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FORCEFUL INTERACTION OF THE CASING STRING WITH THE WALLS OF A CURVILINEAR WELL

Purpose. Developing a method for determining the axial forces and the walls reactions along a casing string, which bend it and make it follow a curved wellbore shape.

Methodology. The casing string is represented as a long elastic rod in the curved well. An inhomogeneous system of four differential equations is developed to describe the rod's deformations. It was reduced to a first-order differential equation with respect to axial force. Its solution was found by the Bernoulli method. The numerical integration of the differential equation is applied.

Findings. The axial force distribution along the casing string was found, taking into account the well curvature and the friction, as well as the reaction forces of the well walls. A method of the table's numerical integration of the well's inclinometric measurements has been developed. The calculating formulas for the reaction forces, axial forces, bending moments and stresses acting in the casing pipes in the well deep are obtained.

Originality. The solved problem takes into account the wall's reaction and the friction forces that create longitudinal bend during the column's movement. The system of differential equations of equilibrium was supplemented by Euler's kinematic equation. The function of zenith angle, which is known due to the table of the directional survey data, was taken as the integration variable. The inverse problem is solved – all unknown internal forces, also such the external distributed reaction, which causes the column to repeat the well's shape, was been determined by the angular deformations of casing string, which are given by the well's shape in the inclinometric table.

Practical value. The developed method allows detecting the areas with a significant local increase in the well's curvature, which indicate their obstructed passability. This allows for accurate determination of depth intervals to increase the borehole diameter, which is necessary before lowering the column. According to the analysis results, it is possible to determine the parameters of the stress-strain state of the casing string, which can be used to predict its working capacity and operating life.

Keywords: *casing string, curved well, inclinometric measurement, wall reaction*

Introduction. The technology of reliable and safe extraction of oil and gas from large depths requires the borehole wall lining by a string of casing. Modern methods of directional and horizontal drilling allow reaching productive layers at a depth of 4–6 km with a string length of 5–8 km, while steel pipes have a diameter of only 140–168 mm with a wall thickness of 10–12 mm. The main production casing, which connects the wellhead with deposits of hydrocarbons, must be continuous, strong and pressure-tight.

A typical well program includes a vertical section, one or more inclined sections (which provide the necessary large deviation from the wellhead) and a vertical bottom-hole section. The straight-line areas are interconnected by transition curved ones, which are described by the circle arc of a constant radius. When being designed, all wells are usually located in one vertical plane.

During drilling there are deviations from the well design profile, which are continuously corrected by technical and technological means. As a result, the area of the real well is not exactly straight or circular arc, but contains local distortions and deviations from the given shape. To establish the real profile of the drilled well, its logging is conducted, during which zenith angle ϑ between the tangent to the curved axis of the well and the vertical is measured. According to the data obtained, an inclinometric table is compiled demonstrating the table dependence of the angle $\vartheta(s)$ on the coordinate s, which is the distance from the earth surface along the curved axis of the well to the specified intersection. Measurements are carried out from the wellhead to the bottom with a certain pitch Δs.

The production casing, lowered into the curved well, enters into force interaction with the wells due to the rigidity of the pipes. Due to the reaction of the walls, the string bends, resembling the profile of the well. As a result, in the pipes body there emerges a complex stress-strain state caused by their bending and axial tension, which greatly affects the reliability and durability of the casing.

Literature review. There is a wide variety of literature on the mechanical deformation of long casing columns in wells. Many researchers have dealt with various aspects of pipe bending in directional wellbores.

In his monograph "Flexible Bars" (1962), R. Frisch-Fay found the solutions of several problems concerning long curved bars of uniform cross-section and uniform loaded cantilever beams. He also studied the uniformly distributed loading by applying the series for a cantilever with one free edge only for vertical and horizontal fixing. His principle of elastic similarity explains how to cope with excessive bends of a flexible bar freely horizontal placed on two supports.

A purpose-built finite-element model is applied by McSpadden, Coker, et al. [1] to simulate radial displacement of a casing string constrained within an outer wellbore. This represents a fully stiff-string model wherein the casing is approximated by general beam elements with 6 degrees of freedom at each node to account for all possible physical displacements and rotations. Results predicted include deflection of the casing centerline from the wellbore centerline, effective dogleg curvature, bending deformation, wall contact forces, and bending stress magnification. But in typical casing and tubular stress design, the proposed "soft-string" model assumes casing strings are coincident with the wellbore centerline. The known or assumed wellbore curvature is applied directly to the casing string. Any effect of casing string stiffness and allowable radial displacement within the outer wellbore is ignored. Likewise, the impact of bending stress magnification is typically ignored along with the effects of centralizer placement.

Fei Yin and Deli Gao [2] performed a mechanical analysis and design of the casing in a directional well under in-situ stresses. They found that the casing in directional well under the action of inelastic surrounding rock displays a complex mechanical state. Taking the in-situ stresses and well trajectory into account, the mechanical model of casing in directional well under in-situ stresses is established. The mechanical interaction of casing and surrounding rock is simplified to a generalized plane strain problem. To analyze the casing behavior, the complicated solution is divided into three simpler problems: the elastic mechanics analyses under normal stress, torsion stress and shear stress respectively. The analytical expressions of casing stress and load in directional well under in-situ stresses and inside hydrostatic pressure are deduced. This analytical solution is verified by numerical simulation. Furthermore, casing design for the directional well through complex formation in an oilfield is conducted.

The article [3] is devoted to the methods for calculating the axial load in the columns of rods, drill pipes, casings and tubing strings. Results of the analysis of solution of the tasks to determine the loss of axial load along the length of different types of columns based on their longitudinal and transverse strain in directional wells are presented. It is shown that the neglect of the magnitudes of transverse loads and shear forces leads to an error of calculation of loss of axial load on the friction along the length of the oil field column at its bottom spirally deformed area. The irrationality of such methods of calculation for directional and horizontal wells is confirmed.

Ai Chi, Yu Fahao, et al. [4] proposed a model for calculating the axial force, and presented a new numerical method for solving it by considering deflection and buckling of casing string, as well as centralizer and washover tubing contacting with well wall. For estimating whether casing string will be cut off after washover head contacts with casing, it is necessary to calculate the axial force. During extracting casing in different well sections, based on calculating results, selecting a reasonable bit weight can prevent casing string from being cut off. As the axial force is known, the bending deflection of washover tubing between two centralizers can be calculated accurately. According to calculating results, we can calculate the proper centralizer spacing as the theoretical basis of the reasonable design of pipe strings for extracting casing in directional well.

The problem about identification of elastic bending deformation of a drill string in curve wells based on the theory of flexible curved rods and the direct inverse problems of drill string bending in the channels of curvilinear bore-holes is stated in the paper [5]. The problem is solved which determines the resistance forces and moments during performing ascending-descending operations in curvilinear bore-holes with trajectories of the second order curve shapes. The sensitivity of the resistance forces relative to geometric parameters of the bore-hole axial line trajectories is analyzed.

The soft string and stiff string models are different casing string methods that have been used by the oil and gas industry to calculate torque and drag. Zhang and Samuel [6] discussed the intrinsic difference between these two models and proposed a criterion for determining which method would deliver the most accurate results. The results demonstrate that borehole tortuosity and the shape of the wellbore can significantly change the status of the string. A string with a large-size section can be soft in a straight wellbore. Likewise, a string with small-size section can be stiff in a wellbore with severe tortuosity. To accurately estimate the drag force, the stiffness, as well as the wellbore shape and its clearance, should be considered. Extensive simulations have been performed and are reviewed in this paper. The results confirm that the soft string model is a better choice when the string is slimmer, the wellbore is in a lower curvature shape, and the clearance is larger. On the contrary, the stiff string model is more useful when the string is stronger, the wellbore is in a high curvature shape, and the clearance is lower. When to use the models depends on the bending shapes of the string in the wellbore. Since neither model is equipped to handle all scenarios, combining the two methods provides better results.

In the article [7], the authors provided digital modeling of the casing process in a directional well. The casing string is subjected to great bending stress and high drag in the curved section of directional well, which may lead to strength failure, seal failure, stability failure and be hindered. Based on the finite element method, the influences of wellbore curvature, friction coefficient, running velocity and different materials on the casing strings running, are simulated dynamically in the curved section of directional well. The numerical results show that the Mises equivalent stress on the casing strings increases obviously after being run on curved section, and the maximum equivalent stress occurs at the bottom of the casing strings when increasing the running velocity. With the increase in wellbore curvature, the drag of casing strings running increases nonlinearly, but the smoother the wellbore wall is, the smaller the friction is. The greater the rigidity of the casing is, the greater the drag in the curved section and the more difficult it is to casing strings running.

Kryzhanivskyi, et al. [8] applied the long rod theory, which receives large nonlinear deformations under the action of its weight, to describe the elastic mechanics of a casing in a deep curved well. It is proved that the elastic rod deformation under the impact of the longitudinal and transverse forces can be calculated by a heterogeneous second order differential equation with variable coefficients. The solution of the equation of long rod angular deformations was found in the form of a linear combination of Airy and Scorer's functions and in the form of three linearly independent polynomial series in the sum with a partial answer. This solution was the clue to the formulas of deflections, angular slopes, internal bending moments and transverse forces in the column with the arbitrary arrangement of supports and boundary conditions in their intersections. But this task did not take into account the reaction of the well walls and the friction forces acting on the casing string during its movement.

The problem of determining the forces acting on the string of pipes in the well when its shape is given is considered in Yu. Pesliak's work "Calculation of stresses in columns of pipes of oil wells" (1973). To solve it, a system of G. Kirchhoff equations describing the spatial deviations of a long elastic rod, which has a finite bending stiffness, is used, and its solution in a vector form is carried out. However, the results in scalar form suitable for engineering calculations are obtained only for the case of a well section, which is curved along a circular arc of a constant radius in one plane, and for a well case, which is presented by a helix with a constant zenith angle and a constant rate of change of the azimuthal angle. In order to determine the axial forces and frictional forces in the case of random deviation of the well, numerical integration is applied on an example of a well with a constant speed of change of the zenith and azimuthal angle to their maximum value of 90°.

The problem of the advance and bending of the string of pipes in the deviated borehole is considered in the monograph "Calculations of boundary states of pipe columns and pipelines" (1997) by P. Vyslobitskyi. For its solution a geometric approach is used to study the force interaction of pipes with well walls. At the same time, the pipe was graphically inscribed into well deviated along the circular arc following several possible, according to the author, schemes of placement of contact points with its walls, in which reactions and frictional forces may occur. The acting forces were determined by the equilibrium equations of the pipe sections between these contact points and the pipe deformation equations, for which the formulas of the small deformations of the cantilever beam were unjustifiably applied. With the general formulation of the problem of bending and casing string drift in the deviated wellbore, a system of differential equations was proposed. The system had to express the bent state of the string, but it does not contain two equations of equilibrium of internal and external forces projections, and therefore it is incomplete and cannot be solved.

Thus, **the unresolved problem** is obtaining a closed system of differential equations, which describes the deformation of the string of pipes in a deviated well under their own weight and the reactions of the walls. The solutions of this problem will determine the distribution of axial forces, bending moments and stresses in the body of the column.

A long casing string in the well behaves like an elastic, solid rod [8], which has sufficient bending stiffness. It is influenced by a vertical weight j, uniformly distributed along the length, which creates variable axial forces in the body of the column. The column of initially rectilinear pipes in the curvilinear well forcibly receives the geometric shape of its curved axis. This is due to the reaction forces of the well walls, which, together with the weight, act on the column and bend it.

In the first approximation, we consider that the column contacts the well walls along its entire length (we neglect small gaps between the wall and the pipe in comparison with large geometric deviations of the axis from the rectilinear form). Consequently, along with the distributed weight j, a long elastic rod is influenced by the reaction of the walls $f(s)$, distributed by its length according to a certain law, as a result of which it acquires a given form. We assume that the distributed load $f(s)$ is directed along the normal to the curvilinear axis of the rod and is positive if its projection to the horizontal has a positive direction.

The purpose of the work is to develop a method for determining the distribution of axial forces and the reactions of the walls, which, together with their own weight, act on a casing string and make it follow a given wellbore shape. To do this, it is necessary to develop and integrate the system of differential equations of equilibrium of a long elastic rod bent in one plane. According to the obtained results, it is necessary to find expressions of force parameters that describe the stress-deformed state of the casing in a deviated well.

The method to obtaining the main system of differential equations and its first integral. The analysis showed that large elastic deformations of the long rod of the unit-value stiffness can be considered for the bend, without losing the universality of the solution [8]. At the same time, the bending moment is numerically equal to the curvature of the rod, and the current force factors differ in size from the estimated ones by the factor EJ (E is the elastic modulus of the material, J is the moment of inertia of the cross-section of the rod).

Let us consider the arc element – the segment of the curved axis of the rod with length ds, at the beginning of which the tangent line is inclined to the vertical under zenith angle ϑ (Fig. 1). In this section, the internal axial t and transverse u forces as well as bending moment q are applied. In the final crosscut of the element, which received an increase in zenith angle $d\vartheta$, the same forces are applied, but with increments dt, du, dq correspondingly, whose direction must balance the initial ones.

The element is also affected by external forces: its weight $j \cdot ds$, the reaction of the wall $f \cdot ds$ and the friction force $k_t \cdot f \cdot ds$, directed along the axis of the string against its motion, where k_t is the coefficient of friction. In the equations of equilibrium, discussed below, the sign of friction corresponds to the descent of the string in the well. For the case of lifting a column in the

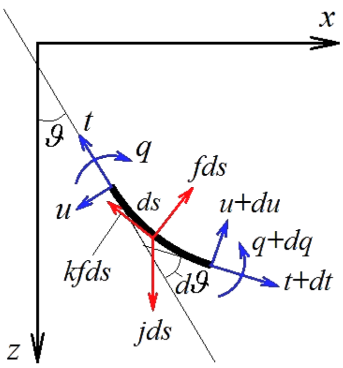

Fig. 1. The scheme of loading of an elastic bent element of a casing string in a curvilinear well

equations and their solutions, the coefficient of friction should be taken with the opposite sign.

The following system of differential equations is obtained from the equilibrium conditions of the bent rod element (Fig. 1)

$$\frac{du}{ds} + t \cdot \frac{d\vartheta}{ds} - j\sin\vartheta + f = 0; \quad (1)$$

$$\frac{dt}{ds} - u \cdot \frac{d\vartheta}{ds} + j\cos\vartheta - k_t f = 0; \quad (2)$$

$$\frac{dq}{ds} + u = 0. \quad (3)$$

Differential (1) was found from the force projections upon the normal, and (2) – upon the tangent (Fig. 1). Equation (3) is based on the equilibrium of boundary moments and moments of edge forces.

A similar system describing the bending deformation of a long elastic rod in one plane was obtained in the work by R. Frisch-Fay. However, it did not take into account the reaction of the wall and friction, and was incomplete. In order for the system to have a solution, the fourth equation is required, which is the kinematic Euler equation. It establishes the connection between the angular deformation of the rod and its curvature q (bending moment)

$$q = \vartheta' = \frac{d\vartheta}{ds} = \frac{1}{R}, \quad (4)$$

where R is the local radius of curvature; the dot denotes the derivative of s.

Due to this, the system of differential equations becomes closed and has the only solution.

Thus, the deformations of casing string, bent due to the deviation of the borehole, are described by a non-uniform system of four differential (1–4). As we see, this system contains three unknown functions t, u and q (which are internal force factors) and an unknown function of the distributed reaction f (which is an external load). The function ϑ is known due to the inclinometric table of well measurement. It is necessary to solve the inverse problem – having the known load j and the deformations ϑ given by the shape of the well, it is necessary to determine the unknown internal distributed forces t, u, q and such an external load function f, which creates a given shape of the rod.

Let us substitute (3 and 4) into (1 and 2) and reduce the system (1–4) to two equations

$$t \cdot \vartheta' - \vartheta''' = j\sin\vartheta - f; \quad (5)$$

$$t' + \vartheta'' \cdot \vartheta' = -j\cos\vartheta + k_t f. \quad (6)$$

The system of differential (5–6), where the zenith function ϑ is known, contains two unknown functions t and f. According to (5), we have

$$f = j\sin\vartheta - t \cdot \vartheta' + \vartheta'''. \quad (7)$$

Thus, the problem of determining the distributed reaction of walls f requires the finding of the axial force t. To do this, let us multiply (6) by k_T, add it to (5) to exclude the function f from the system (5–6)

$$t' + k_t\vartheta' \cdot t = k_t\vartheta''' - \vartheta' \cdot \vartheta'' + k_t j\sin\vartheta - j\cos\vartheta. \quad (8)$$

The resulting differential first-order equation is linear, inhomogeneous and, in general, with variable coefficients. We will study and solve the basic differential equation (8).

Application of the Bernoulli method to integrate the differential equation of axial force. Let us mark the right part (8) in the following way

$$\varphi = k_t\vartheta''' - \vartheta' \cdot \vartheta'' + k_t j\sin\vartheta - j\cos\vartheta. \quad (9)$$

Integration (8) is carried out provided that the coefficient of resistance of the string motion within a single area is constant: $k_t = \text{const}$, $0 < k_t < 1$. Let us solve the Cauchy problem provided that in the established intersection with the coordinate $s = L$, where $z = Z$, $\vartheta = \Theta$ and axial force $t(L) = t_Z$ is applied.

The general solution of the inhomogeneous equation (8) is sought by the Bernoulli method [9] in the form of a product of two functions: $t = v \cdot w$; its substitution in (8) gives

$$v' \cdot w + v \cdot w' + k_t\vartheta' \cdot v \cdot w = \varphi;$$
$$(v' + k_t\vartheta' \cdot v) \cdot w + v \cdot w' = \varphi. \qquad (10)$$

Since we are looking for one function t, then one of the two product functions can be arbitrary. Let us choose v so that it satisfies the homogeneous equation, formed from the expression in brackets (10) and solution of which we can find by separating the variables [9]

$$v' + k_t\vartheta' \cdot v = 0; \qquad (11)$$

$$v = e^{k_t(\Theta-\vartheta)}. \qquad (12)$$

Now let us substitute expressions (9, 11 and 12) into (10) and integrate the resulting differential equation

$$e^{k_t\Theta}(w-c) = k_t\int_L^s e^{k_t\vartheta}\vartheta'''ds - \int_L^s e^{k_t\vartheta}\vartheta'\vartheta''ds + +j\int_L^s e^{k_t\vartheta}(k_t\sin\vartheta - \cos\vartheta)ds, \qquad (13)$$

where c is the constant of integration.

The first of the integrals contained in (13) is found by integrating the parts

$$\int_L^s e^{k_t\vartheta}\vartheta'''ds = \int_L^s e^{k_t\vartheta}d(\vartheta'') = e^{k_t\vartheta}\vartheta'' - e^{k_t\Theta}\vartheta''_\Theta - k_t\int_L^s e^{k_t\vartheta}\vartheta'\vartheta''ds,$$

where $\vartheta'_\Theta = \vartheta'(\Theta)$ is the value of the derived function in the intersection, where $\vartheta = \Theta$.

Now the first and second integrals of (13) can be combined

$$k_t\int_L^s e^{k_t\vartheta}\vartheta'''ds - \int_L^s e^{k_t\vartheta}\vartheta'\vartheta''ds =$$
$$= k_te^{k_t\Theta}(e^{k_t(\vartheta-\Theta)}\vartheta'' - \vartheta''_\Theta) - (1+k_t^2)\int_L^s e^{k_t\vartheta}\vartheta'\vartheta''ds.$$

The second of integrals (13) is also integrated by parts

$$\int_L^s e^{k_t\vartheta}\vartheta'\vartheta''ds = \frac{1}{2}\int_L^s e^{k_t\vartheta}d(\vartheta'^2) =$$
$$= \frac{e^{k_t\Theta}}{2}\left(e^{k_t(\vartheta-\Theta)}\vartheta'^2 - \vartheta'^2_\Theta\right) - \frac{k_t}{2}\int_L^s e^{k_t\vartheta}\vartheta'^3ds.$$

Substituting the resulting integrals in (13), we obtain a function w

$$w = k_t(e^{k_t(\vartheta-\Theta)}\vartheta'' - \vartheta''_\Theta) - -\frac{1+k_t^2}{2}\left(\left(e^{k_t(\vartheta-\Theta)}\vartheta'^2 - \vartheta'^2_\Theta\right) - \frac{k_t}{e^{k_t\Theta}}\int_L^s e^{k_t\vartheta}\vartheta'^3ds\right) + +\frac{j}{e^{k_t\Theta}}\int_L^s e^{k_t\vartheta}(k_t\sin\vartheta - \cos\vartheta)ds + c. \qquad (14)$$

The product of functions (12 and 14) gives a function t

$$t = e^{k_t(\Theta-\vartheta)}w = k_t\left(\vartheta'' - e^{k_t(\Theta-\vartheta)}\vartheta''_\Theta\right) - -\frac{1+k_t^2}{2}\left(\left(\vartheta'^2 - e^{k_t(\Theta-\vartheta)}\vartheta'^2_\Theta\right) - \frac{k_t}{e^{k_t\vartheta}}\int_L^s e^{k_t\vartheta}\vartheta'^3ds\right) + +\frac{j}{e^{k_t\vartheta}}\int_L^s e^{k_t\vartheta}(k_t\sin\vartheta - \cos\vartheta)ds + ce^{k_t(\Theta-\vartheta)}.$$

Under the conditions of the Cauchy problem, we get $c = t_Z$. Consequently, the distribution of the axial force in the body of the column, taking into account the deviation of the well and friction on its walls, has the form

$$t = k_t\left(\vartheta'' - \vartheta''_\Theta e^{k_t(\Theta-\vartheta)}\right) - -\frac{1+k_t^2}{2}\left(\left(\vartheta'^2 - \vartheta'^2_\Theta e^{k_t(\Theta-\vartheta)}\right) + \frac{k_t}{e^{k_t\vartheta}}\int_s^L e^{k_t\vartheta}\vartheta'^3ds\right) - -\frac{j}{e^{k_t\vartheta}}\int_s^L e^{k_t\vartheta}(k_t\sin\vartheta - \cos\vartheta)ds + t_Ze^{k_t(\Theta-\vartheta)}. \qquad (15)$$

In the expression (15) the direction of integration is changed. Transformations helped to get rid of the second and third derivatives under integrals. The last integral (15) cannot be simplified in general case. It can be found in quadratures only for the case of a constant radius of the well curvature when $ds = Rd\vartheta$ [10].

Knowing the axial force t (15), one can find a distributed reaction of the well walls by expression (7).

Method of numerical differentiation and integration of inclinometric table. According to the results of directional survey, that is the table of zenith angles ϑ, measured with the interval Δs, a real well profile is constructed. At first, they determine depth gain Δz, horizontal displacement Δx from the vertical axis of the well in the directional drilling, lateral deviation Δy from the directional orientation

$$\Delta z_n = \Delta s_n\cos\vartheta_n;$$
$$\Delta x_n = \Delta s_n\sin\vartheta_n\cos(A_n - Az);$$
$$\Delta y_n = \Delta s_n\sin\vartheta_n\sin(A_n - Az),$$

where n is the sequence number of measurement; $\Delta s_n = s_n - s_{n-1}$ is coordinate gain s of intersection along the deviated wellbore; A_n is measured magnetic azimuth; Az is azimuth of directional orientation.

According to the calculated gains, absolute values of depth Z_n, horizontal displacement X_n and lateral deviation Y_n as the sum of gains are determined

$$Z_n = \sum_{i=1}^n\Delta z_i; \quad X_n = \sum_{i=1}^n\Delta x_i; \quad Y_n = \sum_{i=1}^n\Delta y_i,$$

by which they build a vertical profile and a horizontal well plan.

For a numerical differentiation of a table-defined function ϑ, a central scheme [11] is used

$$d\vartheta_n = \frac{\vartheta_{n+1} - \vartheta_{n-1}}{s_{n+1} - s_{n-1}}, \qquad (16)$$

where the letter d denotes numerical differentiation.

According to (16), the values of the second $d2$ and the third $d3$ derivatives can be obtained correspondingly

$$d2\vartheta_n = \frac{d\vartheta_{n+1} - d\vartheta_{n-1}}{s_{n+1} - s_{n-1}};$$
$$d3\vartheta_n = \frac{d2\vartheta_{n+1} - d2\vartheta_{n-1}}{s_{n+1} - s_{n-1}}.$$

Applying the (7), the true value of F_n of the distributed reaction of the well wall in the n^{th} section is found by the formula

$$F_n = EJf_n = EJj\sin\vartheta_n - T_n \cdot d\vartheta_n + EJ \cdot d3\vartheta_n, \qquad (17)$$

where $T_n = EJt_n$ is the actual value of the axial force, which must first be found, defining the integrals in the (15).

The numerical integration of tabulated functions is carried out according to the trapezoidal rule [11]. For this, the interval of integration $[s, L]$ is divided into elementary intervals; on each of them, they find the area of the trapezoid, constructed on the ordinates of the function at the edges of the interval. The value of an integral is equal to the sum of the squares of all elementary trapezoids.

For an inclinometric table, for an elementary interval, it is natural to choose the measurement interval Δs, which makes it possible to find the values of the integral functions for the formula (15) at the edges of each interval.

As formula (15) shows, to find the value of the axial force t in the current section s, it is necessary to know its value t_Z at the end of the integration interval. The only cross section of the casing, where the axial force is known in advance, is its free end (casing shoe) – here it is $t_Z = 0$. Proceeding from this, the following method of numerical analysis of inclinometric table is developed.

For the integration interval, we choose the measurement interval Δs. Then in the current section s_n, which is the beginning of the interval and where you need to find the axial force t_n, one can determine all the values of functions and derivatives necessary for (15). The same values at the end of the interval (where $s = L$ and $\vartheta = \Theta$) are found by the data of the next $(n + 1)^{th}$ measurement.

At the same time, for formula (15) the integral value is equal to the trapezoidal area constructed on the ordinates of the integrands determined according to the n^{th} and $(n + 1)^{th}$ measurements. The value of the trapezoidal area is found as the product of the interval Δs to the arithmetic mean of the specified ordinates.

Thus, transforming formula (15) and integrals in it according to the proposed method, the real value of the axial force T_n at each step of integration is determined by the formula

$$T_n = \frac{EJ}{e^{k\vartheta_n}}\left[k(e^{k\vartheta_n} d2\vartheta_n - e^{k\vartheta_{n+1}} d2\vartheta_{n+1}) - \right.$$
$$-\frac{1+k^2}{2}\left(e^{k\vartheta_n}(\mathrm{d}\,\vartheta_n)^2 - e^{k\vartheta_{n+1}}(\mathrm{d}\,\vartheta_{n+1})^2 + \right.$$
$$\left.\left. + k(s_{n+1}-s_n)\frac{e^{k\vartheta_n}(d\vartheta_n)^3 + e^{k\vartheta_{n+1}}(d\vartheta_{n+1})^3}{2}\right)\right] + \frac{EJj}{e^{k\vartheta_n}}\times \quad (18)$$
$$\times\frac{e^{k\vartheta_n}(\cos\vartheta_n - k\sin\vartheta_n) + e^{k\vartheta_{n+1}}(\cos\vartheta_{n+1} - k\sin\vartheta_{n+1})}{2}\times$$
$$\times(s_{n+1}-s_n) + T_{n+1}\frac{e^{k\vartheta_{n+1}}}{e^{k\vartheta_n}}.$$

Beginning with the last N^{th} measurement for which the value $T_N = EJt_Z = 0$ is known, according to (18), we find the previous value T_{N-1}, by which we get value T_{N-2}, and so on. The determination of the distribution of the axial forces in the body of the pipe occurs from the bottom upwards along the casing from its free end, with the preset value of the axial force for the last measurement.

The design of the casing column is described by setting the diameters D_n of the pipes and the thickness δ_n of their walls at each depth interval according to the well program; consequently, we determine the area S_n of the crosscut of the pipes and its moments of inertia J_n.

At each depth interval, we set the mass m_t of one linear meter of the casing pipe, the mass m_m of the collar, the length l_m of the pipes (the distance between the couplings), the mass m_c of the centralizer and the distance l_c between them. Let us determine the combined mass m_n of the linear meter of the casing column by the formula

$$m_n = m_t + \frac{m_m}{l_m} + \frac{m_c}{l_c}.$$

The coefficients of friction are given for each interval of bedding of rocks in accordance with the borehole log. We also set the values of the densities γ_n of the drilling fluid, which is in the well after it was washed out before the casing is lowered. The combined weight j_n of the linear casing meter is calculated by the formula

$$j_n = \frac{9.8(\rho - \gamma_n)}{EJ_n}\cdot\frac{m_n}{\rho},$$

where ρ is the density of the casing material; m_n/ρ is the area of its combined cross-section.

Along the wellbore, we find the values of the axial forces and the reactions of the wall by the (18 and 17). The values of the local radii R_n of the curvature and the internal bending moments M_n are calculated by the formulas

$$R_n = 1/d\vartheta_n; \quad M_n = EJq_n = EJd\vartheta_n. \quad (19)$$

To determine the strength of the casing, we must determine the local maximal values of internal stresses in the body of the pipes by the sum of stresses from tension and bending

$$\sigma_{\max} = \frac{T_n}{S_n} + \frac{|M_n|}{J_n}\frac{D_n}{2} = \frac{T_n}{S_n} + \frac{ED_n}{2|R_n|}. \quad (20)$$

The value of the bending moment and the radius of curvature is taken modulo to obtain the maximum stress value in the pipe, regardless of the direction of its bend and the location of the stretched fibers.

The developed numerical analysis program was tested in a test mode by comparing with the results of analytically found formulas of the axial force t and reaction of walls f for a well section of a constant radius of curvature, taking into account frictional forces [10]. At the same time the error of program calculations was no more than 0.02 %.

Results and discussion. Approbation of the developed methodology is carried out according to the data of the operating well. At first the analysis of its program was carried out using theoretical solutions. For this purpose, in the areas from which the real program is made, the following parameters are calculated according to the formulas obtained analytically in [10]: the distribution of axial forces in the initial and final vertical sections; the distribution of axial forces and the reactions of the walls on the radius of zenith angle buildup, on two inclined rectilinear sections and two radius sections of the zenith angle decline. The values of the radii of curvature and bending moments are calculated according to the (19), the maximum stresses – according to (20). The results of calculations are presented in Fig. 2.

The theoretical analysis of the well design showed that the axial force in the body of the column with increasing depth decreases piecewise linearly on straight sections (vertical and inclined), as well as on the curved ones along a circular arc. The same nature has the distribution of tensile stresses in the body of the pipes. At the same time, a discontinuous change in the stresses of two types was detected. The first type (on the marks of 1200 and 3700 m) is caused by a change in the standard size of the casing pipes. The second type of stress jumping is typical for the column curving intervals (with a constant radius of curvature according to the program) and is determined by the value of the bending moment created by the curvature. In addition, jumps of well wall reactions in the regions of the conjugation of its rectilinear and curved areas occur.

The jump-like nature of the change in the stresses and reactions of the walls is due to the fact that in the transition from rectilinear areas of the design well to those curved along a circular arc there is no geometric break of its axis, since the tangents coincide in the transitional section. However, the jump in the bending moment occurs, which is on the arc and is proportional to the curvature, but is absent on a straight line.

This is a consequence of the idealization of the project, in the first place, through the description of the distorted areas by the arc of the ideal circle. In a real well, whose diameter is slightly larger than the diameter of the pipes, the edges of the column at the conjugated sites due to the elasticity of the pipes receive variable curvatures, which acquire values from R^{-1} on the arc section to 0 on a straight line and vice versa.

A positive well reaction indicates that the casing column rests on its lower wall; this is observed on inclined rectilinear and in the areas of the decline of the zenith angle. The negative reaction of the walls shows that the column rests on the upper wall of the well due to the forces of elasticity of the initially rectilinear casing; this is manifested in the area of the zenith angle buildup. On the inclined sections, the reaction of the well coincides with the reaction of the inclined plane. These results are consistent with the conclusions of [10].

The numerical analysis program also worked out the program of the operating well, given in the form of inclinometric table; the results of this are presented in Fig. 2 with blue lines. Patterns of the distribution of axial forces, the reactions of the well walls, bending moments, maximum stresses in the body of the column, obtained by numerical analysis and calculated by analytical formulas [10], qualitatively coincide completely. The error of the developed numerical analysis method is due to the inaccuracy of numerical differentiation and integration and depends first of all on the choice of the value of the interval [11]. The difference between the numerical and theoretical calculations of the design maximum stresses in the body of the pipes is 0.01–0.03 % along the entire column.

In addition, the developed program of numerical analysis worked out inclinometric table of data of field measurements of the actual well number 1; the results of this are presented in Fig. 2 with red lines. This allowed revealing the following features of the behavior of the casing in a real drilled borehole.

The actual deviation of the real well profile from the design one is shown through a graph of bending moments (Fig. 2, *d*), which can also be considered as a graph for changing the actual curvature of the well, since they are proportional according to (19). As you can see, the axis of an actual drilled well significantly deviates from the design profile (rectilinear or radius one). This is evidenced by the continuous change in bending moments by both magnitude and direction, which is caused by a change in the actual values of the local curvature of the well. This is due to the impact of a large number of technical, technological and geological factors on the drilling process.

Under these conditions, a casing column, trying to preserve its initially rectilinear form, at the expense of the forces of elasticity rests on opposite walls of a stochastically curved well, causing variables in magnitude and direction of reaction

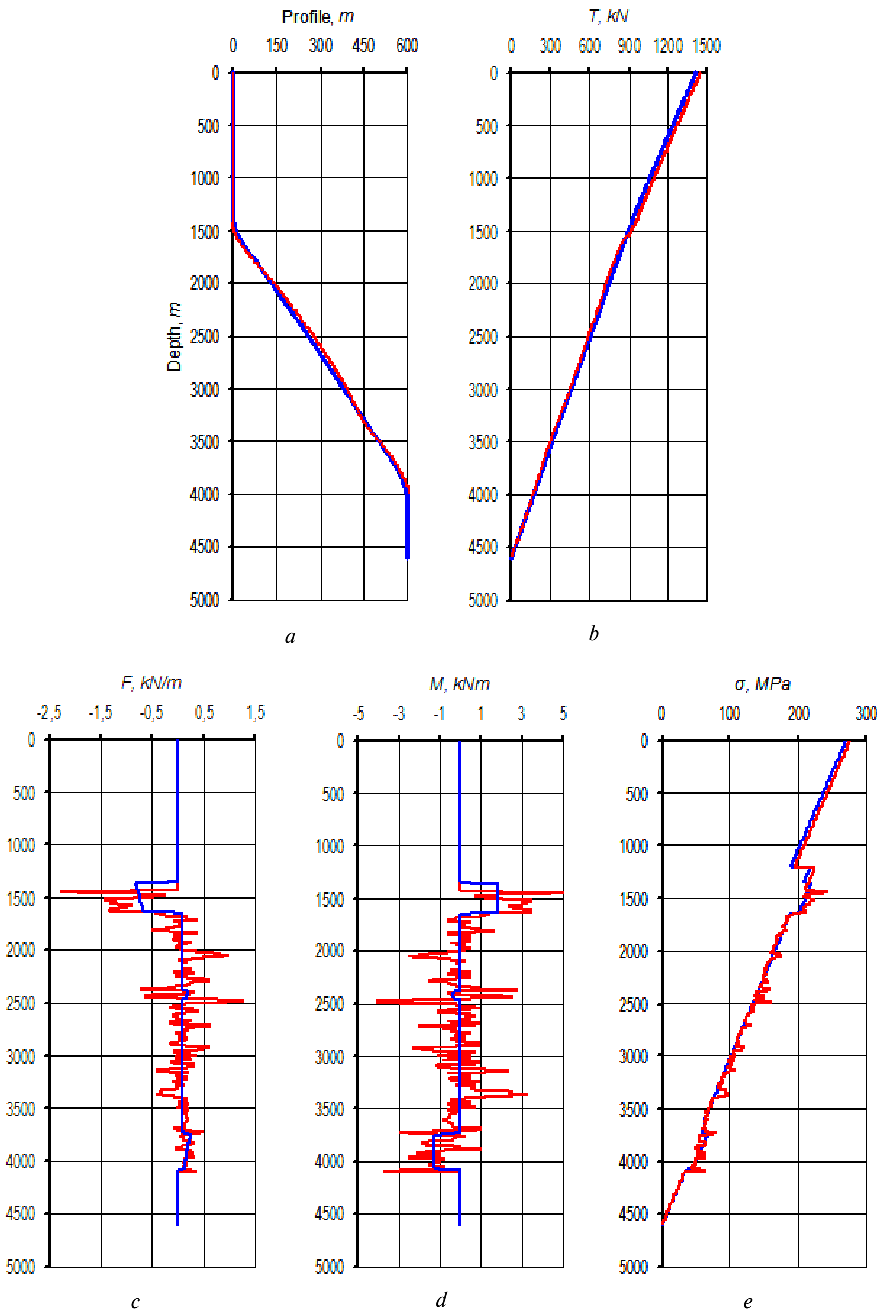

Fig. 2. Wells profile (a), graphs of axial forces T (b), walls reactions F (c), bending moments M (d) and normal tensions σ (e) in column combined at depth intervals:

blue line – according to the well project; red line – according to inclinometric measurements

(Fig. 2, *c*). By comparing Figs. 2, *c, d*, we can see that the magnitude and change in the local curvature of the well causes a proportional value and a change in the reaction of its wall. The reaction of the wall is also proportional to the bending rigidity of the casing. Accordingly, the internal bending moment and bending stress in the body of the pipe also change.

The largest jump in the values of the reaction of the actual well walls, the bending moment and the maximum stresses in the body of the casing is observed at a mark of 1,440 m, where the actual deviation of the well and zenith angle buildup (as opposed to the design of 1,350 m) begin. Along with this, the results of numerical analysis of an actual well made it possible to detect its areas with a significant increase in the curvature and the reaction of the wall. These are areas where the forced deviation of the well (zenith angle buildup and decline) occurred. In addition, in the areas of stabilization of the zenith angle, one can also find a local increase in the curvature and the reaction of the wall.

Numerical analysis of the stresses shows that for this well profile the tensile stress of the column is dominant (Fig. 2, *e*). Local stresses are of fluctuating nature and are related to the increase of local curvature of the well and bending moment in the column.

Conclusions. The stress-strain state of the casing in the curved well can be determined by the non-uniform system of differential equations, which describes the bending of a long elastic rod under the action of distributed forces of its own weight, the reaction of supports and friction. Having the shape of the well with a known function of the zenith angle, we can find the solutions of the system in the form of functions of the distribution of axial forces and bending moments in the body of the column, as well as the reactions of the walls, which lead the column to the actual well profile.

Parameters of the stress-strain state of the casing in an actually drilled well can be determined by the developed methods of numerical integration of the data of inclinometric measurements of the well and the software of their numerical analysis. This allows us to identify areas of local increase in curvature and stresses of the casing pipes in the curved well.

The developed method of numerical analysis of the well allows detecting the areas with a significant local increase in the curvature, which indicates their obstructed passability. It allows one to accurately determine the depth intervals for increasing the well diameter. This must be done before lowering the casing string. In addition, according to the results of the analysis, it is possible to determine the parameters of the stress-strain state of the casing, which can be used to predict its working capacity and operating life in the curved well.

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Силова взаємодія обсадної колони зі стінками криволінійної свердловини

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Мета. Розроблення методики визначення осьових зусиль і реакцій стінок уздовж обсадної колони, що згинають її та надають їй вигнуту форму стовбура свердловини.

Методика. Обсадна колона представлена як довгий пружний стрижень у криволінійній свердловині. Для опису деформацій стрижня розроблена неоднорідна система з чотирьох диференціальних рівнянь. Вона зведена до диференціального рівняння першого порядку відносно осьової сили. Його розв'язок був знайдений методом Бернуллі. Застосовано чисельне інтегрування диференціального рівняння.

Результати. Знайдено розподіл осьових зусиль уздовж обсадної колони з урахуванням кривизни свердловини й тертя, а також сил реакції стінок свердловини. Розроблена методика чисельного інтегрування таблиці інклінометричних вимірювань свердловини. Отримані розрахункові формули для сил реакції, осьових сил, згинальних моментів і напружень, що діють в обсадних трубах у глибині свердловини.

Наукова новизна. Розв'язана задача враховує реакцію стінок і сили тертя, що створюють поздовжній згин під час руху колони. Система диференціальних рівнянь рівноваги доповнена кінематичним рівнянням Ейлера. За змінну інтегрування прийнято функцію зенітного кута, що є відомою завдяки таблиці даних інклінометрії. Розв'язана обернена задача – за кутовими деформаціями обсадної колони, що задані формою свердловини в інклінометричній таблиці, визначені всі невідомі внутрішні сили, а також зовнішня розподілена реакція, яка змушує колону повторювати форму свердловини.

Практична значимість. Розроблена методика дозволяє виявити ділянки зі значним локальним збільшенням кривизни свердловини, що свідчать про їх утруднену прохідність. Це дає змогу точно визначити інтервали глибин для розширення діаметра свердловини, що є необхідним перед опусканням колони. За результатами аналізу можна визначити параметри напружено-деформованого стану обсадної колони, за якими прогнозувати її працездатність і ресурс роботи.

Ключові слова: *обсадна колона, криволінійна свердловина, інклінометричні вимірювання, реакція стінки*

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