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Determination of the coordinate dependence of a pinning potential from the microwave experiment with vortices

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The measurement of the complex impedance response and accompanied power absorption $\mathcal{P}(\omega)$ in the radiofrequency and microwave ranges represents a most popular experimental method to investigate pinning mechanisms and the vortex dynamics in type-II superconductors. In the theory, the pinning potential (PP) well for a vortex must be *a priori* specified in order to subsequently analyze the measured data. We have theoretically solved the inverse problem at $T = 0$ K and exemplify how the coordinate dependence of a PP can be determined from a set of experimental curves $\mathcal{P}(\omega|j_0)$ measured at subcritical dc currents $0 < j_0 < j_c$ under a small microwave excitation $j_1 << j_c$ with frequency ω. We furthermore elucidate how and why the depinning frequency ω_p, which separates the non-dissipative (quasi-adiabatic) and the dissipative (high-frequency) regimes of small vortex oscillations in the PP, is reduced with the increase of j_0. The results can be directly applied to a wide range of conventional superconductors with a PP subjected to superimposed dc and small microwave ac currents at $T << T_c$.

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74.25.nn Surface impedance.

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1. Introduction

One of the most popular experimental methods for the investigation of the vortex dynamics in type-II superconductors is the measurement of the complex ac response in the radiofrequency and microwave ranges [1]. The reason for this is that at frequencies substantially smaller than those invoking the breakdown of the energy gap, the high-frequency and microwave impedance measurements of a mixed state contain information about flux pinning mechanisms, the vortex dynamics, and accompanied with it dissipative processes in a superconductor. It should be noted that this information can not be extracted from the dc resistivity data obtained in the steady state regime when pinning in the sample is strong. In fact, in the last case when the critical current densities j_c are rather large, the realization of the dissipative mode, in which the flux-flow resistivity ρ_f can be measured, requires $j_0 \gtrsim j_c$. This is commonly accompanied by a non-negligible electron overheating in the sample [2,3] which changes the value of the desired ρ_f. At the same time, measurements of the absorbed by vortices power from an ac current with the amplitude $j_1 \ll j_c$ allow one to determine ρ_f at a dissipative power $\mathcal{P}_1 \sim \rho_f j_1^2$ which can be many orders of magnitude less than $\mathcal{P}_0 \sim \rho_f j_0^2$. Consequently, measurements of the complex ac response versus frequency ω practically probe the pinning forces in the absence of overheating effects, otherwise unavoidable at overcritical steady-state dc current densities.

At last years, the appearance of experimental works utilizing the usual four-point scheme [4], strip-line coplanar waveguides (CPWs) [5], the Corbino geometry [6,7], or the cavity method [8] to investigate the microwave vortex response in as-grown thin-film superconductors (or in those containing some nano-tailored pinning potential (PP) landscape) reflects the explosively growing interest to the subject. In fact, such artificially fabricated pinning nanostructures provide a PP of unknown shape that requires certain assumptions concerning its coordinate dependence in order to fit the measured data. At the same time, in a real sample a certain amount of disorder is always presented, acting as pinning sites for a vortex as well. Therefore, an approach how to reconstruct the form of the PP experimentally ensued in the sample is of great demand for both, application-related and fundamental reasons. An early scheme how to reconstruct the coordinate dependence of the pinning force from measurements implying a small ripple magnetic field superposed on a larger dc magnetic field was reported in Ref. 9. Similar problems to reconstruct specific form of a potential subjected to superimposed constant and small alternating signals arise not only in the vortex physics but also in a number of other fields. Mainly due to the closest mathematical analogy we would like to mention the Josephson junction problem wherein a plenty of non-sine forms of the current-phase relation is known to occur [11] and which could in turn benefit from our results reported here.

Turning back to the development of theory of our problem, the very early model to describe the absorbed power by vortices refers to the work of Gittleman and Rosenblum (GR) [12] where a small ac excitation of vortices in the absence of a dc current has been considered. The GR results have been obtained at $T = 0$ K in the linear approximation for the pinning force. We will present briefly their results in the present work since the subsequent description of our new results requires these as the essential background. Later on, the theory accounting also for the vortex creep at non-zero temperature in a one-dimensional cosine PP has been extended by Coffey and Clem (CC) [13]. However, this theory has been developed for a small microwave current in the absence of a dc. Recently, the CC results have been substantially generalized by us [14,15] for a two-dimensional cosine washboard pinning potential (WPP). The washboard form of the PP has enabled an exact theoretical description of the two-dimensional anisotropic nonlinear vortex dynamics for any arbitrary values of ac and dc amplitudes, temperature, the Hall constant, and the angle between the transport current direction with respect to the guiding direction of the WPP. Among other nontrivial results obtained, an enhancement [14] and a sign change [15] in the power absorption for $j_0 \gtrsim j_c$ have been predicted. Whereas the general solution of the problem in Refs. 14, 15 has been obtained in terms of a matrix continued fraction and is suitable for the analysis mainly in the form of figure data due to a large number of variable parameters, an analytical implementation of the solution at $T = 0$ K, $j_0 < j_c$, and $j_1 \to 0$ has been performed in Refs. 16, 17, taking the anisotropy of the vortex viscosity and an arbitrary Hall constant also into account.

In the present work, we report the possibility of reconstruction the coordinate dependence of a PP if a set of $\mathcal{P}_0(\omega)$ curves has been measured at different dc current amplitudes in the whole range $0 \le j_0 \lesssim j_c$ at a small microwave amplitude $j_1 \to 0$. Whereas a preliminary communication on this matter can be found in Ref. 18, here we provide a detailed description of the PP reconstruction procedure. The geometry of the problem implies a standard four-point microstrip bridge of a thin-film superconductor placed into a small perpendicular magnetic field with a magnitude $B \ll B_{c2}$ at $T \ll T_c$. The sample is assumed to have at least one pinning site and dc and ac currents are directed collinearly. The theoretical treatment of the problem is detailed next.

2. Dynamics of pinned vortices on a small microwave current

The GR model [12] considers oscillations of damped vortex in a parabolic PP. They measured the power absorption by vortices in PbIn and NbTa films over a wide range of frequencies ω and successfully analyzed their data on the basis of a simple equation for a vortex moving with the velocity $v(t)$ along the x axis

$$\eta\dot{x} + k_p x = f_L, \tag{1}$$

where x is the vortex displacement, η is the vortex viscosity, k_p is the constant which characterizes the restoring force f_p in the PP well $U_p(x) = (1/2)k_p x^2$ and $f_p = -dU_p/dx = -k_p x$. In Eq. (1) $f_L = (\Phi_0/c) j_1(t)$ is the Lorentz force acting on a vortex, Φ_0 is the magnetic flux quantum, c is the speed of light, and $j_1(t) = j_1 \exp(i\omega t)$ is the density of a small microwave current with the amplitude j_1. Looking for the solution of Eq. (1) in the form $x(t) = x \exp(i\omega t)$, where x is the complex amplitude of the vortex displacement, one immediately gets $\dot{x}(t) = i\omega x(t)$ and

$$x = \frac{(\Phi_0/\eta c) j_1}{i\omega + \omega_p}, \tag{2}$$

where $\omega_p \equiv k_p/\eta$ is the depinning frequency. To calculate the magnitude of the complex electric field arising due to the vortex on move, one takes $E = B\dot{x}/c$. Then

$$E(\omega) = \frac{\rho_f j_1}{1 - i\omega_p/\omega} \equiv Z(\omega) j_1. \tag{3}$$

Here $\rho_f = B\Phi_0/\eta c^2$ is the flux-flow resistivity and $Z(\omega) \equiv \rho_f/(1 - i\omega_p/\omega)$ is the microwave impedance of the sample.

In order to calculate the power $\mathcal{P}$ absorbed per unit volume and averaged over the period of an ac cycle, the standard relation $\mathcal{P} = (1/2)\mathrm{Re}(EJ^*)$ is used, where E and J are the complex amplitudes of the ac electric field and current density, respectively. The asterisk denotes the complex conjugate. Then, from Eq. (3) it follows

$$\mathcal{P}(\omega) = \frac{1}{2}\mathrm{Re}\, Z(\omega) j_1^2 = \frac{1}{2}\frac{\rho_f j_1^2}{1 + (\omega_p/\omega)^2}. \tag{4}$$

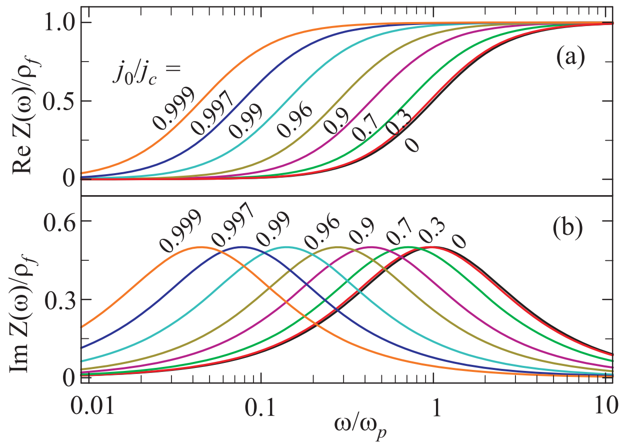

Fig. 1. The frequency dependences of real (a) and imaginary (b) parts of the ac impedance calculated for a cosine pinning potential $U_p(x) = (U_p/2)(1 - \cos kx)$ at a series of dc current densities, as indicated. In the absence of a dc current, the GR results are revealed in accordance with Eqs. (5).

For the subsequent analysis, it is convenient to write out real and imaginary parts of the impedance $Z = \mathrm{Re}Z + i\mathrm{Im}Z$, namely

$$\mathrm{Re}Z(\omega) = \frac{\rho_f}{1+(\omega_p/\omega)^2}, \quad \mathrm{Im}Z(\omega) = \frac{\rho_f(\omega/\omega_p)}{1+(\omega/\omega_p)^2}. \quad (5)$$

The frequency dependences (5) are plotted in dimensionless units Z/ρ_f and ω/ω_p in Fig. 1 (see the curve for $j_0 = 0$). From Eqs. (1), (2), and (4) it follows that pinning forces dominate at low frequencies ($\omega \ll \omega_p$), where $Z(\omega)$ is nondissipative with $\mathrm{Re}\, Z(\omega) \approx (\omega/\omega_p)^2$, whereas at higher frequencies ($\omega \gg \omega_p$) frictional forces dominate and $Z(\omega)$ is dissipative with $\mathrm{Re}\, Z(\omega) \approx \rho_f[1-(\omega_p/\omega)^2]$. In other words, due to the reduction of the amplitude of the vortex displacement with the increase of the ac frequency, a vortex is getting not influenced by the pinning force. This can be seen from Eq. (2) where $x \sim 1/\omega$ for $\omega \gg \omega_p$; this is accompanied, however, with the independence of the vortex velocity of ω in this regime in accordance with Eq. (3).

3. Influence of a dc current on the depinning frequency

When an arbitrary dc current is superimposed on a small microwave signal, the GR model can be generalized, for an arbitrary PP. For definiteness sake, let us consider a subcritical dc current with the density $j_0 < j_c$, where j_c is the critical current density in the absence of a microwave current. Our aim now is to determine the changes in the PP parameters the superimposition of the dc current leads. In the presence of $j_0 \neq 0$, the effective PP becomes $\tilde{U}(x) \equiv U_p(x) - xf_0$, where $U_p(x)$ is the x-coordinate dependence of the PP when $j_0 = 0$. Note also that $f_0 < f_c$ where f_0 and f_c are the Lorentz forces which correspond to the current densities j_0 and j_c, respectively.

In the presence of a dc current, the equation of motion for a vortex has the form

$$\eta v(t) = f(t) + f_p, \quad (6)$$

where $f(t) = (\Phi_0/c)j(t)$ is the Lorentz force with $j(t) = j_0 + j_1(t)$, where $j_1(t) = j_1 \exp(i\omega t)$, and j_1 is the amplitude of a small microwave current. Due to the fact that $f(t) = f_0 + f_1(t)$, where $f_0 = (\Phi_0/c)j_0$ and $f_1(t) = (\Phi_0/c)j_1(t)$ are the Lorentz forces for the subcritical dc and microwave currents, respectively, one can naturally assume that $v(t) = v_0 + v_1(t)$, where v_0 does not depend on the time, whereas $v_1(t) = v_1 \exp(i\omega t)$. In Eq. (6) the pinning force is $f_p = -dU_p(x)/dx$, where $U_p(x)$ is a PP of some form. Our aim is to determine $v(t)$ from Eq. (6) which, taking into account the considerations above, acquires the following form:

$$\eta[v_0 + v_1(t)] = f_0 + f_p + f_1(t). \quad (7)$$

Let us consider the case when $j_1 = 0$. If $j_0 < j_c$, i.e., $f_0 < f_c$, where f_c is the maximal value of the pinning force, then $v_0 = 0$, i.e., the vortex is in rest. As it is seen from Fig. 2 the rest coordinate x_0 of the vortex in this case depends on f_0 and is determined from the condition of equality to zero of the effective pinning force $\tilde{f}(x) = -d\tilde{U}(x)/dx = f_p(x) + f_0$, which reduces to the equation $f_p(x_0) + f_0 = 0$, or

$$f_0 = \frac{dU_p(x)}{dx}\Big|_{x=x_0}, \quad (8)$$

the solution of which is the function $x_0(f_0)$.

Let us now add a small oscillation of the vortex in the vicinity of x_0 under the action of the small external alternating force $f_1(t)$ with the frequency ω. For this we expand the effective pinning force $\tilde{f}(x)$ in the vicinity of

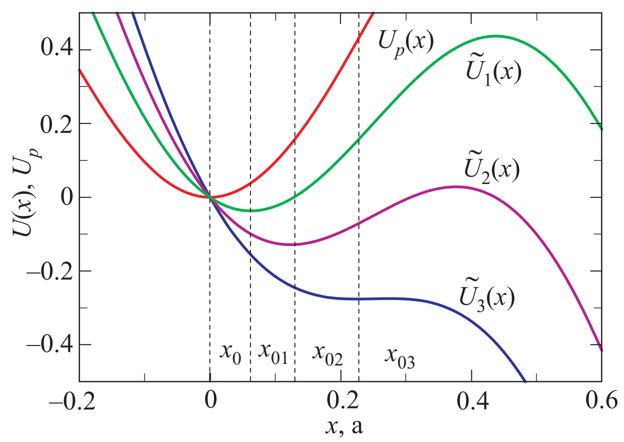

Fig. 2. Modification of the effective PP $\tilde{U}_i(x) \equiv U_p(x) - f_{0i}x$ where $U_p(x) = (U_p/2)(1 - \cos kx)$ is the WPP, with the gradual increase of f_0 such as $0 = f_0 < f_{01} < f_{02} \lesssim f_{03} = f_c$, i.e., a vortex is oscillating in the gradually tilting pinning potential well in the vicinity of the rest coordinate x_{0i}.

$x = x_0$ into a series in terms of small displacements $u \equiv x - x_0$ which gives

$$\tilde{f}(x - x_0) \simeq \tilde{f}(x_0) + \tilde{f}'(x_0)u + \ldots \tag{9}$$

Then, taking into account that $\tilde{f}(x_0) = 0$ and $\tilde{f}'(x_0) = U_p''(x_0)$, Eq. (7) acquires the form

$$\eta \dot{u}_1 + \tilde{k}_p u = f_1, \tag{10}$$

where $\tilde{k}_p(x_0) = U_p''(x_0)$ is the effective constant characterizing the restoring force $\tilde{f}(u)$ at small oscillations of a vortex in the effective PP $\tilde{U}(x)$ close by $x_0(f_0)$ and $v_1 = \dot{u} = i\omega u$. Equation (10) for the determination of v_1 is physically equivalent to GR Eq. (1) with the only distinction that the vortex depinning frequency $\tilde{\omega}_p \equiv \tilde{k}_p/\eta$ now depends on f_0 through Eq. (8), i.e., on the dc transport current density j_0. Thereby, all the results of the previous section [see Eqs. (2)–(5)] can be repeated here with the changes $x \to u$ and $\omega_p \to \tilde{\omega}_p$.

In order to discuss the changes in the dependences $\mathrm{Re}\, Z(\omega)$ and $\mathrm{Im}\, Z(\omega)$ caused by the dc current, the PP must be specified. As usually [13–15], we take a cosine WPP of the form $U_p(x) = (U_p/2)(1 - \cos kx)$, where $k = 2\pi/a$ and a is the period; though any other non-periodic PP can also be used. Then, as it has been previously shown for the cosine WPP [16], $\tilde{\omega}_p(j_0/j_c) = \omega_p\sqrt{1 - (j_0/j_c)^2}$ and the appropriate series of the curves $\mathcal{P}(\omega \mid j_0)$ is plotted in Fig. 1. As evident from the figure data, the curves shift to the left with the increase of j_0. The reason for this is that with the increase of j_0 the PP well while tilted is broadening, as evident from Fig. 2. Thus, during the times shorter than $\tau_p = 1/\omega_p$ (i.e., for $\omega > \omega_p$) a vortex can no longer non-dissipatively oscillate in the PP well. As a consequence, the enhancement of $\mathrm{Re}\, Z(\omega)$ occurs at lower frequencies. At the same time, the curves in Fig. 1 maintain their original shape. Thus, the only universal parameter to be found experimentally is the depinning frequency ω_p. For a fixed frequency and different j_0, real part of $Z(\omega)$ always acquires larger values for larger j_0, whereas the maximum in imaginary part of $Z(\omega)$ corresponds precisely to the middle point of the nonlinear transition in $\mathrm{Re} Z(\omega)$. It should be noted that even for $T = 0$ K the dissipation, though is small, still remains non-zero even at very low frequencies.

4. Reconstruction of a pinning potential from microwave absorption data

We now turn to a detailed analytical description how to reconstruct the coordinate dependence of a PP experimentally ensued in the sample, on the basis of microwave power absorption data in the presence of a subcritical dc transport current. It will be shown that from the dependence of the depinning frequency $\tilde{\omega}_p(j_0)$ as a function of the dc transport current j_0 one can determine the coordinate dependence of the PP $U_p(x)$. The physical background for the possibility to solve such a problem is Eq. (8) which gives the correlation of the vortex rest coordinate x_0 with the value of the static force f_0 acting on the vortex and arising due to the dc current j_0.

4.1. General scheme of the reconstruction

From Eq. (8) it follows that while increasing f_0 from zero to its critical value f_c one in fact "probes" all the points of the dependence $U_p(x)$. Taking the x_0-coordinate derivative in Eq. (8), one obtains

$$\frac{dx_0}{df_0} = \frac{1}{U_p''(x_0)} = \frac{1}{\tilde{k}_p(x_0)}, \tag{11}$$

where the relation $U''(x_0) = \tilde{k}_p(x_0)$ has been used [see Eq. (10) and the text below]. By substituting $x_0 = x_0(f_0)$, Eq. (11) can be rewritten as $dx_0/df_0 = 1/\tilde{k}_p[x_0(f_0)]$, and thus,

$$\frac{dx_0}{df_0} = \frac{1}{\eta\tilde{\omega}_p(f_0)}. \tag{12}$$

If the dependence $\tilde{\omega}(f_0)$ has been deduced from the experimental data, i.e., fitted by a known function, then Eq. (12) allows one to derive $x_0(f)$ by integrating

$$x_0(f_0) = \frac{1}{\eta}\int_0^{f_0} \frac{df}{\tilde{\omega}_p(f)}. \tag{13}$$

Then, having calculated the inverse function $f_0(x_0)$ to $x_0(f_0)$ and using the relation $f_0(x_0) = U_p'(x_0)$, i.e., Eq. (8), one finally obtains

$$U_p(x) = \int_0^x dx_0\, f_0(x_0). \tag{14}$$

4.2. Example procedure to reconstruct a WPP

Here we would like to support the above-mentioned considerations by giving an example of the reconstruction procedure for a WPP. Let us suppose that a series of power absorption curves $\mathcal{P}(\omega)$ has been measured for a set of subcritical dc currents j_0. Then for determinacy, let us imagine that each *i*-curve of $\mathcal{P}(\omega \mid j_0)$ like those shown in Fig. 1 has been fitted with its fitting parameter $\tilde{\omega}_p$ so that one could map the points $[(\tilde{\omega}_p/\omega_p)_i, (j_0/j_c)_i]$, as shown by triangles in Fig. 3.

We fit the data in Fig. 3 by the function $\tilde{\omega}_p/\omega_p = \sqrt{1 - (j_0/j_c)^2}$ and then substitute it into Eq. (13) from which one calculates $x_0(f_0)$. In this case, the function has a simple analytical form, namely $x_0(f_0) = (f_c/k_p)\arcsin(f_0/f_c)$. Evidently, the inverse to it func-

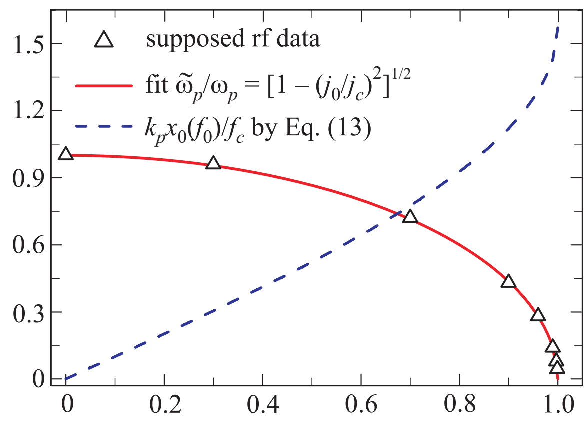

Fig. 3. The pinning potential reconstruction procedure: step 1. A set of $[(\tilde{\omega}_p/\omega_p)_i, (j_0/j_c)_i]$ points (△) has been deduced from the supposed measured data and fitted as $\tilde{\omega}_p/\omega_p = \sqrt{1-(j_0/j_c)^2}$ (solid line). Then by Eq. (13) $x_0(f_0) = (f_c/k_p)\arcsin(f_0/f_c)$ (dashed line).

tion is $f_0(x_0) = f_c \sin(x_0 k_p/f_c)$ with the period $a = 2\pi f_c/k_p$ (see also Fig. 4). By taking the integral (14) one finally gets $U_p(x) = (U_p/2)(1-\cos kx)$, where $k = 2\pi/a$ and $U_p = 2f_c^2/k_p$.

5. Conclusion

In this paper we have shown how from data on the microwave power absorption by vortices in the presence of a subcritical dc transport current the coordinate dependence of the PP in the sample can be determined. The proposed procedure can be used at $T \ll T_c$ and implies a small microwave current density $j_1 \ll j_c$. In order to keep the transport current distribution in the sample as homogeneous as possible the pinning potential is assumed to be not very "strong" in the sense that vortex pinning is caused by, e.g., the vortex length reduction rather than the superconducting order parameter suppression. Though the potential reconstruction scheme has been exemplified for a cosine WPP, i.e., for a periodic and symmetric PP, the elucidated procedure in the general case does not require periodicity of the potential and can account also for asymmetric ones. If this is the case, one has to perform the reconstruction procedure under the dc current reversal, i.e., two times: for $+j_0$ and $-j_0$. The scheme to reconstruct the WPP $U_p(x)$ from the experimental data on $\tilde{\omega}_p(j_0)$ can be briefly summarized as follows: a) by using the data $\mathcal{P}(\omega/\tilde{\omega}_p(j_0))$ to find $\tilde{\omega}_p(j_0)$; b) taking the integral (13) to calculate $x_0(f_0)$; c) then from $x_0(f_0)$ to find the inverse to it function $f_0(x_0)$; and finally d) to integrate $f_0(x_0)$ and by using Eq. (14) to recover the PP $U_p(x)$.

Theoretically, we have limited our consideration by $T = 0$ K, $j_0 < j_c$, and $j_1 \to 0$ because this has allowed us to provide a clear reconstruction procedure in terms of elementary functions accompanied by a simple physical interpretation. Experimentally, adequate measurements can be performed, i.e., on conventional thin-film superconductors (e.g., Nb, NbN) at $T \ll T_c$. These are suitable due to substantially low temperatures of the superconducting state and that relatively strong pinning in these materials allows one to neglect thermal fluctuations of a vortex with regard to the PP depth $U_p \simeq$ 1000–5000 K [19,20]. It should be stressed that due to the universal form of the dependences $\mathcal{P}(\omega \mid j_0)$, the depinning frequency ω_p plays a role of the only fitting parameter for each of the curves $\mathcal{P}(\omega \mid j_0)$, thus fitting of the measured data seems to be uncomplicated. However, one of most crucial issues for the experiment is to superimpose adequately the applied currents and then to uncouple the picked-up dc and microwave signals maintaining the matching of the impedances of the line and the sample. Quantitatively, experimentally estimated values of the depinning frequency in the absence of a dc current and a temperature of about $0.6T_c$ are $\omega_p \approx 7$ GHz for a 20 nm-thick [7] and a 40 nm-thick [8] Nb films. This value is strongly suppressed with the increase of both, the field magnitude and the film thickness.

Concerning the general validity of the results obtained, three remarks should be given. First, though the figure data have been provided here for a cosine WPP as for the most commonly used potential, the coordinate dependence can be reconstructed for not only periodic potentials. In fact, single PP wells, like one used in Ref. 5, can also be proven in accordance with the provided approach. Second, if a PP is periodic, however, it should be noted that the theoretical consideration here has been performed in the single-vortex approximation, i.e., is valid only at small magnetic fields $B \ll B_{c2}$, when the distance between two neighboring vortices, i.e., the period of a PP is larger as compared with the effective magnetic field penetration depth.

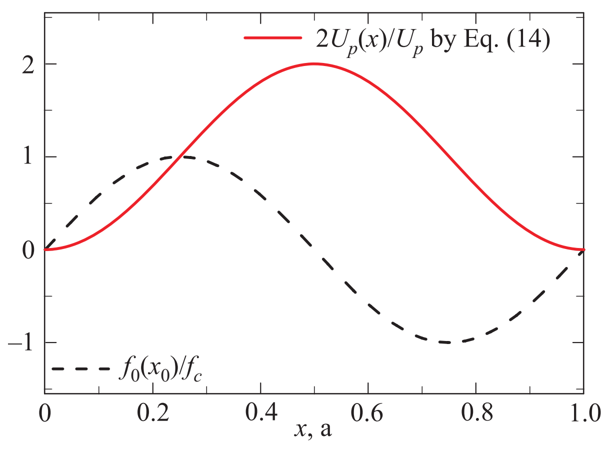

Fig. 4. The pinning potential reconstruction procedure: step 2. The inverse function to $x_0(f_0)$ is $f_0(x_0) = f_c \sin(x_0 k_p/f_c)$ (dashed line). Then by Eq. (14) $U_p(x) = (U_p/2)(1-\cos kx)$, is the PP sought (solid line).

Finally, the results can be directly verified in, e.g., the microstrip geometry for combined microwave and dc electrical transport measurements. Whereas experimental works on this matter have started to appear [4–7], we hope to have stimulated further developments in the field. Furthermore, due to the mathematical analogy between the equation of motion for a vortex used in this work and the equation for the phase difference in the Josephson junction problem, we believe that the proposed scheme of the reconstruction can also be adopted for that case.

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