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TOPOLOGY ON THE SPECTRUM OF THE ALGEBRA OF ENTIRE SYMMETRIC FUNCTIONS OF BOUNDED TYPE ON THE COMPLEX L_∞

It is known that the so-called elementary symmetric polynomials $R_n(x) = \int_{[0,1]}(x(t))^n\,dt$ form an algebraic basis in the algebra of all symmetric continuous polynomials on the complex Banach space L_∞, which is dense in the Fréchet algebra $H_{bs}(L_\infty)$ of all entire symmetric functions of bounded type on L_∞. Consequently, every continuous homomorphism $\varphi : H_{bs}(L_\infty) \to \mathbb{C}$ is uniquely determined by the sequence $\{\varphi(R_n)\}_{n=1}^{\infty}$. By the continuity of the homomorphism φ, the sequence $\{\sqrt[n]{|\varphi(R_n)|}\}_{n=1}^{\infty}$ is bounded. On the other hand, for every sequence $\{\xi_n\}_{n=1}^{\infty} \subset \mathbb{C}$, such that the sequence $\{\sqrt[n]{|\xi_n|}\}_{n=1}^{\infty}$ is bounded, there exists $x_\xi \in L_\infty$ such that $R_n(x_\xi) = \xi_n$ for every $n \in \mathbb{N}$. Therefore, for the point-evaluation functional δ_{x_ξ} we have $\delta_{x_\xi}(R_n) = \xi_n$ for every $n \in \mathbb{N}$. Thus, every continuous complex-valued homomorphism of $H_{bs}(L_\infty)$ is a point-evaluation functional at some point of L_∞. Note that such a point is not unique. We can consider an equivalence relation on L_∞, defined by $x \sim y \Leftrightarrow \delta_x = \delta_y$. The spectrum (the set of all continuous complex-valued homomorphisms) M_{bs} of the algebra $H_{bs}(L_\infty)$ is one-to-one with the quotient set $L_\infty/_\sim$. Consequently, M_{bs} can be endowed with the quotient topology. On the other hand, it is naturally to identify M_{bs} with the set of all sequences $\{\xi_n\}_{n=1}^{\infty} \subset \mathbb{C}$ such that the sequence $\{\sqrt[n]{|\xi_n|}\}_{n=1}^{\infty}$ is bounded.

We show that the quotient topology is Hausdorff and that M_{bs} with the operation of coordinate-wise addition of sequences forms an abelian topological group.

Key words and phrases: symmetric function, topology on the spectrum.

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INTRODUCTION

Algebras of symmetric functions on the spaces of Lebesgue-measurable functions were studied by a number of authors [1], [4], [5], [6], [7] (see also a survey [2]). In [3] the spectrum of the algebra $H_{bs}(L_\infty)$ of entire symmetric functions of bounded type on L_∞ (see definition below) is described. In this paper the topology on the spectrum of $H_{bs}(L_\infty)$ is investigated.

Let L_∞ be the complex Banach space of all Lebesgue measurable essentially bounded complex-valued functions x on $[0,1]$ with norm

$$\|x\|_\infty = \operatorname{ess\,sup}_{t\in[0,1]}|x(t)|.$$

Let Ξ be the set of all measurable bijections of $[0,1]$ that preserve the measure. A function $f : L_\infty \to \mathbb{C}$ is called symmetric if for every $x \in L_\infty$ and for every $\sigma \in \Xi$

$$f(x \circ \sigma) = f(x).$$

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Let $H_{bs}(L_\infty)$ be the Fréchet algebra of all entire symmetric functions $f : L_\infty \to \mathbb{C}$ which are bounded on bounded sets endowed with the topology of uniform convergence on bounded sets. By [3, Theorem 4.3], polynomials $R_n : L_\infty \to \mathbb{C}$, $R_n(x) = \int_{[0,1]}(x(t))^n\,dt$ for $n \in \mathbb{N}$, form an algebraic basis in the algebra of all symmetric continuous polynomials on L_∞. Since every $f \in H_{bs}(L_\infty)$ can be described by its Taylor series of continuous symmetric homogeneous polynomials, it follows that f can be uniquely represented as

$$f(x) = f(0) + \sum_{n=1}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1,\ldots,k_n} R_1^{k_1}(x)\cdots R_n^{k_n}(x).$$

Consequently, for every non-trivial continuous homomorphism $\varphi : H_{bs} \to \mathbb{C}$, taking into account $\varphi(1) = 1$, we have

$$\varphi(f) = f(0) + \sum_{n=1}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1,\ldots,k_n} \varphi(R_1)^{k_1}\cdots \varphi(R_n)^{k_n}.$$

Therefore φ is completely determined by the sequence of its values on R_n :

$$(\varphi(R_1), \varphi(R_2), \ldots).$$

By the continuity of φ, the sequence $\{\sqrt[n]{|\varphi(R_n)|}\}_{n=1}^{\infty}$ is bounded. On the other hand we have following statement.

Theorem 1 ([3]). *For every sequence $\xi = \{\xi_n\}_{n=1}^{\infty} \subset \mathbb{C}$ such that $\sup_{n\in\mathbb{N}} \sqrt[n]{|\xi_n|} < +\infty$, there exists $x_\xi \in L_\infty$ such that $R_n(x_\xi) = \xi_n$ for every $n \in \mathbb{N}$ and $\|x_\xi\|_\infty \le \frac{2}{M} \sup_{n\in\mathbb{N}} \sqrt[n]{|\xi_n|}$, where*

$$M = \prod_{n=1}^{\infty} \cos\left(\frac{\pi}{2}\frac{1}{n+1}\right). \tag{1}$$

Hence, for every sequence $\xi = \{\xi_n\}_{n=1}^{\infty}$ such that $\sup_{n\in\mathbb{N}} \sqrt[n]{|\xi_n|} < +\infty$, there exists the point-evaluation functional $\varphi = \delta_{x_\xi}$ such that $\varphi(R_n) = \xi_n$ for every $n \in \mathbb{N}$. Since every such a functional is a continuous homomorphism, it follows that the spectrum (the set of all continuous complex-valued homomorphisms) of the algebra $H_{bs}(L_\infty)$, which we denote by M_{bs}, can be identified with the set of all sequences $\xi = \{\xi_n\}_{n=1}^{\infty} \subset \mathbb{C}$ such that $\{\sqrt[n]{|\xi_n|}\}_{n=1}^{\infty}$ is bounded.

There are different approaches to the topologization of the spectra of algebras. The most common approach is to endow the spectrum by the so-called Gelfand topology (the weakest topology, in which all the functions $\widehat{f} : M_{bs} \to \mathbb{C}$, $\widehat{f}(\varphi) = \varphi(f)$, where $f \in H_{bs}(L_\infty)$, are continuous). We consider another natural topology on M_{bs}. Let $\nu : L_\infty \to M_{bs}$ be defined by

$$\nu(x) = (R_1(x), R_2(x), \ldots).$$

Let τ_∞ be the topology on L_∞, generated by $\|\cdot\|_\infty$. Let us define an equivalence relation on L_∞ by $x \sim y \Leftrightarrow \nu(x) = \nu(y)$. Let τ be the quotient topology on M_{bs} :

$$\tau = \{\nu(V) : V \in \tau_\infty\}.$$

Note that ν is a continuous open mapping. Therefore, τ contains the Gelfand topology.

In this work we show that $(M_{bs}, +, \tau)$ is an abelian topological group, where "+" is the operation of coordinate-wise addition.

1 THE MAIN RESULT

Let us denote $B(x,r)$ the open ball with center at $x \in L_\infty$ and radius $r > 0$ in L_∞.

Theorem 2. *(M_{bs},τ) is a Hausdorff topological space.*

Proof. Let $a = (a_1, a_2, \ldots), b = (b_1, b_2, \ldots) \in M_{bs}$ such that $a \neq b$. Let $m = \min\{j \in \mathbb{N} : a_j \neq b_j\}$. By Theorem 1, there exist $x_a, x_b \in L_\infty$ such that $\nu(x_a) = a$ and $\nu(x_b) = b$. Let

$$\varepsilon = \min\left\{1, \frac{|a_m - b_m|}{3m} \min\left\{\frac{1}{(\|x_a\|_\infty + 1)^{m-1}}, \frac{1}{(\|x_b\|_\infty + 1)^{m-1}}\right\}\right\}.$$

Note that $V_1 = \nu(B(x_a,\varepsilon))$ and $V_2 = \nu(B(x_b,\varepsilon))$ are neighborhoods of a and b respectively. Let us prove that V_1 and V_2 are disjoint. Let $y \in B(x_a,\varepsilon)$ and $z \in B(x_b,\varepsilon)$. Let us show that $R_m(y) \neq R_m(z)$. Note that

$$|a_m - b_m| = |R_m(x_a) - R_m(x_b)| \le |R_m(x_a) - R_m(y)| + |R_m(y) - R_m(z)| + |R_m(z) - R_m(x_b)|. \quad (2)$$

Since $\|y - x_a\|_\infty < \varepsilon$,

$$\begin{aligned}
|R_m(x_a) - R_m(y)| &\le \int_{[0,1]} |(x_a(t))^m - (y(t))^m|\, dt \\
&= \int_{[0,1]} |x_a(t) - y(t)| \left|(x_a(t))^{m-1} + (x_a(t))^{m-2}(y(t)) + \ldots + (x_a(t))(y(t))^{m-2} + (y(t))^{m-1}\right| dt \\
&\le \varepsilon \int_{[0,1]} \left(|x_a(t)|^{m-1} + |x_a(t)|^{m-2}|y(t)| + \ldots + |x_a(t)||y(t)||^{m-2} + |y(t)|^{m-1}\right) dt \\
&\le \varepsilon \int_{[0,1]} \left(\|x_a\|_\infty^{m-1} + \|x_a\|_\infty^{m-2}\|y\|_\infty + \ldots + \|x_a\|_\infty\|y\|_\infty^{m-2} + \|y\|_\infty^{m-1}\right) dt \\
&\le \varepsilon \int_{[0,1]} \left(\|x_a\|_\infty^{m-1} + \|x_a\|_\infty^{m-2}(\|x_a\|_\infty + \varepsilon) + \ldots + \|x_a\|_\infty(\|x_a\|_\infty + \varepsilon)^{m-2} + (\|x_a\|_\infty + \varepsilon)^{m-1}\right) dt \\
&\le \varepsilon m(\|x_a\|_\infty + \varepsilon)^{m-1} \le \varepsilon m(\|x_a\|_\infty + 1)^{m-1}.
\end{aligned}$$

Since $\varepsilon \le \frac{|a_m - b_m|}{3m(\|x_a\|_\infty + 1)^{m-1}}$, it follows that $|R_m(x_a) - R_m(y)| \le \dfrac{1}{3}|a_m - b_m|$. Analogously, we obtain $|R_m(z) - R_m(x_b)| \le \frac{1}{3}|a_m - b_m|$. Therefore, by (2),

$$|a_m - b_m| \le \frac{2}{3}|a_m - b_m| + |R_m(y) - R_m(z)|.$$

Hence,

$$|R_m(y) - R_m(z)| \ge \frac{1}{3}|a_m - b_m| > 0.$$

Therefore, $R_m(y) \neq R_m(z)$, and, consequently, $\nu(y) \neq \nu(z)$. Hence, V_1 and V_2 are disjoint. □

The operation of coordinate-wise addition $+ : M_{bs}^2 \to M_{bs}$ is defined by

$$a + b = (a_1 + b_1, a_2 + b_2, \ldots)$$

for $a = (a_1, a_2, \ldots), b = (b_1, b_2, \ldots) \in M_{bs}$. Note that $(M_{bs}, +)$ is an abelian group.

Theorem 3. *The operation of coordinate-wise addition $+ : M_{bs}^2 \to M_{bs}$ is continuous with respect to the topology τ.*

Proof. Let $a, b \in M_{bs}$. Let us show that for every neighborhood U of the point $a+b$ there exist neighborhoods V_a and V_b of points a and b respectively, such that $a' + b' \in U$ for every $a' \in V_a$ and $b' \in V_b$.

By Theorem 1, there exist functions $x_{4a}, x_{4b} \in L_\infty$ such that $\nu(x_{4a}) = (4a_1, 4a_2, \ldots)$ and $\nu(x_{4b}) = (4b_1, 4b_2, \ldots)$. Let

$$x_a(t) = \begin{cases} x_{4a}(4t), & \text{if } t \in [0, \frac{1}{4}], \\ 0, & \text{if } t \in (\frac{1}{4}, 1] \end{cases}$$

and

$$x_b(t) = \begin{cases} x_{4b}(4t-2), & \text{if } t \in [\frac{1}{2}, \frac{3}{4}], \\ 0, & \text{if } t \in [0, \frac{1}{2}) \cup (\frac{3}{4}, 1]. \end{cases}$$

Then $\nu(x_a) = a$ and $\nu(x_b) = b$. Note that $\nu(x_a + x_b) = \nu(x_a) + \nu(x_b)$. Hence, $\nu(x_a + x_b) = a + b$. Therefore, $x_a + x_b \in \nu^{-1}(U)$. Since the set $\nu^{-1}(U)$ is open in L_∞, it follows that there exists $\varepsilon > 0$ such that $B(x_a + x_b, \varepsilon) \subset \nu^{-1}(U)$. Let

$$r = \frac{\varepsilon}{2} \frac{M}{M+8},$$

where M is defined by (1). Let $V_a = \nu(B(x_a, r))$ and $V_b = \nu(B(x_b, r))$. Let us show that $a' + b' \in U$ for every $a' \in V_a$ and $b' \in V_b$. Let $y \in B(x_a, r)$ and $z \in B(x_b, r)$ such that $\nu(y) = a'$ and $\nu(z) = b'$. Let

$$y_1(t) = \begin{cases} y(t), & \text{if } t \in [0, \frac{1}{4}], \\ 0, & \text{if } t \in (\frac{1}{4}, 1], \end{cases} \qquad y_2(t) = \begin{cases} 0, & \text{if } t \in [0, \frac{1}{4}], \\ y(t), & \text{if } t \in (\frac{1}{4}, 1], \end{cases}$$

$$z_1(t) = \begin{cases} z(t), & \text{if } t \in [\frac{1}{2}, \frac{3}{4}], \\ 0, & \text{if } t \in [0, \frac{1}{2}) \cup (\frac{3}{4}, 1], \end{cases} \quad z_2(t) = \begin{cases} 0, & \text{if } t \in [\frac{1}{2}, \frac{3}{4}], \\ z(t), & \text{if } t \in [0, \frac{1}{2}) \cup (\frac{3}{4}, 1]. \end{cases}$$

Since $x_a(t) = 0$ for $t \in (\frac{1}{2}, 1]$ and $x_b(t) = 0$ for $t \in [0, \frac{1}{2}) \cup (\frac{3}{4}, 1]$, it follows that

$$\|y - x_a\|_\infty = \max\{\|y_1 - x_a\|_\infty, \|y_2\|_\infty\} \qquad \text{and} \qquad \|z - x_b\|_\infty = \max\{\|z_1 - x_b\|_\infty, \|z_2\|_\infty\}.$$

Since $y \in B(x_a, r)$ and $z \in B(x_b, r)$, it follows that $\|y - x_a\|_\infty < r$ and $\|z - x_b\|_\infty < r$. Consequently,

$$\|y_1 - x_a\|_\infty < r, \quad \|y_2\|_\infty < r, \quad \|z_1 - x_b\|_\infty < r \quad \text{and} \quad \|z_2\|_\infty < r.$$

By Theorem 1, for sequences $\xi = 4\nu(y_2)$ and $\eta = 4\nu(z_2)$ there exist functions $u_\xi, v_\eta \in L_\infty$ such that $\nu(u_\xi) = \xi$, $\nu(u_\eta) = \eta$, $\|u_\xi\|_\infty \le \frac{2c}{M}$ and $\|v_\eta\|_\infty \le \frac{2d}{M}$, where $c = \sup_{n\in\mathbb{N}} \sqrt[n]{|\xi_n|}$ and $d = \sup_{n\in\mathbb{N}} \sqrt[n]{|\eta_n|}$. Note that

$$|\xi_n| = |4R_n(y_2)| \le 4\|y_2\|_\infty^n < 4r^n \qquad \text{and} \qquad |\eta_n| = |4R_n(z_2)| \le 4\|z_2\|_\infty^n < 4r^n.$$

Therefore, $c, d \le \sup_{n\in\mathbb{N}} \sqrt[n]{4}r \le 4r$. Consequently, $\|u_\xi\|_\infty < \frac{8r}{M}$ and $\|v_\eta\|_\infty < \frac{8r}{M}$. Let

$$\widetilde{u}(t) = \begin{cases} 0, & \text{if } t \in [0, \frac{1}{4}] \cup [\frac{1}{2}, 1], \\ u_\xi(4t-1), & \text{if } t \in (\frac{1}{4}, \frac{1}{2}) \end{cases}$$

and

$$\widetilde{v}(t) = \begin{cases} 0, & \text{if } t \in [0, \frac{3}{4}], \\ v_\eta(4t-3), & \text{if } t \in (\frac{3}{4}, 1]. \end{cases}$$

Then

$$\nu(\widetilde{u}) = \nu(y_2) \qquad \text{and} \qquad \nu(\widetilde{v}) = \nu(z_2). \tag{3}$$

Note that $\|\widetilde{u}\|_\infty = \|u_\xi\|_\infty$ and $\|\widetilde{v}\|_\infty = \|v_\eta\|_\infty$. Let $\widetilde{y} = y_1 + \widetilde{u}$ and $\widetilde{z} = z_1 + \widetilde{v}$. Note that

$$\|\widetilde{y} - x_a\|_\infty = \max\{\|y_1 - x_a\|_\infty, \|\widetilde{u}\|_\infty\} \le \|y_1 - x_a\|_\infty + \|\widetilde{u}\|_\infty < r + \frac{8r}{M} = r\frac{M+8}{M} = \frac{\varepsilon}{2}.$$

Analogously, $\|\widetilde{z} - x_b\|_\infty < \frac{\varepsilon}{2}$. Therefore,

$$\|\widetilde{y} + \widetilde{z} - (x_a + x_b)\|_\infty \le \|\widetilde{y} - x_a\|_\infty + \|\widetilde{z} - x_b\|_\infty < \varepsilon.$$

Hence, $\widetilde{y} + \widetilde{z} \in B(x_a + x_b, \varepsilon)$. Therefore, $\nu(\widetilde{y} + \widetilde{z}) \in U$. Note that

$$\nu(\widetilde{y} + \widetilde{z}) = \nu(\widetilde{y}) + \nu(\widetilde{z}).$$

By (3),

$$\nu(\widetilde{y}) = \nu(y_1) + \nu(\widetilde{u}) = \nu(y_1) + \nu(y_2) = \nu(y) = a'$$

and

$$\nu(\widetilde{z}) = \nu(z_1) + \nu(\widetilde{v}) = \nu(z_1) + \nu(z_2) = \nu(z) = b'.$$

Therefore, $\nu(\widetilde{y} + \widetilde{z}) = a' + b'$. Hence, $a' + b' \in U$. □

Theorem 4. *The group's inverse operation $\xi \mapsto -\xi$ on $(M_{bs}, +)$ is continuous with respect to the topology τ.*

Proof. Let us prove that the inverse operation is continuous at the identity element $(0, 0, \ldots)$ of M_{bs}. Let U be a neighborhood of $(0, 0, \ldots)$. Then $\nu^{-1}(U)$ contains $0 \in L_\infty$. Since $\nu^{-1}(U)$ is open, it follows that there exists $\varepsilon > 0$ such that $B(0, \varepsilon) \subset \nu^{-1}(U)$. Let $0 < r < \frac{1}{2}M\varepsilon$, where M is defined by (1), and $V = \nu(B(0, r))$. Note that V is a neighborhood of $(0, 0, \ldots)$. Let us show that $-\xi \in U$ for every $\xi \in V$. Let $\xi = (\xi_1, \xi_2, \ldots) \in V$. Then there exists $y_\xi \in B(0, r)$ such that $\nu(y_\xi) = \xi$. Note that

$$|\xi_n| = |R_n(y_\xi)| \le \|y_\xi\|_\infty < r^n$$

for every $n \in \mathbb{N}$. Therefore,

$$\sup_{n \in \mathbb{N}} \sqrt[n]{|\xi_n|} \le r.$$

By Theorem 1, there exists $x_{-\xi} \in L_\infty$ such that $\nu(x_{-\xi}) = -\xi$ and

$$\|x_{-\xi}\|_\infty < \frac{2}{M} \sup_{n \in \mathbb{N}} \sqrt[n]{|-\xi_n|}.$$

Since

$$\sup_{n \in \mathbb{N}} \sqrt[n]{|-\xi_n|} = \sup_{n \in \mathbb{N}} \sqrt[n]{|\xi_n|} \le r$$

and $r < \frac{1}{2}M\varepsilon$, it follows that $\|x_{-\xi}\|_\infty < \varepsilon$, i.e. $x_{-\xi} \in B(0, \varepsilon)$. Therefore, $x_{-\xi} \in \nu^{-1}(U)$ and, consequently, $\nu(x_{-\xi}) \in U$, i.e. $-\xi \in U$. Hence, for every neighborhood U of $(0, 0, \ldots)$ there exists neighborhood V of $(0, 0, \ldots)$ such that $-\xi \in U$ for every $\xi \in V$. In other words, the inverse operation is continuous at $(0, 0, \ldots)$.

For $\eta \in M_{bs}$ let $f_\eta : M_{bs} \to M_{bs}$ be defined by $f_\eta : \xi \mapsto \xi + \eta$. By Theorem 3, f_η is a continuous function for every $\eta \in M_{bs}$. Let ζ be an arbitrary element of M_{bs}. By the continuity of the inverse operation at $(0, 0, \ldots)$ and by the continuity of functions $f_{-\zeta}$ and f_ζ at ζ and $(0, 0, \ldots)$ respectively, the inverse operation is continuous at ζ as a composition of continuous functions. Hence, the inverse operation is continuous at every point of M_{bs}. □

Corollary 1. *$(M_{bs}, +, \tau)$ is an abelian topological group.*

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Василишин Т.В. *Топологія на спектрі алгебри цілих симетричних функцій обмеженого типу на комплексному просторі L_∞* // Карпатські матем. публ. — 2017. — Т.9, №1. — C. 22–27.

Відомо, що так звані елементарні симетричні поліноми $R_n(x) = \int_{[0,1]}(x(t))^n\,dt$ утворюють алгебраїчний базис алгебри усіх симетричних неперервних поліномів на комплексному банаховому просторі L_∞, яка є скрізь щільною в алгебрі Фреше $H_{bs}(L_\infty)$ усіх цілих симетричних функцій обмеженого типу на L_∞. Як наслідок, кожен неперервний гомоморфізм $\varphi : H_{bs}(L_\infty) \to \mathbb{C}$ однозначно визначається послідовністю $\{\varphi(R_n)\}_{n=1}^\infty$. За неперервністю гомоморфізму φ, послідовність $\{\sqrt[n]{|\varphi(R_n)|}\}_{n=1}^\infty$ є обмеженою. З іншого боку, для кожної послідовності $\{\xi_n\}_{n=1}^\infty \subset \mathbb{C}$, такої, що послідовність $\{\sqrt[n]{|\xi_n|}\}_{n=1}^\infty$ є обмеженою, існує $x_\xi \in L_\infty$ така, що $R_n(x_\xi) = \xi_n$ для кожного $n \in \mathbb{N}$. Тому для функціонала обчислення значення в точці δ_{x_ξ} буде $\delta_{x_\xi}(R_n) = \xi_n$ для кожного $n \in \mathbb{N}$. Отже, кожен неперервний комплекснозначний гомоморфізм алгебри $H_{bs}(L_\infty)$ збігається із функціоналом обчислення значення в деякій точці простору L_∞. Зауважимо, що така точка не є єдиною. Розглянемо відношення еквівалентності на L_∞, визначене правилом $x \sim y \Leftrightarrow \delta_x = \delta_y$. Тоді спектр (множина усіх неперервних комплекснозначних гомоморфізмів) M_{bs} алгебри $H_{bs}(L_\infty)$ є у взаємно однозначній відповідності із фактор-множиною $L_\infty/_\sim$. Відповідно, на M_{bs} можна розглянути фактор-топологію. З іншого боку, природно ототожнити M_{bs} із множиною усіх послідовностей $\{\xi_n\}_{n=1}^\infty \subset \mathbb{C}$ таких, що послідовність $\{\sqrt[n]{|\xi_n|}\}_{n=1}^\infty$ є обмеженою.

У роботі показано, що фактор-топологія є гаусдорфовою і що M_{bs} з операцією покоординатного додавання послідовностей утворює абелеву топологічну групу.

Ключові слова і фрази: симетрична функція, топологія на спектрі.