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ENERGETIC EFFECTS ANALYSIS OF SHEAR WAVES AND THEIR SECOND HARMONICS IN A WAVEGUIDE WITH NON-IDEAL COMPONENT CONTACT

In this article the energetic and kinematic effects that occur in the elastic shear wave and its second harmonics propagation are investigated. The waveguide consists of anisotropic elastic crystal layer of cubic system m3m class enclosed between crystal halfspaces of same anisotropy class. A slipping contact is assumed in the contact zone of waveguide parts. The research is based on a model of general geometrical end physical nonlinearity in dynamic deformation processes. It allows to use elastic potential with the quadratic and cubic deformation components and the deformations with nonlinear terms. The approach of nonlinear elastic wave characteristics expansion into rows of a small parameter is used. Due to this approach at the first stage it's necessary to solve the problem of finding the components of the localized shear wave displacement vector (the problem of the first approximation). In the second stage, using the obtained results of the first approximation problem, the representation of the components of the displacement vector for the second harmonics of the localized elastic wave is solved in analytical form (the problem of the second approximation). By using the obtained kinematic results, the energy effects can be evaluated in the form of a vector of the average for the period of power flow. Specific results of the study of the amplitude-frequency and energy characteristics of the localized shear type elastic waves in the considered waveguide structure were obtained using computer algebra methods.The calculations of cinematic and energetic characteristics (that in contrast to linear SH harmonic are P-SV type waves) have been carried out for NaCl layer and germanium halfspaces waveguide.

MSC: 74J05, 74B05.

Keywords: *elastic waves, cubic crystals, nonlinear second harmonics, anharmonic effects, layer between halfspaces, slip contact of the waveguide components.*

1. Introduction.

The waveguide is related to rectangular coordinate system $Ox_1x_2x_3$ and consists of the layer V_1 surrounded by two halfspaces V_2 and V_3. The layer fills area $V_1 = \{-\infty < x_1,\ x_2 < \infty,\ -h \le x_3 \le h\}$ and the halfspaces fill $V_2 = \{-\infty < x_1,\ x_2 < \infty,\ -\infty < x_3 < -h\}$ and $V_3 = \{-\infty < x_1,\ x_2 < \infty,\ h < x_3 < \infty\}$. The waveguide materials are anisotropic cubic crystals of $m3m$ class described by three independent matrix constants of second order $c_{11}^{(p)}$, $c_{12}^{(p)}$, $c_{44}^{(p)}$; by five independent matrix constants of third order $c_{111}^{(p)}$, $c_{112}^{(p)}$, $c_{114}^{(p)}$, $c_{155}^{(p)}$, $c_{123}^{(p)}$, $c_{456}^{(p)}$ and by density $\rho^{(p)}$. The superscript p shows that the corresponding characteristic relates to the waveguide component V_p in the following ratios. The crystallographic directions of layer and halfspaces are collinear. The SH wave is supposed to flow in the middle of layer along Ox_1 direction. The model of physical and geometrical nonlinear deformation is used in this work. It based on the elastic potential U with quadratic and cubic terms of finite deformations:

$$U = \frac{1}{2} c_{jqrk} \varepsilon_{jq} \varepsilon_{rk} + \frac{1}{6} c_{jqrklm} \varepsilon_{jq} \varepsilon_{rk} \varepsilon_{lm} \qquad (j, q, r, k, l, m = \overline{1,3}) \tag{1}$$

and based on nonlinear representations of the components of the mechanical deformations tensor

$$\varepsilon_{jk} = \frac{1}{2}(u_{l,k} + u_{k,j} + u_{l,j} u_{l,k}), \tag{2}$$

where $u_{r,k} = \partial u_r / \partial x_k$, u_r are the components of wave elastic displacement vector.

The components of mechanical stress tensor σ_{jd} that related to the choice of elastic potential above are represented as the sum of linear and nonlinear components

$$\sigma_{jd} = \sigma_{jd}^{(l)} + \sigma_{jd}^{(n)}, \tag{3}$$

where

$$\sigma_{jd}^{(l)} = c_{jdrk} u_{r,k}, \quad \sigma_{jd}^{(n)} = \frac{1}{2} c_{jdrk} u_{l,r} u_{l,k} + c_{pdrk} u_{j,p} u_{r,k} + \frac{1}{2} c_{jdrklm} u_{r,k} u_{l,m}. \tag{4}$$

Equation of motion for the elastic medium can be represented in a tensor form

$$\rho \ddot{u}_j^{(p)} - \sigma_{jd,d}^{(p,l)} = \sigma_{jd,d}^{(p,n)}, \quad (j = \overline{1,3}). \tag{5}$$

The research of small nonlinear wave effects is based on small parameter method where the normalised complex wave displacement functions $u_j^{(p)}$ $(j = \overline{1,3})$ are sought as the representation $u_j = u_j^{(l)} + \delta u_j^{(n)}$. Where $\delta = u_* / R_* << 1$ is implied to be a small parameter.

2. Obtaining a numerical-analytical solution of the problem.

In the considered problem about generalized linear shear waves spreading in a m3m class cubic system monocrystal layer between halfspaces of the same anysotropic class upon condition of nonideal slipping contact of waveguide components the linear components of the investigated wave field are determined from homogeneous spectral boundary value problem.

$$\sigma_{2j,j}^{(p,l)} - \rho_p \ddot{u}_2^{(p)} = 0, \quad (p = 1, 3), \tag{6}$$

$$\begin{gathered}
(u_3^{(1,l)})_{x_3=-1} = (u_3^{(2,l)})_{x_3=-1}, \quad (u_3^{(1,l)})_{x_3=1} = (u_3^{(3,l)})_{x_3=1}, \\
(\sigma_{33}^{(1,l)})_{x_3=-1} = (\sigma_{33}^{(2,l)})_{x_3=-1}, \quad (\sigma_{33}^{(1,l)})_{x_3=1} = (\sigma_{33}^{(3,l)})_{x_3=1}, \\
(\sigma_{3j}^{(1,l)})_{x_3=-1} = 0, \quad (\sigma_{3j}^{(1,l)})_{x_3=1} = 0, \\
(\sigma_{3j}^{(2,l)})_{x_3=-1} = 0, \quad (\sigma_{3j}^{(3,l)})_{x_3=1} = 0, \quad (j = (\overline{1,2}))
\end{gathered} \tag{7}$$

The complex vector-functions of linear wave displacement $\vec{u}^{(p,l)}$ are characterised by single nonlinear component $u_2^{(p,l)}$. The boundary conditions of linear approximation problem are transformed into stress-free conditions of middle layer boundary surfaces V_1. Thereby the representations of complex functions of wave displacements $u_2^{(p,l)}$ with a normalizing non-dimensional parameter $u_{2q}^{(0)}$ for the component V_p can be written:

$$u_{2q}^{(l,1)} = u_{2q}^{(0)} cos(\alpha^{(q)} x_3) e^{-i(\omega t - k_q x_1)},$$
$$u_{2q}^{(l,2)} = 0, \tag{8}$$
$$u_{2q}^{(l,3)} = 0,$$

here $\alpha^{(q)} = q\pi/2 \quad (q = \overline{0,\infty})$. The dispersion relation that associates frequency ω and wave number k_q has the form

$$((\Omega_1^2 - k_q^2)/(c_{44}^{(1)})^{1/2} = q\pi/2, \quad \Omega_1^2 = \rho_1 \omega^2 R_*^2/c_*. \tag{9}$$

We will consider cases $q \geq 1$ below.

The structure (8), (9) will be used for the definition of relations of the problem of searching the appropriate nonlinear anharmonic perturbations for the localised SH waves. Relations of the second approximation have the form:

$$(\sigma_{ij.j}^{(p,l)})_{\vec{u}^{(p)}=\vec{u}^{(p,n)}} - \rho_p \ddot{u}_i^{(p,n)} = -(\sigma_{ij.j}^{(p,n)})_{\vec{u}^{(p)}=\vec{u}^{(p,l)}}. \tag{10}$$

The boundary conditions on the contacting borders $x_1 = \pm 1$ in the considered waveguide are presented:

$$u_1^{(2,n)} = u_1^{(1,n)}, \quad u_3^{(2,n)} = u_3^{(1,n)} \text{ where } x_3 = -1,$$
$$(\sigma_{3i}^{(1,l)})_{\vec{u}^{(1)}=\vec{u}^{(1,n)}} + (\sigma_{3i}^{(1,n)})_{\vec{u}^{(1)}=\vec{u}^{(1,l)}} = 0, \quad (i = 1, 2)$$
$$(\sigma_{33}^{(1,l)})_{\vec{u}^{(1)}=\vec{u}^{(1,n)}} + (\sigma_{33}^{(1,n)})_{\vec{u}^{(1)}=\vec{u}^{(1,l)}} = (\sigma_{33}^{(2,l)})_{\vec{u}^{(2)}=\vec{u}^{(2,n)}} + (\sigma_{33}^{(2,n)})_{\vec{u}^{(2)}=\vec{u}^{(2,l)}};$$
$$u_1^{(3,n)} = u_1^{(1,n)}, \quad u_3^{(3,n)} = u_3^{(1,n)} \text{ where } x_3 = 1,$$
$$(\sigma_{3i}^{(1,l)})_{\vec{u}^{(1)}=\vec{u}^{(1,n)}} + (\sigma_{3i}^{(1,n)})_{\vec{u}^{(1)}=\vec{u}^{(1,l)}} = 0, \quad (i = 1, 2) \tag{11}$$
$$(\sigma_{33}^{(1,l)})_{\vec{u}^{(1)}=\vec{u}^{(1,n)}} + (\sigma_{33}^{(1,n)})_{\vec{u}^{(1)}=\vec{u}^{(1,l)}} = (\sigma_{33}^{(3,l)})_{\vec{u}^{(2)}=\vec{u}^{(2,n)}} + (\sigma_{33}^{(3,n)})_{\vec{u}^{(2)}=\vec{u}^{(2,l)}}.$$

The components of complex stress vector of second harmonics are determined from the relations of boundary value problem (11), (12) in analytic form by using computer algebra methods. From the structure of boundary value problem analysis (11), (12) it appears that the second harmonics of discovered linear localised waves are the waves of P-SV type. In this case the second harmonics for the layer are represented as the sum of partial and general solutions of corresponding inhomogeneous boundary value problem. For the halfspaces the anharmonic perturbations are described only by general solve of the problem for the system of homogeneous differential equations.

The structure of the analytical solution of the problem of the search for elastic wave displacements $u_j^{(n,p)} \quad (j=1, j=3)$ in the nonlinear second harmonics of investigated waves can be written in form:

$$\begin{aligned}
u_1^{(1,n)} &= (u_2^{(0)})^2(\tilde{\lambda}_{11}cos(\zeta_1^{(1)}x_3)+\tilde{\lambda}_{12}cos(\zeta_2^{(1)}x_3))+\tilde{\mu}_{11}\sin(\zeta_1^{(1)}x_3)+\tilde{\mu}_{12}\sin(\zeta_2^{(1)}x_3)+\\
&\quad +\tilde{\nu}_1+\tilde{\chi}_1\cos(2\alpha^{(1)}x_3)+\tilde{\xi}_1\sin(2\alpha^{(1)}x_3)\big)exp(-2i(\omega t-kx_1)),\\
u_3^{(1,n)} &= (u_2^{(0)})^2(\tilde{\lambda}_{31}\sin(\zeta_1^{(1)}x_3)+\tilde{\lambda}_{32}\sin(\zeta_2^{(1)}x_3)+\tilde{\mu}_{31}\cos(\zeta_1^{(1)}x_3)+\tilde{\mu}_{32}\cos(\zeta_2^{(1)}x_3)+\\
&\quad +\tilde{\nu}_3+\tilde{\chi}_3\sin(2\alpha^{(1)}x_3)+\tilde{\xi}_3\cos(2\alpha^{(1)}x_3))exp(-2i(\omega t-kx_1)), \qquad (12)\\
u_1^{(2,n)} &= (u_2^{(0)})^2(\tilde{\beta}_{11}^{(2)}exp(\zeta_1^{(2)}x_3)+\tilde{\beta}_{12}^{(2)}exp(\zeta_2^{(2)}x_3))exp(-2i(\omega t-kx_1)),\\
u_3^{(2,n)} &= (u_2^{(0)})^2(\tilde{\beta}_{31}^{(2)}exp(\zeta_1^{(2)}x_3)+\tilde{\beta}_{32}^{(2)}exp(\zeta_2^{(2)}x_3))exp(-2i(\omega t-kx_1)),\\
u_1^{(3,n)} &= (u_2^{(0)})^2(\tilde{\beta}_{11}^{(3)}exp(\zeta_1^{(3)}x_3)+\tilde{\beta}_{12}^{(3)}exp(\zeta_2^{(3)}x_3))exp(-2i(\omega t-kx_1)),\\
u_3^{(3,n)} &= (u_2^{(0)})^2(\tilde{\beta}_{31}^{(3)}exp(\zeta_1^{(3)}x_3)+\tilde{\beta}_{32}^{(3)}exp(\zeta_2^{(3)}x_3))exp(-2i(\omega t-kx_1)).
\end{aligned}$$

Coefficients $\tilde{\lambda}_{ij}, \quad \tilde{\mu}_{ij}, \quad \tilde{\beta}_{ij}^{(p)}$ in the representation of general solution and coefficients $\nu_i, \quad \chi_i, \quad \xi_i$ in the representation of partial solution are obtained in analytic form by the computer algebra methods and contain extremely cumbersome expressions.

The energy effects analysis by the propagation of elastic waves is mainly based on the calculation of the vector components of the average for a period density of power flow. Representations are obtained for dimensionless normalized values of the vector components of the average for a period density of power flow:

$$P_j = P_j^{(l)}\delta + P_j^{(n)}\delta^3, \qquad (13)$$

$$\begin{aligned}
P_j^{(l)} &= \frac{\omega}{2}Im\left[\overline{u}_i^{(l)}(x_3)\sigma_{ji}^{(ll)}\right],\\
P_j^{(n)} &= \omega Im\left[\overline{u}_i^{(n)}(x_3)\left(\sigma_{ji}^{(ln)}+\sigma_{ji}^{(nl)}\right)\right]. \qquad (14)
\end{aligned}$$

Here $P_j^{(l)}$ characterizes first approximation the linear wave energy and $P_j^{(n)}$ characterizes energy of the nonlinear anharmonic perturbations.

3. Numerical results.

An analysis of the kinematic and energetic characteristics properties of the discovered localized waves and their second harmonics is implemented for the case of a waveguide from a layer V_1 of sodium chloride crystal located between half-spaces V_2 and V_3 of germanium crystal. Physical and mechanical properties of these components are characterized by the independent constants below:

$$\text{monocrystal NaCl} - c_{11}^{(1)} = 4,958c_*,\ c_{12}^{(1)} = 1,306c_*,\ c_{44}^{(1)} = 1,279c_*,$$
$$c_{111}^{(1)} = -86,36c_*,\ c_{112}^{(1)} = -4,96c_*,\ c_{123}^{(1)} = 0,93c_*,\ c_{144}^{(1)} = 1,32c_*,$$
$$c_{456}^{(1)} = 0,71c_*,\ c_{155}^{(1)} = -5,87c_*,\ \rho_1 = 2,1678\rho_*;$$
$$\text{monocrystal Ge} - c_{11}^{(2)} = 16,7c_*,\ c_{12}^{(2)} = 7,9_*,\ c_{44}^{(2)} = 6,5c_*,$$
$$c_{111}^{(2)} = -82,5c_*,\ c_{112}^{(2)} = -45,1c_*,\ c_{123}^{(2)} = -6,4c_*,\ c_{144}^{(2)} = 1,2c_*,$$
$$c_{456}^{(2)} = -6,4c_*,\ c_{155}^{(2)} = -31,0c_*,\ \rho_2 = 2,33\rho_*;$$

The parameter values c_*, ρ_* are $c_* = 10^{10}\ (N/m^2)$, $\rho_* = 10^3\ (kg/m^3)$.

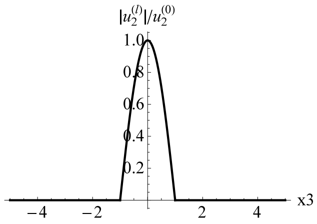

Fig. 1 Spreading of normalized values $|u_2^{(l)}|/u_2^{(0)}$

For a comparative frequency parametric analysis of the discovered nonlinear wave effects, the calculations of dimensionless normalized amplitudes of elastic lateral shifts in linear waves $|u_2^{(l)}|/u_2^{(0)}$ and their corresponding characteristics $|u_1^{(n)}|/(u_2^{(0)})^2$, $|u_3^{(n)}|/(u_2^{(0)})^2$ in their second harmonics were made in the region along the thickness of the waveguide, that includes the layer area $x_3/h \in [-1;1]$ and the areas $x_3/h \in [-5;-1)$ and $x_3/h \in (1;5]$ of half-spaces. The distributions $|u_1^{(n)}|/(u_2^{(0)})^2$, $|u_3^{(n)}|/(u_2^{(0)})^2$ are presented in Fig. 2 – Fig. 5 for cases of frequency value $\Omega(k_q) \in \{2.0;3.0;4.0;5.0\}$. Here $\Omega(k_q)$ are the normalized frequencies of localized waves that belong to the real branch with a number q in the corresponding dispersion spectrum.

It is worth emphasizing that the amplitude characteristics of non-linear second harmonics are proportional to the square of the linear harmonics normalized amplitude parameter, i.e. they are proportional to the value, which is a small quantity for localized waves at $\delta << 1$.

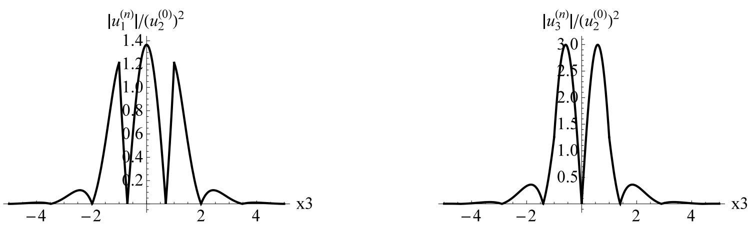

Fig. 2 Spreading of normalized values $|u_1^{(n)}|/(u_2^{(0)})^2$, $|u_3^{(n)}|/(u_2^{(0)})^2$ with $\Omega = 2$

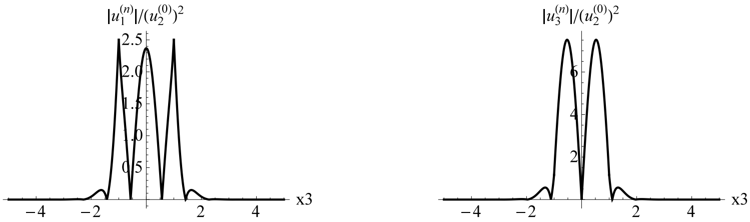

Fig. 3 Spreading of normalized values $|u_1^{(n)}|/(u_2^{(0)})^2$, $|u_3^{(n)}|/(u_2^{(0)})^2$ with $\Omega = 3$

It may be got such conclusions from the analysis of aforecited distributions. A qualitative effect for linear distributions $|u_2^{(l)}|/u_2^{(0)}$ is the linear shifts localization in the layer, the monotonic extinction of the wave displacements intensity and a decrease to 0 in the zone of layer and half-spaces contact (Fig. 1). But considering the geometric and physical nonlinearity for a waveguide with a slipping contact of its components reveals for this case the existence of double-frequency wave motions (nonlinear second harmonics) in half-spaces with localization near contact surfaces.

The characteristics comparison of $|u_1^{(n)}|/(u_2^{(0)})^2$ and $|u_3^{(n)}|/(u_2^{(0)})^2$ indicates that the relative intensity indices of the second harmonics grow with increasing frequency parameter Ω. The maximum intensity of the component $|u_3^{(n)}|/(u_2^{(0)})^2$ (SV – component of the nonlinear anharmonic disturbance) over the entire studied frequency range is in $2-4$ times greater than the maximum of $|u_1^{(n)}|/(u_2^{(0)})^2$.

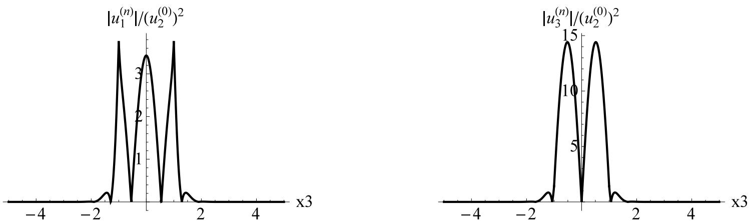

Fig. 4 Spreading of normalized values $|u_1^{(n)}|/(u_2^{(0)})^2$, $|u_3^{(n)}|/(u_2^{(0)})^2$ with $\Omega = 4$

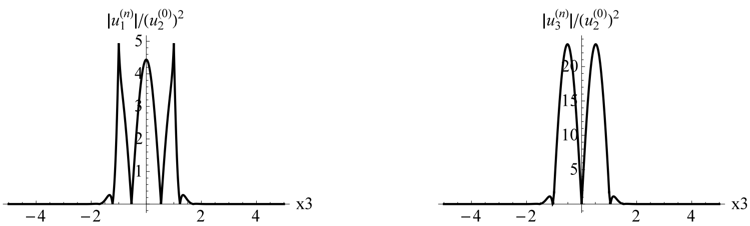

Fig. 5 Spreading of normalized values $|u_1^{(n)}|/(u_2^{(0)})^2$, $|u_3^{(n)}|/(u_2^{(0)})^2$ with $\Omega = 5$

The distributions of $|u_1^{(n)}|/(u_2^{(0)})^2$are characterized by three maximum peaks: in

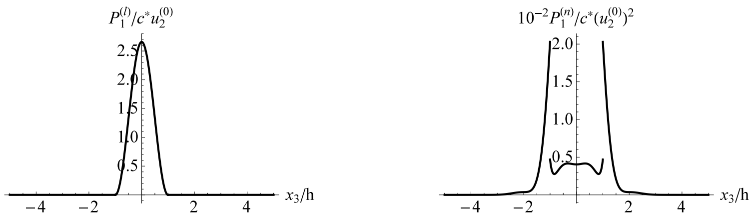

Fig. 6 Spreading of normalized values $|P_1^{(l)}|/u_2^{(0)}$, $|P_1^{(n)}|/(u_2^{(0)})^2$ with $\Omega = 2$

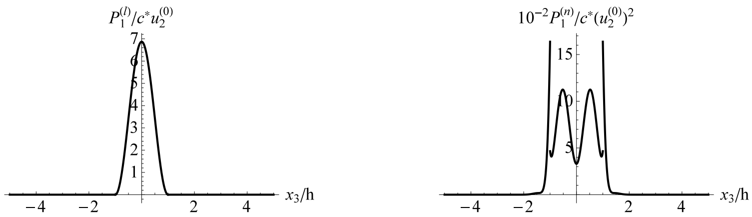

Fig. 7 Spreading of normalized values $|P_1^{(l)}|/u_2^{(0)}$, $|P_1^{(n)}|/(u_2^{(0)})^2$ with $\Omega = 3$

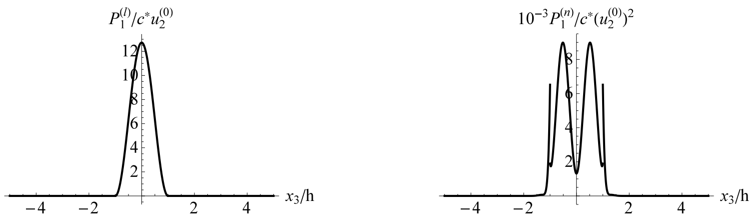

Fig. 8 Spreading of normalized values $|P_1^{(l)}|/u_2^{(0)}$, $|P_1^{(n)}|/(u_2^{(0)})^2$ with $\Omega = 4$

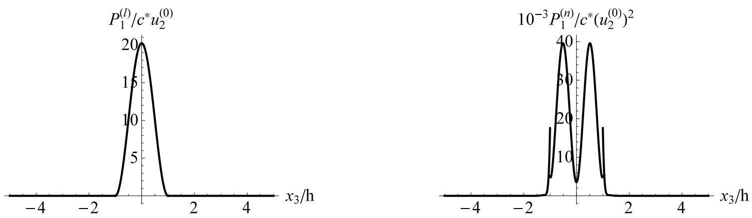

Fig. 9 Spreading of normalized values $|P_1^{(l)}|/u_2^{(0)}$, $|P_1^{(n)}|/(u_2^{(0)})^2$ with $\Omega = 5$

waveguide components contact zone and in the middle surface of the layer $x_3 = 0$. The distributions of $|u_3^{(n)}|/(u_2^{(0)})^2$ are characterized by two peaks localized in zone $x = \pm 1$. With frequency increasing the oscillation displacements increase in amplitude, but the penetration depth decreases. Also displacements shift to the layer zone, and when they permeate to half-spaces, they almost immediately fall off.

The energy properties of localized shear waves and their nonlinear second harmonics are characterized by the components of the vector of the average power flow over a period according to formulas [13-14]. Energy flows are characterized by non-zero components $P_1^{(l)}$, $P_1^{(n)}$. The results of illustrative calculations of normalized distributions $P_1^{(l)}$, $P_1^{(n)}$ with $\Omega(k_q) \in \{2.0; 3.0; 4.0; 5.0\}$ are presented in Fig 6 - Fig 9.

For the case of linear localized waves, $P_1^{(l)}$ reaches its maximum value in the middle plane of the layer $x_3 = 0$ and monotonically fades away and reaches 0 when it comes to the half-space zone $V2$, $V3$. The afore indicated components of energy flows in the general case have a first-order discontinuity — a jump at the contact boundary of the components of the waveguide. The nonzero component of the power flux for the second harmonic of localized shear waves gives values corresponding to the values of $\Omega \in \{2; 3\}$ in the half-space zone $x_3 = \pm 1$, and for cases of $\Omega \in \{4; 5\}$ the maximum is in the layer level $x_3 \approx \pm 0.6$.

References

1. Blistanov, A.A., Bondarenko, V.S., Chkalova, V.V. (1982). *Akusticheskiye kristally.* M., Nauka. Pod redaktsiey M.P. Shashkol'skoy (in Russian).
2. Zhogoleva, N.V., Shevchenko, V.P. (2016). Nonlinear second harmonics of localized shear waves in anisotropic layer between anisotropic half-spaces under condition of imperfect contact. *Mat. Metody Fiz.-Mekh. Polya, 59*(3), 169–179.
3. Shcherbak, N.V., Storozhev, V.I. (2009). Analiz nelineynykh angarmonicheskikh vozmuscheniy dlya uprugikh SH-voln, lokalizovannykh v kristallicheskom sloye mezhdu anizitropnymi poluprostranstvami. *Trudy instituta prikladnoy matematiki i mehaniki, 19*, 234–243 (in Russian).
4. Chattopadhyay, A., Gupta, S., Pato, Kumari, Sharma, V.K. (2013). Torsional wave propagation in non-homogeneous layer between non-homogeneous half-spaces. *International Journal for Numerical and Analytical Methods in Geomechanics, 37*(10), 1280–1291.
5. Kumon, R.E., Hamilton, M.F. (2002). Directional dependence of nonlinear surface acoustic waves in the (001) plane of cubic crystals. *J. Acoust. Sos. Am., 111*(1), 2060–2069.
6. Niklasson, J., Datta, S.K., Dunn, M.L. (2000). On ultrasonic guided waves in a thin anisotropic layer between two isotropic layers. *J. Acoust. Soc. Am., 108*(I.3), 924–933.
7. Sadler, J., O'Neill, B., Maev, R.G. (2005). Ultrasonic wave propagation across a thin nonlinear anisotropic layer between two half-spaces. *J. Acoust. Soc. Am., 118*, 51–59.

Н.В. Жоголева

Аналіз енергетичних ефектів зсувних хвиль та їх других гармонік при неідеальному контакті складових хвилеводу.

В роботі проводиться дослідження енергетичних та кінематичних ефектів, що виникають при поширенні локалізованої пружної зсувної хвилі та її других гармонік. Хвилевод складається з кристалічного шару класу m3m кубічної системи, розташованого з умовою ковзного контакту проміж двома однотипними кристалічними півпросторами класу m3m кубічної системи. Дослідження грунтуються на моделі геометричної та фізичної нелінійності в динамічних деформаційних процесах. Це дозволяє використовувати пружний потенціал з квадратичними та кубічними компонентами деформацій та деформації з нелінійними членами. Використовується підхід розкладання нелінійних пружних хвильових характеристик в ряди за малим параметром. Завдяки такому підходу, в першу чергу, розв'язується задача по знаходженню компонентів вектору переміщення локалізованої зсувної хвилі (задача першого наближення). На другому етапі, з використанням результатів задачі першого наближення, отримуються в аналітичній формі представлення компонентів вектору переміщень для других гармонік локалізованих пружних хвиль (задача другого наближення). За допомогою отриманих кінематичних результатів проведено оцінку енергетичних ефектів у вигляді вектору середнього за період потоку потужності. Конкретні результати дослідження амплітудно-частотних та енергетичних характеристик пружних хвиль локалізованого типу зсуву в даній хвилеводній структурі отримано за допомогою методів комп'ютерної алгебри. Розрахунки кінематичних та енергетичних характеристик (які на відміну від лінійної гармоніки SH типу є хвилями P-SV типу) були проведені для хвилевода у вигляді шару хлориду натрію та півпросторів з германію.

Ключові слова: *пружні хвилі, кубічні кристали, нелінійні другі гармоніки, ангармонічні ефекти, шар між півпросторами, ковзний контакт компонент хвилеводу.*

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