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FUSION REACTIVITY ENHANCEMENT AT D-^{3}He PLASMAS DUE TO ^{3}He ADDITIVE HEATING

О.О. Шишкін, А.О. Москвітін, Ю.К. Москвітіна. **Підсилення термоядерної реактивності у D-^{3}He плазмі при нагріванні малої домішки іонів ^{3}He.** Вивчено можливість підсилення термоядерної реактивності у D-^{3}He тороідній плазмі при використанні іонного циклотронного нагрівання малої домішки іонів ^{3}He. Для проведення числового дослідження розроблено код, який базується на наближенні тестових частинок. Цей код розв'язує систему рівнянь руху у дрейфовому наближенні для іонів ^{3}He у тороїдному магнітному полі з урахуванням кулонівських зіткнень із частинками основної плазми. У код введено спрощений Монте Карло метод для моделювання іонного циклотронного нагрівання. Перехід функції розподілу іонів ^{3}He від маквелівської до немаксвелівської відіграє ключову роль у підвищені термоядерної реактивності. Формування значного хвоста функції розподілу призводить до підвищення реактивності. Це є важливим чинником для функціонування термоядерного реактора з іонним циклотронним нагріванням малої домішки іонів.

Ключові слова: плазма, термоядерний синтез, Монте Карло, немаксвелівська функція розподілу.

О.А. Шишкин, А.А. Москвитин, Ю.К. Москвитина. **Усиление термоядерной реактивности в D-^{3}He плазме при нагреве малой добавки ионов ^{3}He.** Изучена возможность усиления термоядерной реактивности в D-^{3}He тороидальной плазме при использовании ионного циклотронного нагрева малой добавки ионов ^{3}He. С целью проведения численного исследования разработан код, основанный на приближении тестовых частиц. Этот код решает систему уравнений движения в дрейфовом приближения для ионов ^{3}He в тороидальном магнитном поле с учетом кулоновских столкновений с частицами основной плазмы. Также в код введен упрощенный Монте Карло метод для моделирования ионного циклотронного нагрева. Переход функции распределения ионов ^{3}He от максвелловской к немаксвелловской играет ключевую роль в усилении термоядерной реактивности. Формирование значительного хвоста функции распределения приводит к усилению реактивности. Это является важным фактом для функционирования термоядерного реактора с использованием ионного циклотронного резонансного нагрева малой добавки ионов.

Ключевые слова: плазма, термоядерный синтез, метод Монте Карло, немаксвелловская функция распределения.

O.A. Shyshkin, A.O. Moskvitin, Yu.K. Moskvitina. **Fusion reactivity enhancement in D-^{3}He plasma due to ^{3}He additive heating.** Possible fusion reactivity enhancement due to ^{3}He minor additive ion cyclotron range frequency (ICRF) heating in D-^{3}He toroidal plasma is demonstrated through the exposed present numerical simulations. For this purpose the particle code based on test-particle approach is developed. This code solves guiding center equations for ^{3}He ions in toroidal magnetic field including Coulomb collisions of these ions with the particles of background plasms. A simple Monte Carlo model for ICRF heating is implemented in this code as well. The transformation of ^{3}He distribution function from Maxwellian to non-Maxwellian due to heating plays the key role for reactivity enhancement. The formation of significant energetic tail gives rise to the reactivity enhancement. This is an important issue for the performance of fusion reactors with minority ICRF heating.

Keywords: plasma, nuclear fusion, Monte Carlo simulation, non-Maxwellian distribution function.

One of the possible techniques to decrease neutron load on plasma facing components and superconducting coils in fusion reactors refers to users fuel cycle based on D-^{3}He reaction as alternative to D-T. Taking into account that the thermal reactivity of D-^{3}He is much lower than that of D-T, new approach such as ICRF catalyzed fusion should be developed. The main idea of this technique is to modify reagent distribution function in order to achieve favorable reaction rate for nuclear fusion energy production. The effect of transformation from the Maxwellian to non-Maxwellian plasma is essential for reactor aspects studies both in tokamaks and heliotrons.

To provide ignition analysis for D-^{3}He plasma with ^{3}He minor addition the set of particle and power balance equations should be solved [1]. The main point is that these equations include the term, which is the volume averaged reaction rate. This term gives the fusion reaction intensity and is proportional to the product of densities of fusion reagents and the averaged reactivity. The reactivity itself depends on the distribution functions of fusion reagents.

In this paper we demonstrate the possibility to increase the averaged reactivity by distribution function modification of ^{3}He minority due to ^{3}He selective ICRF heating. This study is done by means of numerical code, based on test-particle approach. A simple model for ICRF heating is included in code as well.

This paper structure is the following: firstly we introduce the particle code, and secondly we present the calculation results of ^{3}He minority distribution function under RF heating. Thirdly, we show the data from reactivity enhancement calculations, and then we summarize our study and describe the directions for the further research.

To calculate the distribution function of ^{3}He minor addition in D plasma we developed a numerical code based on the test-particle approach [2]. This code solves the guiding center equation of a general vector form given by

$$\mathbf{v}_g = v_\parallel \frac{\mathbf{B}}{B} + \frac{c}{B^2}\mathbf{E}\times\mathbf{B} + \frac{m_a c(2v_\parallel^2 + v_\perp^2)}{2Z_a e B^3}\mathbf{B}\times\nabla B + \frac{m_a c\, v_\parallel^2}{Z_a e B^4}(\mathbf{B}\times \mathrm{rot}\mathbf{B})\times\mathbf{B}. \tag{1}$$

We solve Eq. (1) by means of the Runge-Kutta integrating scheme for the test-particle of mass m_a and charge $Z_a e$ in toroidal magnetic field $\mathbf{B}$, taking into account the effect of the electric field $\mathbf{E}$. It is considered that the adiabatic invariant of motion $\mu = v_\perp^2 / B = \mathrm{const}$ together with the total energy $W = m_a(v_\perp^2 + v_\parallel^2)/2 + Z_a e\Phi = \mathrm{const}$ are conserved in the absence of collisions and RF heating.

To simulate the Coulomb collisions of test-particle with the other species the discretized collision operator based on binomial distribution is used [3]. The idea of this operator is that after each integration time step, the test-particle gets a collision kick in pitch-angle $\lambda \equiv v_\parallel / v$ and kinetic energy $K \equiv m_a v^2/2$. In case of ^{3}He ions colliding with the background deuterons and electrons the operator reads for pitch-angle scattering as

$$\lambda_n = (1 - \nu_d\,\tau)\lambda_0 \pm \sqrt{\nu_d(1-\lambda_0^2)\,\tau}\,, \tag{2}$$

and for kinetic energy slowing down and scattering

$$K_n = K_o - 2\,\nu_K\left[K_o - T\frac{x\psi'(x)}{\psi(x)}\right]\tau \pm 2\sqrt{K_o \nu_K T\,\tau}, \tag{3}$$

where ν_d is the deflection frequency, $\nu_K = \nu_S - x\nu_\parallel$ is the combination of slowing down and parallel velocity diffusion frequencies. The Maxwell integral $\psi(x)$ is a function of the square of ratio of test-particle velocity to the background species thermal velocity. The subscripts '*n*' and '*o*' stand for the new and old values respectively, and τ is the integration time step [4].

The ICRF heating of minorities is modeled by modifying the perpendicular velocity of ^{3}He when it passes the resonant layer $2\pi F_{RF} = n\omega_c$ by the value

$$\Delta v_\perp = \frac{Z_a e}{2m_a} I\left|E_+\right| J_{n-1}(k_\perp\rho_L)\cos(\varphi_r) + \frac{Z_a e}{8m_a^2 v_\perp}\left(I\left|E_+\right| J_{n-1}(k_\perp\rho_L)\right)^2 \sin^2\varphi_r, \tag{4}$$

where E_+ and φ_r are the left-circularly polarized component of RF wave electric field and random phase respectively [5]. The argument of Bessel function J_{n-1} is the product of test-particle Larmor radius ρ_L and wave field parameter $k_\perp$. At the same time $k_\parallel = 0$ for our further treatment. The time that a particle needs to pass the resonant layer is given in terms of time derivatives of cyclotron frequency ω_c and harmonic number n as $I = \min(\sqrt{2\pi/n\dot{\omega}_c}, 2\pi(n\ddot{\omega}_c/2)^{-1/3} Ai(0))$, where $Ai(0)$ is Airy function.

A simple electric field distribution is assumed throughout this study $E_+ = E_{+0}\tanh((1-r/a_{pl})/l)\cos\vartheta$ with a plasma minor radius a_{pl}, poloidal angle variable ϑ and wave field structure parameter l.

At our simulations we employ a simple toroidal magnetic field model with magnetic field components presented in the toroidal coordinates as

$$\mathbf{B} = B_0\frac{1}{h}\left(\mathbf{e}_\vartheta\,\varepsilon_t\frac{1}{q} + \mathbf{e}_\phi\right), \tag{5}$$

where $h = 1-\varepsilon_t\cos\vartheta$, $\varepsilon_t = r/R_0$ and q is the safety factor.

Parameters of fusion plasma

Major radius R_0	6,2 m
Minor radius of plasma a_{pl}	2 m
Deuterium density n_D	5×10^{19} m^{-3}
Electron density n_e	1.2 n_D
Plasma temperature T_{pl}	15 keV
Toroidal magnetic field on axis B_0	5,3 T
RF electric field amplitude E_{+0}	12 kV/m

In the present analysis, two cases of ^{3}He ion fractions have been traced during 1 second in a fusion plasma with the following parameters given in table.

In the first fraction, 24000 ions were heated on the main harmonic with the RF frequency 50 MHz at the magnetic field of 5,3 T [6]. This is the condition planned for the ICRF heating at ITER. We also employ $k_\perp = 62{,}8$ m^{-1} and $l = 0{,}2$ in the present calculations. In this case, the resonance layer R_{res} = 6,69 m appears in greater value of the periodic wave electric field and the time averaged acceleration of the particles is higher than in the other case with 15000 ions under the RF frequency 51 MHz having the resonance layer at R_{res} = 6,55 m.

At Fig. 1 the averaged ion kinetic energy for both particle fractions is displayed versus the tracing time. As one can see for both cases the energy is growing up to 600 keV for 50 MHz heating whereas up to 300 keV for 51 MHz. The kinetic energy decreases after reaching the maximum by the energy transfer to the background deuterons and electrons, which are assumed Maxwellian with a constant temperature 15 keV. The other reason of the energy decay is the escaping of energetic particles from the confinement volume. At Fig. 2 the number of ^{3}He ions in the plasma is plotted versus the tracing time.

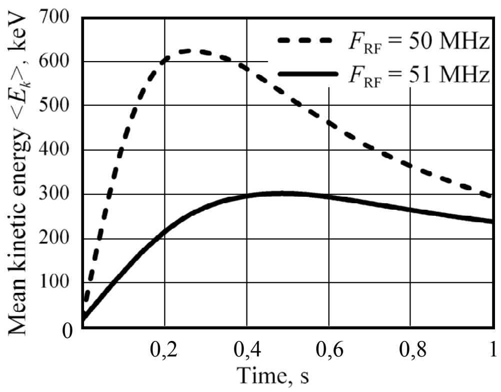

Fig. 1. The averaged kinetic energy for two ^{3}He minority fractions under different heating frequencies versus the tracing time

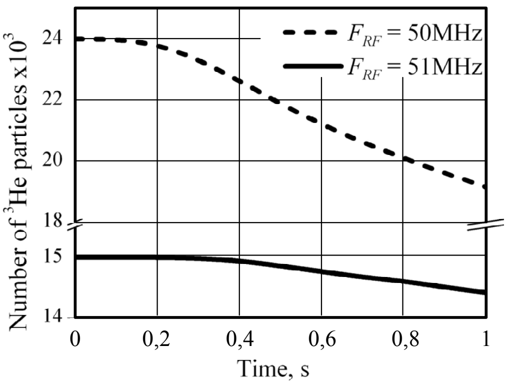

Fig. 2. The number of ^{3}He ions in the confinement volume versus the tracing time

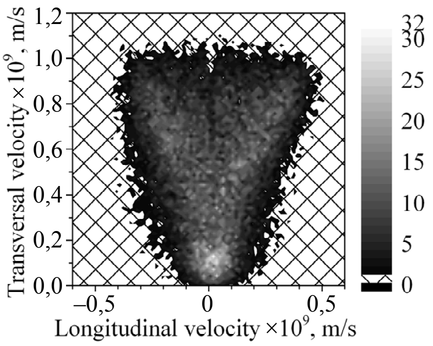

Fig. 3. Distribution function of ^{3}He ions in ($v_{||}$,$v_{\perp}$) velocity space under FRF = 50 MHz heating at the time slice t = 0,3 s

As it was pointed out in the previous section, the energy from the RF heating is deposited in the perpendicular velocity of the test-particles. This effect is included in the model by means of perpendicular velocity alteration given by (4). Hence the distribution function of ^{3}He is modified to anisotropic shape with an elongated tail in the $v_{\perp}$ direction.

The further elongation is prevented by collisions of test-species with background plasma. The energy from the test-particles transfers to the background species and spreads in the pitch-space. This is called the gyro-relaxation effect, which is included in our model by means of expressions (2) and (3). As an example, at Fig. 3 the distribution function of ^{3}He ions in ($v_{||}$, $v_{\perp}$) velocity space under $F_{RF} = 50$ MHz heating at the time slice $t = 0{,}3$ s is demonstrated. The size and the shape of the energetic tail depend on the heating efficiency, particle loses and energy transfer from the energetic fraction to the background plasma.

The distribution functions of ^{3}He ions under the $F_{RF} = 50$ MHz heating versus the absolute velocity value for several time slices starting from 0.1 s up to 1 s are displayed in Fig. 4. At each time slice there is a certain energy, which is gained from RF wave and stored in ^{3}He fraction. With the gray curves on Fig. 4, the Maxwellian distributions of ^{3}He ions are given for the same stored energy values as in the non-Maxwellian case.

The similar plot is presented at Fig. 5 with black curves for ^{3}He ions under the $F_{RF} = 51$ MHz heating. The energetic ion tail is still observed, but as it was mentioned above due to less effective heating the number of particles accelerated up to high energies is smaller compared to the previous case. Collisions with the background plasma particles (deuterons and electrons) truncate the tail as well. Despite the losses of energetic particles, displayed at Fig. 2, up to 1 s of tracing we still observe a number of particles that form energetic tails in both heating scenarios.

The fusion reactivity is calculated in general as a sixfold integral

$$\left\langle \sigma \mathrm{v} \right\rangle_{\mathrm{D}^3\mathrm{He}} = \int\int f_{\mathrm{D}}(\mathbf{v}_{\mathrm{D}}) f_{^3\mathrm{He}}(\mathbf{v}_{^3\mathrm{He}}) \sigma(\mathrm{v}) \mathrm{v}\, d\mathbf{v}_{\mathrm{D}}\, d\mathbf{v}_{^3\mathrm{He}}, \qquad (6)$$

where $\mathrm{v} = \left|\mathbf{v}_{\mathrm{D}} - \mathbf{v}_{^3\mathrm{He}}\right|$ is the relative particle velocity, f_{D} and $f_{^3\mathrm{He}}$ are the distribution functions for deuterium and ^{3}He respectively and $\sigma(\mathrm{v})$ is the fusion cross-section [7]. By means of this expression we are able to calculate the reactivity for modified distribution functions displayed at Figs. 4 and 5.

On this purpose we include in expression (6) the non-Maxwellian ^{3}He distribution functions $f_{^3\mathrm{He}}$ for both heating scenarios. At the same time deuterium distribution function f_{D} is assumed to stay in the Maxwellian with the constant temperature 15 keV. The reactivity values plotted at Fig. 6 (black) are related to the distribution functions at four time slices displayed at Fig. 4 for the $F_{RF} = 50$MHz heating scenario. The reactivity rates for $F_{RF} = 51$ MHz scenario are displayed at Fig. 7 (black). Each point on this figure is related to the distribution function shape shown (in black) at Fig. 5.

Then assuming that the ^{3}He fraction in spite of heating could have Maxwellian distribution function but with the stored energy equal to that gained from heating and these reactivities in this case are pointed with gray color at Figs. 6 and 7.

The reactivity depends on the relative velocity of the interacting particles. When we increase the energetic tail for one reacting specie (^{3}He) we increase the relative velocity and hence the reactivity by itself. In our case we observe an increment by a factor of 10.

To summarize the ICRF heating of ^{3}He minority in D-^{3}He toroidal plasma is studied by means of numerical simulations. On this purpose the particle code based on test-particle approach is developed. This code solves the guiding center equations for ^{3}He test particles taking into account Coulomb collisions by means of a discretized collision operator. A simple Monte Carlo model for ICRF heating is implemented in this code as well.

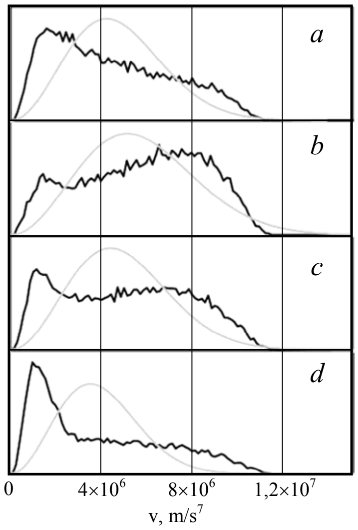

Fig. 4. Distribution functions of ^{3}He ions under F_{RF} = 50 MHz heating (black curves) in time slices 0,1, 0,3, 0,6 and 1 seconds on graphs a), b), c) and d) respectively. Maxwellian plasma distribution functions (grey curves) with the same energy stored as in cases of heating are also plotted. Vertical axes are in arbitrary units

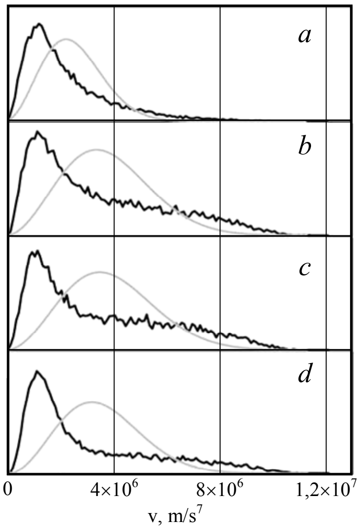

Fig. 5. Distribution functions of ^{3}He ions under FRF = 51MHz heating (black curves) in time slices 0,1, 0,3, 0,6 and 1 seconds on graphs a), b), c) and d) respectively. Maxwellian plasma distribution functions (green curves) with the same energy stored as in cases of heating are also plotted. Vertical axes are in arbitrary units

Two heating scenarios with 50 MHz and 51 MHz antenna frequency are examined at the magnetic field of 5,3 T and ITER-like plasma. It is observed that the effective heating of ^{3}He on the main harmonic with 50 MHz frequency is followed by formation of non-Maxwellian distribution function with a significant energetic tail. The same situation but with a less energetic tail appears under 51 MHz heating.

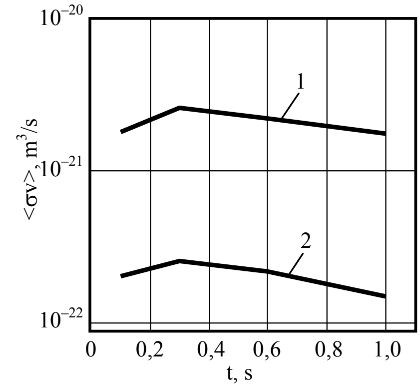

Fig. 6. Reactivity rates ^{3}He(d,p)α for non-Maxwellian ^{3}He distribution function (1) under F_{RF}=50MHz heating and corresponding Maxwellian distribution (2) with the same amount of stored energy versus the tracing time

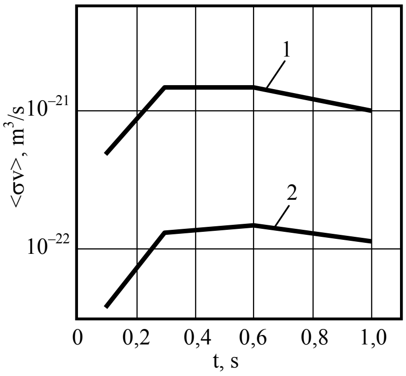

Fig. 7. Reactivity rates ^{3}He(d,p)α for non-Maxwellian ^{3}He distribution function (1) under F_{RF}=51MHz heating and corresponding Maxwellian distribution (2) with the same amount of stored energy versus the tracing time

The non-Maxwellian shape of the ^{3}He distribution function plays the key role for reactivity enhancement. It is calculated that the formation of the energetic tail gives rise to the reactivity increase of factor of 10 for both heating scenarios in the considered fusion plasma. The increase of reactivity rate is an important issue for the performance of fusion reactors, which needs further detailed studies.

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