Influence of polarization of conduction electrons in semiconductor on their light absorption

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Abstract. The influence of polarization of conduction electrons in semiconductor on their light absorption is examined. The Drude model is used for the description of an electron motion inside conduction band. Account of electron screening the external electric field essentially modifies the expression for the light absorption coefficient in comparison with that in the traditional theory. As a result, the absorption spectrum becomes "resonance" like.

Keywords: dielectric permittivity, polarization, optical properties, intraband optical absorption.

1. Introduction

Light absorption of free electrons within conduction band of semiconductor is used as a research method for this band and allows finding the relaxation time by momentums of conduction electrons τ [1−3]. Optical properties of substance are determined by its dielectric permittivity $\varepsilon(\omega)$ that depends on this substance conductivity. The Drude model is traditionally used to describe the electrical conductivity in the indicated frequency range ω [1−3].

The Drude model considers free electrons as electron gas whose charge is neutralized by the positive background of lattice ions. This lattice is considered as having the frequency independent dielectric constant ε_0. Moreover, the equation of motion of the electron having the mass m being under the influence of external electric field **D**, which is applied to the substance, is of the form

$$\frac{d\mathbf{v}(t)}{dt} = \frac{e}{m}\frac{\mathbf{D}(t)}{\varepsilon_0} - \frac{1}{\tau}\mathbf{v}(t). \qquad (1)$$

Here, t is time; e is an electron charge; $\mathbf{v}(t)$ is its velocity; $\tau = 1/\gamma$ is the relaxation time of the electron momentum.

The electric current density of electrons is taken in the form of

$$\mathbf{j}(t) = e n_e \mathbf{v}(t). \qquad (2)$$

Here, n is the concentration of electrons. The external alternating field $\mathbf{D}(t) = \mathbf{D}_\omega \exp(i\omega t)$, having the amplitude $\mathbf{D}_\omega$, creates the alternating current having the amplitude of electric current density $\mathbf{j}_\omega$. In accordance with the formulae (1), (2), the linear response of this current on the external field **D** is given by the expression

$$\mathbf{j}_\omega = s(\omega)\frac{1}{\varepsilon_0}\mathbf{D}_\omega. \qquad (3)$$

Where

$$s(\omega) = \frac{n_e e^2 \tau}{m(1+i\omega\tau)} = \varepsilon_0 \frac{\omega_p^2}{4\pi}\frac{1}{\gamma+i\omega} \qquad (4)$$

is the conductivity of the Drude model caused by electrons. Here, $\omega_p^2 = 4\pi e^2 n_e/(\varepsilon_0 m)$ is the square of plasma frequency for conduction electrons [4].

In view of light absorption by semiconductors, the following form of dielectric permittivity is usually used [1−3]

$$\varepsilon_S(\omega) = \varepsilon_0 + \frac{4\pi}{i\omega} s(\omega) . \qquad (5)$$

The Drude theory does not consider screening the external field by free electrons. This screening begins to appear at the time of the order of $t_p = \omega_p^{-1}$. As a result, at the light frequency $\omega \le \omega_p$ or the wavelengths $\lambda \ge \lambda_p = 2\pi c / \omega_p$, where c is the velocity of light, conduction electrons of semiconductor will be under the action of not the external in relation to them field $\mathbf{D}/\varepsilon_0$, but they will be under the action of the internal field in the system of these electrons $\mathbf{E} \ne \mathbf{D}/\varepsilon_0$.

The difference between the fields $\mathbf{E}$ and $\mathbf{D}/\varepsilon_0$ is due to the polarization vector of electric charges of substance $\mathbf{P}$ [$4-10$]. This difference is out of view both in the Drude model (1)–(4) and derivation of the expression (5). Screening the external electric field by substance charges essentially influences on optical properties of this substance [$11-15$].

The influence of mentioned polarization on the optical absorption by free electrons of semiconductor is studied in this work. The method of account of screening the external field by substance charges (i.e., the mentioned polarization) under the examination of its dielectric permittivity is given in the second section of the paper. In its third section, this method is implemented for conduction electrons of semiconductor. The feature of this method is that it uses the conductivity (4) derived in the Drude model. The coefficient of light absorption by free electrons of semiconductor, which takes into account the mentioned polarization, is studied in the fourth section.

2. Dielectric permittivity of substance in view of polarization

The electric field in substance $\mathbf{E}$ does not coincide with the external field $\mathbf{D}$, which acts on it, because of the presence of electric charges in the substance

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} , \qquad (6)$$

where

$$\mathbf{j} = \frac{d\mathbf{P}}{dt} ; \qquad (7)$$

$\mathbf{P}$ is the vector of polarization of substance; $\mathbf{j}$ is the density of electric current created by the field $\mathbf{D}$ [$4-10$]. It is noticed in §15.6 of [10] that the consideration of linear reaction of substance on the external field is not always take into account the difference between fields $\mathbf{E}$ and $\mathbf{D}$. It is the field $\mathbf{E}$ that acts on electric charges of substance.

Let us consider the external field $\mathbf{D}$ that act on the substance, in the form of the wave $\mathbf{D}_{\mathbf{k},\omega} \exp(i(\omega t - \mathbf{k}\omega))$, where $\mathbf{k}$ is its wave vector; $\mathbf{x}$ is

the spatial coordinate; $\mathbf{D}_{\mathbf{k},\omega}$ is the amplitude of wave. This field creates waves of field, current density and polarization, which amplitudes are $\mathbf{E}_{\mathbf{k},\omega}$, $\mathbf{j}_{\mathbf{k},\omega}$ and $\mathbf{P}_{\mathbf{k},\omega}$ in the substance. In accordance with the formulae (6) and (7), these amplitudes are related via the following expressions:

$$\mathbf{D}_{\mathbf{k},\omega} = \mathbf{E}_{\mathbf{k},\omega} + 4\pi\mathbf{P}_{\mathbf{k},\omega} ; \qquad (8)$$

$$\mathbf{j}_{\mathbf{k},\omega} = i\omega\mathbf{P}_{\mathbf{k},\omega} . \qquad (9)$$

The formula (8) can be presented in the form of

$$\mathbf{D}_{\mathbf{k},\omega} = \varepsilon(\mathbf{k},\omega)\mathbf{E}_{\mathbf{k},\omega} , \qquad (10)$$

where the following formula

$$\varepsilon(\mathbf{k},\omega) = 1 + 4\pi\mathbf{P}_{\mathbf{k},\omega} / \left(\mathbf{E}_{\mathbf{k},\omega}\right)$$

defines the dielectric permittivity of substance $\varepsilon(\mathbf{k},\omega)$.

It is possible to express the current density both by means of the field $\mathbf{D}$, and by means of the field $\mathbf{E}$

$$\mathbf{j}_{\mathbf{k},\omega} = \sigma(\mathbf{k},\omega)\mathbf{E}_{\mathbf{k},\omega} = s(\mathbf{k},\omega)\mathbf{D}_{\mathbf{k},\omega} . \qquad (12)$$

The conductivity s determines the current response to the external field $\mathbf{D}$, and it is given by the known formula for linear response by Kubo [4, 5, $7-10$]. It should be noted that in this section the conductivity s considers the contribution to the current of all charges, unlike the previous section where the contribution to the current of a subsystem of mobile electrons is only considered. The conductivity σ gives the current response to the internal field $\mathbf{E}$, which differs from the external field $\mathbf{D}$. Therefore, the conductivity s is named as the substance external conductivity, and σ is named as the substance internal conductivity [7, 9].

The formulae (8), (12) give that the difference between conductivities σ and s conditioned by the polarization. The formulae (9), (11), (12) give the relation

$$\varepsilon(\mathbf{k},\omega) = 1 + \frac{4\pi}{i\omega}\sigma(\mathbf{k},\omega) . \qquad (13)$$

Consequently, according to the definition of the dielectric permittivity ε, it includes the conductivity σ, but not s. Because optical properties of semiconductor are determined by ε, then to study the light absorption is necessary to know σ, but not s.

In a more general case, when the spatial dispersion is accounted, the problem to find the relation between σ and s requires the simultaneous solution of Maxwell equations and equations of motion for charges [4, 5, $7-10$]. But in the long-wave approximation $k \to 0$, it is easy to establish this relationship. Let us denote by $\sigma(\omega)$, $s(\omega)$, and $\varepsilon(\omega)$ the values $\sigma(\mathbf{k},\omega)$, $s(\mathbf{k},\omega)$, and $\varepsilon(\mathbf{k},\omega)$ in the long-wave approximation ($k \to 0$). In this case, the following simple arguments are in the work (p. 150 [10]). The substitution of (10) into (12) gives

$$\sigma(\omega) = s(\omega)\,\varepsilon(\omega)\,. \qquad (14)$$

From the formulae (13) and (14), we obtain

$$\sigma(\omega) = \frac{1}{1 - \dfrac{4\pi}{i\omega} s(\omega)}\, s(\omega)\,. \qquad (15)$$

This formula allows using the conductivity $s(\omega)$ obtained without polarization to consider optical properties of semiconductor, which are determined by the conductivity $\sigma(\omega)$. Here and below, for simplicity, the value $s(\omega)$ is assumed as scalar.

Spatial dispersion essentially complicates finding the relation between $\sigma(\omega)$ and $s(\omega)$. In this relation, it is convenient to consider the electric polarizability instead of conductivity. Polarization, as well as the current density can be expressed through the external field **D**, and through the internal field **E**

$$\mathbf{j}_{\mathbf{k},\omega} = \xi(\mathbf{k},\omega)\mathbf{E}_{\mathbf{k},\omega} = \beta(\mathbf{k},\omega)\mathbf{D}_{\mathbf{k},\omega}\,, \qquad (16)$$

where ξ is the electric polarizability of semiconductor by the field **E**, and β – by the field **D** [5]. According to the formulae (9), (12), (16) it is received

$$\sigma(\mathbf{k},\omega) = i\omega\xi(\mathbf{k},\omega)\,,$$
$$s(\mathbf{k},\omega) = i\omega\beta(\mathbf{k},\omega)\,. \qquad (17)$$

The system of Maxwell equations and equations of motion of charges give (the formula (6.14) on p. 38 [5])

$$\xi(\boldsymbol{k},\omega) =$$
$$= \left[1 + 4\pi\left(\frac{\omega}{c}\right)^2 \beta(\boldsymbol{k},\omega)\,\Omega(\boldsymbol{k},\omega)\right]^{-1} \beta(\boldsymbol{k},\omega)\,, \qquad (18)$$

where the value $\Omega(\mathbf{k},\omega)$ is of the form

$$\Omega(\mathbf{k},\omega) = \left(O\!\left(k^2\right) - \left(\frac{\omega}{c}\right)^2\right)^{-1}. \qquad (19)$$

As a result, in the long-wave approximation $k \to 0$, the relation

$$\sigma(\mathbf{k},\omega) = \frac{1}{1 - \dfrac{4\pi}{i\omega} s(\mathbf{k},\omega)\left(1 + O\!\left(k^2\right)\right)}\, s(\mathbf{k},\omega)\,. \qquad (20)$$

follows from the formulae (17) – (19).

In this approximation, the relationships (15) and (20) coincide. Hence, the more general theory that considers spatial dispersion confirms the result (15) obtained in a simple way.

3. Dielectric permittivity of semiconductor caused by conduction electrons in view of polarization

In the previous section, the conductivities $\sigma(\omega)$ and $s(\omega)$ account the contribution to current of all charges. For the case of light absorption by free electrons in the conduction band of semiconductor, these charges can be divided into two groups. First, we will include free electrons of the conduction band that are active in the optical process, and will reserve the notations $\sigma(\omega)$, $s(\omega)$ for them. Second, we will include all other charges that create the real component of dielectric permittivity of semiconductor ε_0 being independent on the light frequency.

Now, the field $\mathbf{D}(t)/\varepsilon_0$ will be the external field with respect to free electrons of the semiconductor conduction band. Here, $\mathbf{D}(t)$ is the field that acts on semiconductor. The field internal to the system of these electrons is $\mathbf{E}(t) \neq \mathbf{D}(t)/\varepsilon_0$. The difference between the fields $\mathbf{E}(t)$ and $\mathbf{D}(t)/\varepsilon_0$ is caused by the polarization vector for these electrons of semiconductor [4–9, 11–16]:

$$\mathbf{D}(t) = \varepsilon_0 \mathbf{E}(t) + 4\pi \mathbf{P}(t)\,. \qquad (21)$$

In this case, in the approximation $k \to 0$, instead of the formulae (12) (13), we have the following relations:

$$\mathbf{j}_\omega = \sigma(\omega)\mathbf{E}_\omega = s(\omega)\mathbf{D}_\omega\frac{1}{\varepsilon_0}\,, \qquad (22)$$

$$\varepsilon(\omega) = \varepsilon_0 + \frac{4\pi}{i\omega}\sigma(\omega)\,. \qquad (23)$$

Here, $\mathbf{E}_\omega$ and $\mathbf{j}_\omega$ are the amplitudes of waves of the field and current density that are created in semiconductor by the external field having the amplitude $\mathbf{D}_\omega/\varepsilon_0$, in the approximation $k \to 0$.

Taking into account the relationship $\mathbf{D}_\omega = \varepsilon(\omega)\mathbf{E}_\omega$ that determines the dielectric permittivity of semiconductor, we have the following expression from the relationship (22) instead of the formula (14)

$$\sigma(\omega) = s(\omega)\,\varepsilon(\omega)\frac{1}{\varepsilon_0}\,. \qquad (24)$$

The relationships (23) and (24) give the following formula instead of the formula (15)

$$\sigma(\omega) = \frac{1}{1 - \dfrac{4\pi}{i\,\omega\varepsilon_0} s(\omega)}\, s(\omega)\,. \qquad (25)$$

Substitution of the conductivity of conduction electrons of the Drude model (4) into the formula (25) gives the appropriate external electrical conductivity:

$$\sigma(\omega) = \varepsilon_0 \frac{\omega_p^2}{4\pi}\frac{\omega}{\gamma\omega + i\left(\omega^2 + \omega_p^2\right)}\,. \qquad (26)$$

The same result can be obtained from the equation for electron motion (1) using the following simple approximate considerations. By definition, the polarization vector of substance **P** is the density of dipole moments of charges in this substance [6, 16]:

$$\mathbf{P}(t) = \frac{1}{V} \sum_{i=1}^{N} q_i \mathbf{x}_i(t) . \tag{27}$$

Here, V is the volume of substance; N is the total number of charges in substance; q_i is the charge of $i-$th charge; $\mathbf{x}_i(t)$ is its coordinate.

In this paper, light is considered in the long-wave approximation. So, the polarization vector $\mathbf{P}$ of the system of N identical conduction electrons distributed uniformly in space can be considered approximately as follows:

$$\mathbf{P}(t) = \frac{1}{V} \sum_{i=1}^{N} e \mathbf{x}_i(t) \approx \frac{e}{V} N \mathbf{x}(t) = e\, n_e \mathbf{x}(t) . \tag{28}$$

Here, $\mathbf{x}(t)$ is the coordinate of a single electron.

Because of the relationships (21) and (28), the field $\mathbf{D}(t)$ has the form:

$$\mathbf{D}(t) = \varepsilon_0 \mathbf{E}(t) + 4\pi \mathbf{P}(t) =$$
$$= \varepsilon_0 \mathbf{E}(t) + \varepsilon_0 \frac{m}{e} \omega_p^2 \mathbf{x}(t). \tag{29}$$

Having substituted the expression (29) for the field $\mathbf{D}(t)$ into the electron motion equation (1), we obtain:

$$\frac{d\mathbf{v}(t)}{dt} = \frac{e}{m} \mathbf{E}(t) - \gamma \mathbf{v}(t) + \omega_p^2 \mathbf{x}(t) . \tag{30}$$

The electric field $\mathbf{D}(t) = \mathbf{D}_\omega \exp(i\omega t)$ having the amplitude $\mathbf{D}_\omega$ creates in the system of conduction electrons the internal electric field $\mathbf{E}(t) = \mathbf{E}_\omega \exp(i\omega t)$ having the amplitude $\mathbf{E}_\omega$; oscillation of the coordinate of one electron is $\mathbf{x}(t) = \mathbf{x}_\omega \exp(i\omega t)$ and has the amplitude $\mathbf{x}_\omega$; its velocity oscillation is $\mathbf{v}(t) = \mathbf{v}_\omega \exp(i\omega t)$ with the amplitude $\mathbf{v}_\omega$. Taking into account the relationship $\mathbf{v}(t) = (d\mathbf{x}(t))/dt$, we get $\mathbf{x}_\omega = -i\mathbf{v}_\omega/\omega$. Therefore, the equation (30) can be represented as follows:

$$\left(\gamma + i\left(\omega + \frac{\omega_p^2}{\omega} \right) \right) \mathbf{v}_\omega = \frac{e}{m} \mathbf{E}_\omega . \tag{31}$$

The equation (31) gives

$$\mathbf{v}_\omega = \frac{e}{m} \frac{\omega}{\gamma\omega + i\left(\omega^2 + \omega_p^2 \right)} \mathbf{E}_\omega . \tag{32}$$

According to the equation (32), the amplitude of electric current density $\mathbf{j}_\omega = e n_e \mathbf{v}_\omega$ can be represented by the formula $\mathbf{j}_\omega = \delta(\omega)\mathbf{E}_\omega$, in which the quantity $\delta(\omega)$ is given by the following relation:

$$\delta(\omega) = \varepsilon_0 \frac{\omega_p^2}{4\pi} \frac{\omega}{\gamma\omega + i\left(\omega^2 + \omega_p^2 \right)} . \tag{33}$$

The comparison of the formulae (26) and (33) indicates that $\delta(\omega) = \sigma(\omega)$.

In view of the formula (25), the expression (23) takes the following form:

$$\varepsilon(\omega) = \varepsilon_0 \frac{1}{1 - \dfrac{4\pi}{i\omega\varepsilon_0} s(\omega)} . \tag{34}$$

This expression essentially differs from the expression (5), in which the influence of polarization is not considered.

Having substituted the formula (4) into the formulae (5) and (34), we get

$$\varepsilon_s(\omega) = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega(\omega - i\gamma)} \right) . \tag{35}$$

– the dielectric permittivity of semiconductor of the Drude model, which does not consider the polarization of conduction electrons;

$$\varepsilon(\omega) = \varepsilon_0 \frac{1}{1 + \dfrac{\omega_p^2}{\omega(\omega - i\gamma)}} \tag{36}$$

– the dielectric permittivity of semiconductor that uses the Drude model and takes into account the polarization of conduction electrons.

Having expanded the right side of the formula (36) into a series by powers of $(\omega_p/\omega)^2$, we verify the validity of the following relation

$$\varepsilon(\omega) = \varepsilon_s(\omega) + O\left(\left(\frac{\omega_p}{\omega} \right)^4 \right) . \tag{37}$$

As a result, the dielectric permittivity (35), which ignores the effect of polarization, can be applied only in approximation of large frequencies $\omega^4 \gg \omega_p^4$.

4. Absorption of light by free electrons

The light absorption coefficient of substance $\alpha(\omega)$ is given by the expression [1]

$$\alpha(\omega) = \frac{2\omega\chi(\omega)}{c} = \frac{\omega}{c} \frac{\varepsilon_2(\omega)}{n(\omega)} . \tag{38}$$

Here,

$$n(\omega) = \sqrt{\frac{1}{2} \left(\varepsilon_1(\omega) + \sqrt{\varepsilon_1^2(\omega) + \varepsilon_2^2(\omega)} \right)}$$

– the refractive index of substance;

$$\chi(\omega) = \sqrt{\frac{1}{2} \left(-\varepsilon_1(\omega) + \sqrt{\varepsilon_1^2(\omega) + \varepsilon_2^2(\omega)} \right)}$$

– the absorption index of substance; $\varepsilon_1(\omega) = \mathrm{Re}\,\varepsilon(\omega)$; $\varepsilon_2(\omega) = -\mathrm{Im}\,\varepsilon(\omega)$.

Let us designate $\alpha_s(\omega)$ as the light absorption coefficient in the Drude model, which is obtained using the dielectric permittivity $\varepsilon_s(\omega)$ (35) instead of the dielectric permittivity $\varepsilon(\omega)$ (36) in the formula (38).

Within the region of weak light absorption, when there is the condition $n^2(\omega) \gg \chi^2(\omega)$, it can be assumed $n = \sqrt{\varepsilon_0}$ [1]. In this case, in the approximation of $\omega^2\tau^2 \gg 1$ (which is usually well executed), $\alpha_s(\omega)$ has the following form [1]

$$\alpha_s(\omega) = \frac{1}{L}\left(\frac{\omega_p}{\omega}\right)^2, \qquad (39)$$

where $L = c\tau/\sqrt{\varepsilon_0}$. This expression is obtained from the formula (38) using the value (35) that is given by the Drude model as the dielectric constant. Nevertheless, the polarization is not taken into account.

Let us apply the dielectric permittivity $\varepsilon(\omega)$ (36) to calculate the light absorption coefficient. Derivation of the formula (36) also uses the Drude model but with account of polarization.

Let us represent the formula (38) as follows:

$$\alpha(\omega) = $$
$$= \frac{\omega}{c}\,\frac{\varepsilon_2(\omega)}{\sqrt{\varepsilon_1(\omega)}}\,\frac{\sqrt{2}}{\sqrt{1+\sqrt{1+(\varepsilon_2(\omega)/\varepsilon_1(\omega))^2}}}. \qquad (40)$$

The formula (36) for the dielectric permittivity of semiconductor $\varepsilon(\omega)$, which uses the Drude model and allows for the polarization of conduction electrons, gives

$$\varepsilon_1(\omega) = \varepsilon_0\,\frac{\gamma^2 + \omega^2\left(1 + \dfrac{\omega_p^2}{\omega^2}\right)}{\gamma^2 + \omega^2\left(1 + \dfrac{\omega_p^2}{\omega^2}\right)^2}, \qquad (41)$$

$$\varepsilon_2(\omega) = \varepsilon_0\,\frac{\omega_p^2}{\omega}\,\frac{\gamma}{\gamma^2 + \omega^2\left(1 + \left(\dfrac{\omega_p}{\omega}\right)^2\right)^2}.$$

In the approximation $\omega^2\tau^2 \gg 1$, we have from the formulae (41)

$$\varepsilon_1(\omega) = \varepsilon_0\,\frac{1}{1 + \left(\dfrac{\omega_p}{\omega}\right)^2}, \qquad (42)$$

$$\varepsilon_2(\omega) = \varepsilon_0\left(\frac{\omega_p}{\omega}\right)^2\,\frac{1}{\omega\tau\left(1 + \left(\dfrac{\omega_p}{\omega}\right)^2\right)^2}. \qquad (43)$$

According to the formula (39), we have $(\omega_p/\omega)^2 = L\alpha_s(\omega)$. Therefore, using the formulae (40), (42), (43), we obtain the light absorption coefficient in the form of

$$\alpha(\omega) = \alpha_s(\omega)\,\frac{1}{(1 + L\alpha_s(\omega))^{3/2}}\,\eta(\omega). \qquad (44)$$

Where

$$\eta^2(\omega) = $$
$$= 2\left\{1 + \left[1 + \frac{(L\alpha_s(\omega))^2}{\omega^2\tau^2(1 + L\alpha_s(\omega))^2}\right]^{1/2}\right\}^{-1}. \qquad (45)$$

The formula (45) gives that it can be assumed $\eta(\omega) = 1$ in the approximation of $\omega^2\tau^2 \gg 1$. As the value $\alpha_s(\omega)$ can be represented in the following form

$$\alpha_s(\omega) = \frac{1}{L}\left(\frac{\lambda}{\lambda_p}\right)^2, \qquad (46)$$

the light absorption coefficient (44) is given by the expression

$$\alpha = \frac{1}{L}\left(\frac{\lambda}{\lambda_p}\right)^2\,\frac{1}{\left(1 + \left(\left(\dfrac{\lambda}{\lambda_p}\right)^2\right)\right)^{3/2}}. \qquad (47)$$

The formula (47) demonstrate that the light absorption coefficient α coincides with the coefficient $\alpha_s(\omega)$ (46), which is obtained without considering the polarization, only at sufficiently short light wavelengths λ, for which there is the inequality $\lambda^2 \ll \lambda_p^2$. With increasing the value λ, when this inequality is no longer true, the light absorption coefficient is no longer given by the expression (46).

It is convenient to represent the light absorption coefficient (47) as

$$\alpha = \frac{1}{L}B(z), \qquad (48)$$

where

$$B(z) = \frac{z^2}{(1 + z^2)^{3/2}} \qquad (49)$$

is a dimensionless quantity and $z = \lambda/\lambda_p$. The function $B(z)$ has the maximum at the value of z, which is equal
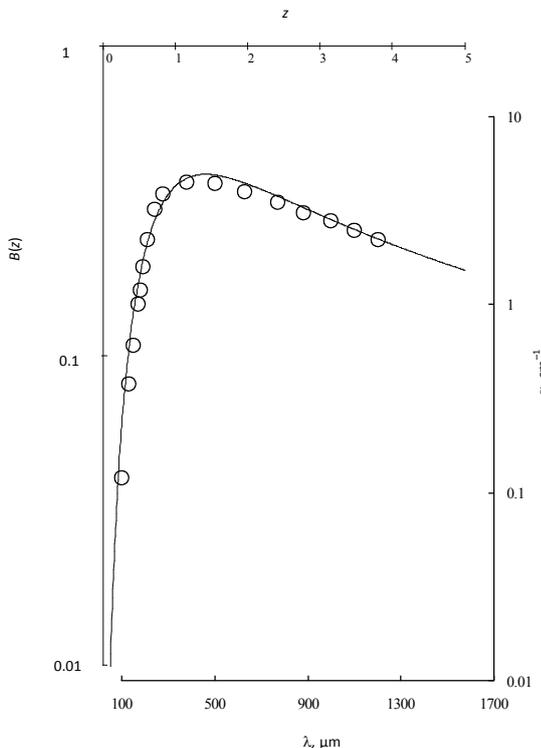
to $z_{max} = \sqrt{2}$. In the considered case, the value L does not depend on λ, and the whole dependence of the light absorption coefficient α on λ is given by the function $B(z)$.

5. Discussion

This light absorption predicted by the formulae (48) and (49) is observed experimentally in *n*-type silicon at the temperature 2.5 K [17, 18]. However, these experimental results were interpreted as those, which are caused by electrons jumps over impurity centers under the action of light [17, 18]. Such interpretation is not based on sequential calculations.

In the figure, the form of function $B = B(z)$ is represented by the solid line, and the experimental results for the dependence of the light absorption coefficient on the light wavelength λ in *n*-type silicon at the temperature 2.5 K [17, 18] are given by circles (in the form given in Fig. 10.3 [17]). It is seen that the dependence of the coefficient α on λ and the dependence $B = B(z)$ are identical within the range $100 < \lambda < 1200$ μm.

The experiment gives the shift of the maximum value of α toward shorter wavelengths with increasing N_d [18]. But the accuracy of these experimental results is insufficient to determine the dependence of this shift on N_d. It follows from the figure that should be $L = 0.1$ cm in order to obtain agreement between calculated and experimental values. If we take $\varepsilon_0 = 11.8$; then the value of L gives $\tau = 10^{-11}$ s.

This article uses results obtained by the simple Drude model having the frequency independent relaxation time τ. More rigorous methods (kinetic equation, quantum mechanical calculation) prove that this approximate model can be used under the following two conditions [1, 2]. First, the electron momentum relaxation time τ is independent on its energy. Second, it must be $\hbar\omega << E_F$ (for degenerate electrons), where E_F is the Fermi energy of electrons. In the considered experiment, semiconductor has a great concentration of donors, and it is cooled to very low temperatures. In this case, degenerate conduction electrons are mainly scattered on neutral donors, and the relaxation time τ is independent of their energy [3]. So, the first of these conditions is satisfied. The second condition has the form $\lambda >> ch/E_F$. If we set for evaluating $E_F = 0.05$ eV, it gives $\lambda >> 22$ μm. Wavelengths used in the above-mentioned experiment satisfy this condition. Therefore, the second condition is satisfied, too.

In addition, used in this paper approximation is $\omega^2\tau^2 >> 1$, which has the following form $\lambda << 2\pi L\sqrt{\varepsilon_0}$. When using $\varepsilon_0 = 11.8$ and $L = 0.1$ cm, the latter condition gives the inequality $\lambda << 2.1$ cm that is performed in the discussed experiment. Thus, the use of the simple Drude model is justified.

Because of substance polarization, the electric field that acts on its electrical charges is different from the external electric field. The known methods by Clausius-Mossotti, Lorentz-Lorenz to account for this polarization in the dielectric permittivity [2, 5, 19] essentially use the model of spatial localization of electric charges and can not be applied to the free conduction electrons, which are not localized in space.

The method offered in this paper takes polarization into account and can be applied to free conduction electrons, which enables to use the results of Drude's model. Its application to the light absorption by conduction electrons gives the dependence of light absorption coefficient on the light wavelength, which differs significantly from results of the theory, which does not take into account the effect of polarization. This difference is small at low values of the wavelength λ, which satisfy the condition $\lambda^2 << \lambda_p^2$, but it becomes essential when this inequality is violated.

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