pi}=0,\quad 0\le x\le\pi,\ 0\le y\le\pi,\ 0\le t\le T.\end{aligned}\tag{11}$$

The solution can be sought in the form $u(t,x,y)=\sum\limits_{n=1}^{\infty}w_n(t,x)\sin 2ny$. Functions on the right side of the equation and the initial conditions decomposed in such series

$$\begin{aligned}&y^5-\frac{5}{2}\pi y^4+\frac{5}{3}\pi^2y^3-\frac{1}{6}\pi^4 y=-\sum_{n=1}^{\infty}\frac{15}{2n^5}\sin 2ny,\\ &f(t,x,y)=\sum_{n=1}^{\infty}f_n(t,x)\sin 2ny,\\ &-\frac{\pi^2}{6}+\pi y-y^2=-\sum_{m=1}^{\infty}\frac{1}{m^2}\cos 2my,\\ &\Big(-\frac{\pi^2}{6}+\pi y-y^2\Big)u=\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\frac{w_k}{m^2}\delta_n^{k,m}\sin 2ny,\end{aligned}$$

where $\delta_n^{k,m}=\begin{cases}\frac{1}{2}, & \text{if } k+m-n=0,\\ -\frac{1}{2}, & \text{if } (k-m+n)(m-k+n)=0.\end{cases}$

So, we obtain the countable system of second order differential equations

$$\frac{\partial^2 w_n}{\partial t^2}-x^2\frac{\partial^2 w_n}{\partial x^2}+\omega_n^2w_n-2\omega_n\frac{\partial w_n}{\partial t}=-x\frac{\partial w_n}{\partial x}+\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\frac{w_k}{m^2}\delta_n^{k,m}+f_n,\quad n\in\mathbb{N},\tag{12}$$

with initial conditions

$$w_n|_{t=0}=0,\quad \frac{\partial w_n}{\partial t}\Big|_{t=0}=-\frac{15}{2n^5}(\pi x-x^2),\qquad 0\le x\le\pi,\ n\in\mathbb{N},$$

and homogeneous boundary conditions.

Perform a change of variables $w_n=v_ne^{\omega_n t}$. The system (12) will be rewritten in a form

$$\frac{\partial^2 v_n}{\partial t^2}-x^2\frac{\partial^2 v_n}{\partial x^2}+x\frac{\partial v_n}{\partial x}=\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\frac{v_ke^{(\omega_k-\omega_n)t}}{m^2}\delta_n^{k,m}+\frac{f_n}{e^{\omega_n t}},\quad n\in\mathbb{N},$$

with initial and homogeneous boundary conditions

$$v_n|_{t=0}=0,\quad \frac{\partial v_n}{\partial t}\Big|_{t=0}=-\frac{15}{2n^5}(\pi x-x^2),\qquad 0\le x\le\pi,\ n\in\mathbb{N}.$$

In this case $\Delta=4x^2$, that is

$$\begin{aligned}v_{1,n}&=\frac{\partial v_n}{\partial t}+x\frac{\partial v_n}{\partial x},\\ v_{2,n}&=\frac{\partial v_n}{\partial t}-x\frac{\partial v_n}{\partial x}.\end{aligned}$$

As a result, we obtain the countable system of first order differential equations

$$\begin{cases} \dfrac{\partial v_{1,n}}{\partial t} - x\dfrac{\partial v_{1,n}}{\partial x} = \sum\limits_{k=1}^{\infty}\sum\limits_{m=1}^{\infty} \dfrac{v_k e^{(\omega_k-\omega_n)t}}{m^2}\delta_n^{k,m} + \dfrac{f_n}{e^{\omega_n t}}, \\ \dfrac{\partial v_{2,n}}{\partial t} + x\dfrac{\partial v_{2,n}}{\partial x} = \sum\limits_{k=1}^{\infty}\sum\limits_{m=1}^{\infty} \dfrac{v_k e^{(\omega_k-\omega_n)t}}{m^2}\delta_n^{k,m} + \dfrac{f_n}{e^{\omega_n t}}, \\ \dfrac{\partial v_n}{\partial t} = \dfrac{v_{1,n}+v_{2,n}}{2}. \end{cases} \tag{13}$$

Since $\lambda_1 = x > 0, \lambda_2 = -x < 0$, initial and boundary conditions will be rewritten in a form:

$$v_n|_{t=0} = 0,\ v_{1,n}|_{t=0} = -\frac{15}{2n^5}(\pi x - x^2),\ v_{2,n}|_{t=0} = -\frac{15}{2n^5}(\pi x - x^2), \quad (t,x) \in \Pi_0; \tag{14}$$

$$v_n|_{t=0} = 0,\ v_{2,n}|_{t=0} = -\frac{15}{2n^5}(\pi x - x^2),\ v_{1,n}|_{x=0} = -v_{2,n}|_{x=0}, \quad (t,x) \in \Pi_1; \tag{15}$$

$$v_n|_{t=0} = 0,\ v_{1,n}|_{t=0} = -\frac{15}{2n^5}(\pi x - x^2),\ v_{2,n}|_{x=\pi} = -v_{1,n}|_{x=\pi}, \quad (t,x) \in \Pi_2. \tag{16}$$

After solving the problem (13)–(16) (see [9]), we will obtain a system of functions

$$\begin{aligned} v_n &= -\frac{15t}{2n^5 e^{\omega_n t}}(\pi x - x^2), \\ v_{1,n} &= -\frac{15t}{2n^5 e^{\omega_n t}}\big((1-\omega_n t)(\pi x - x^2) + t(\pi x - 2x^2)\big), \\ v_{2,n} &= -\frac{15t}{2n^5 e^{\omega_n t}}\big((1-\omega_n t)(\pi x - x^2) - t(\pi x - 2x^2)\big). \end{aligned}$$

So $w_n = \frac{-15t}{2n^5}(\pi x - x^2)$.

Therefore $u(t,x,y) = \frac{15t}{2}(x^2 - \pi x)\sum\limits_{n=1}^{\infty}\frac{\sin 2ny}{n^5}$ is the exact solution of the problem (10)–(11). In the Figure 4 we can see 3D-graphics of the solution in the case of $t = 0.25$ and $t = 0.5$.

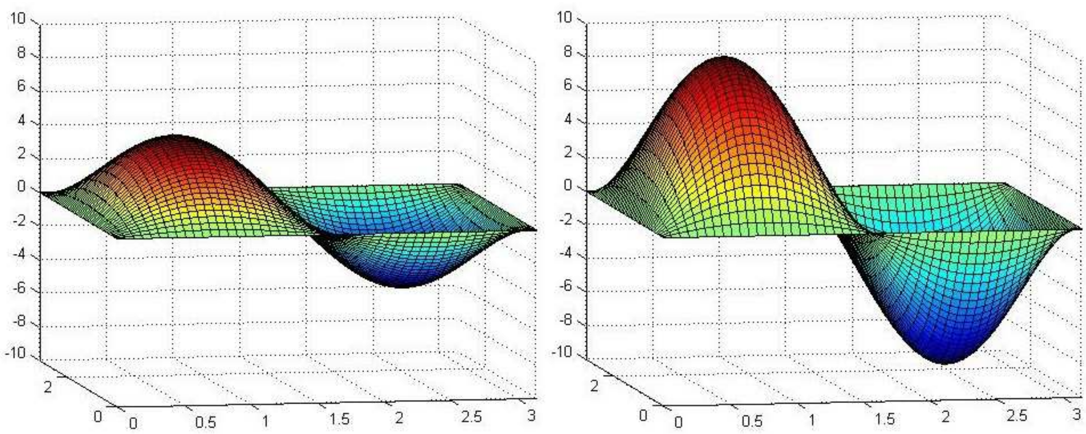

Figure 4: Graphics of solutions at $t = 0.25$ and $t = 0.5$.

Together with the problem (13)–(16), we consider truncated system

$$\begin{cases} \dfrac{\partial v_{1,n}}{\partial t} - x\dfrac{\partial v_{1,n}}{\partial x} = \sum\limits_{k=1}^{N}\sum\limits_{m=1}^{\infty} \dfrac{v_k e^{(\omega_k-\omega_n)t}}{m^2}\delta_n^{k,m} + \dfrac{f_n}{e^{\omega_n t}}, \\ \dfrac{\partial v_{2,n}}{\partial t} + x\dfrac{\partial v_{2,n}}{\partial x} = \sum\limits_{k=1}^{N}\sum\limits_{m=1}^{\infty} \dfrac{v_k e^{(\omega_k-\omega_n)t}}{m^2}\delta_n^{k,m} + \dfrac{f_n}{e^{\omega_n t}}, \\ \dfrac{\partial v_n}{\partial t} = \dfrac{v_{1,n}+v_{2,n}}{2}, \end{cases} \tag{17}$$

with the initial and the boundary conditions (14)–(16). With some suppositions [10], the solutions of the problems (17), (14)–(16) and (13)–(16) will be as close as possible.

Let v_n^N is the solution of the problem (17), (14)–(16) and $u^N(t,x,y) = \sum\limits_{n=1}^{N} w_n^N \sin 2ny$. Figure 5 shows a graph of $\dfrac{\max\limits_{t,x,y}\{|u^N(t,x,y)-u(t,x,y)|\}}{\max\limits_{t,x,y}\{|u(t,x,y)|\}}$.

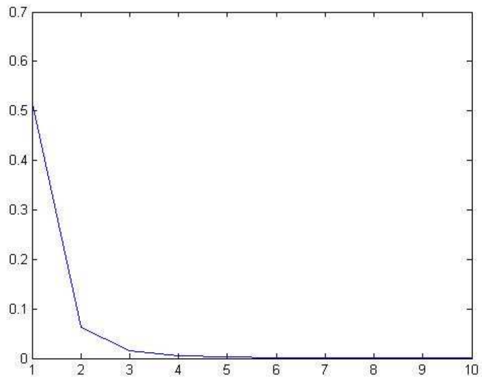

Figure 5: Dependence of difference between exact and approximate solution by N.

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У цій роботі на модельному прикладі мішаної задачі для диференціального рівняння четвертого порядку показано, як таку задачу можа звести до задачі для зліченної гіперболічної системи зв'язних рівнянь першого порядку.

Ключові слова і фрази: зліченна гіперболічна система, мішана задача.