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CALCULATION OF THE CONVEXITY ADJUSTMENT TO THE FORWARD RATE IN THE VASICEK MODEL FOR THE FORWARD IN-ARREARS CONTRACTS ON LIBOR RATE

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Abstract. We calculate the convexity adjustment to the forward rate in the Vasicek model for the in-arrears forward contracts. With the help of the no-arbitrage market condition it is shown that such adjustment should be non-negative. Analytical formulas are found for the in-arrears interest rate options.

Key words and phrases. Convexity adjustment, forward rate agreement, Vasicek model, no-arbitrage market, in-arrears LIBOR, iFRA.

2010 *Mathematics Subject Classification.* Primary 91G20; Secondary 91-02.

1. Introduction

Forward contracts are widely used financial instruments used for purchase/sale of some asset at some future date at the specified fixed price. An example of forward contract is a forward rate agreement (FRA) on interest rate as an underlying asset.

FRA is a cash settled contract with the payment based on the net difference between the floating interest rate and the fixed rate [1] . For example, assume the following data:

- fixed rate $K = 3.5\,\%$,
- floating rate $L = 4\,\%$,
- nominal $N = \$5\text{m}$,
- days in contract period $\tau = 181$.

Then at the end of the forward period there will be a settlement

$$\text{FRA payment} = (0.04 - 0.035) \cdot \$5m \cdot \frac{181}{360} = \$12,569.44.$$

Fixed rate which makes the initial price of the FRA to be equal 0 is called forward rate.

There exists an exotic in-arrears contract which is settled at the beginning of the forward period not at the end. The forward rate of an in-arrears contract is greater than the forward rate of a vanilla contract and the difference between these two rates depends on stochastic model used to simulate financial processes and called convexity adjustment. The purpose of this work is to study in-arrears contracts and calculate this adjustment.

Similar studies may be found in [2] where LIBOR in-arrears rate was considered. The adjustment was calculated using the replication strategy and solving stochastic differential equation in the Libor market model. Another approach using the change of measure was studied in [3]. However, simple lognormal stochastic model was chosen to calculate an in-arrears forward LIBOR rate. There are also researches on in-arrears options – caps and floors [4] where prices of options were found using the replication strategy for option-like pay-off. In [5] and [6], authors explored the Vasicek and Cox–Ingersoll–Ross models within LIBOR in-arrears rate. They obtained the adjustment from numerical solution of convexity term SDE.

We took a single-factor Vasicek stochastic model for the instantaneous interest spot-rate [7] and analytically calculated the adjustment using the change of measure framework. As a result, it was shown that the convexity adjustment is an increasing and convex function of mean-reversion. Moreover, we proved using the no-arbitrage principle that the convexity adjustment is non-negative regardless the stochastic model used in its modeling. Similar proof may be found in [8], but the author considers scenario with three given particular cases, not the general one. At the end, we briefly studied in-arrears options to find out that their prices also appear to be greater than those of vanilla options due to the adjustment as in the case of in-arrears forward LIBOR rate.

This paper is organised as follows. Section 2 introduces some basic definitions which we use further deriving the adjustment. Section 3 contains proof for no-arbitrage interval of rates. In Section 4, we calculate the convexity adjustment and include some figures which are consistent with results obtained in [5]. Section 5 summarises the properties of the adjustment. Finally, in Section 6, in-arrears options on interest rate are considered.

2. Definitions

We introduce some definitions which we will use further in this paper. Zero-coupon bond is often considered as a basic contract for interest rate derivative.

Definition 2.1. Zero-coupon bond (ZCB) with maturity T is a security which promises to pay owner 1 currency unit at T. We denote ZCB price at the moment t by $P(t,T)$, where $P(t,T)$ is an $\mathcal{F}_t$-measurable function and $P(T,T)=1$.

LIBOR is the indicative rate on which banks are willing to lend money each other, LIBID is the indicative rate on which banks are willing to borrow money. We assume equivalence of LIBID and LIBOR. We also make standard "Black–Sholes–Merton model" assumptions:

- no transaction costs,
- no default risk,
- no funding risk,
- no liquidity risk.

Now we define LIBOR rate and forward rate agreement more precisely.

Definition 2.2. We denote LIBOR spot rate at the moment t for a time period $\alpha > 0$ by $L(t,t,t+\alpha)$. Bank can lend (or borrow) N currency units at the time t for a period α and get (return) $N(1+\alpha L(t,t,t+\alpha))$ currency units at the moment $t+\alpha$.

Definition 2.3. Forward rate agreement (FRA) is an over-the-counter contract for the exchange of two cash flows at some date. Floating reference rate is fixed at T_1. Buyer of this contract at $t \le T_1$ with maturity T_2, fixed rate K and principal N, agrees on following obligation between counterparties at T_2:

- **pay** $(T_2 - T_1)K \cdot N$ currency units to contract counterparty,
- **receive** $(T_2 - T_1)L(T_1, T_1, T_2) \cdot N$ currency units from contract counterparty.

The price of the FRA at T_2 is equal to $(T_2 - T_1)(L(T_1,T_1,T_2) - K)N$.

For simplicity, we will assume that principal amount $N = 1$.

Definition 2.4. Forward rate $L(t,T_1,T_2)$ is the fixed rate K which makes price of the FRA contract at t equal to 0 for $t \le T_1 \le T_2$.

It can be shown [1] that $L(t,T_1,T_2) = \frac{P(t,T_1)-P(t,T_2)}{(T_2-T_1)P(t,T_2)}$.

Now, we consider exotic in-arrears FRA: this contract is settled at time T_1, rather than T_2 as plain vanilla FRA.

Definition 2.5. In-arrears FRA (iFRA) is an over-the-counter contract for the exchange of two cash flows at some date. Floating reference rate is fixed at T_1. Buyer of this contract at $t \le T_1$ with maturity T_1, fixed rate K and principal N, agrees on following obligation between counterparties at T_1 (not T_2):

- **pay** $(T_2 - T_1)K \cdot N$ currency units to counterparty,
- **receive** $(T_2 - T_1)L(T_1, T_1, T_2) \cdot N$ currency units from counterparty.

The price of the iFRA at T_1 is equal to $(T_2 - T_1)(L(T_1, T_1, T_2) - K)N$.

We will denote K which makes iFRA to have a 0 price at t by $iL(t, T_1, T_2)$.

A portfolio of assets is called self-financed if its value changes only due to changes in the asset prices.

Definition 2.6. Self-financed portfolio A is called an arbitrage portfolio on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$ if its price (value) at the time t is $V^A(t) \le 0$ and $\exists T > t$: $\mathbb{P}(V^A(T) \ge 0) = 1$ and $\mathbb{P}(V^A(T) > 0) > 0$.

We use the assumption of absence of any arbitrage portfolio on the market.

3. No-arbitrage values of in-arrears forward LIBOR rate

Statement 3.1. *Suppose that* $\mathbb{P}(L(T_1, T_1, T_2) \ne L(t, T_1, T_2)) > 0$ *under real-word measure. Then the in-arrears forward rate* $iL(t, T_1, T_2) >$ *forward rate* $L(t, T_1, T_2)$, $t < T_1 \le$ $\le T_2$.

Opportunity to get the same payoff in the iFRA as in the FRA should be more expensive because it can be quickly used for one's operation needs. We will prove statement by contradiction assuming opposite and constructing an arbitrage portfolio.

Proof. Assume that $iL(t, T_1, T_2) \le L(t, T_1, T_2)$. Without loss of generality, let $(T_2 - T_1) =$ $= 1$ year. Consider the following strategy:

- t: buy iFRA contract with $K = iL(t, T_1, T_2)$, principal amount $N = \$1$, sell FRA contract with $K = L(t, T_1, T_2)$ and principal $N = \$1 \cdot (1 + L(t, T_1, T_2))$. Portfolio value $V_t = 0$ because the contracts are made with forward rates.
- T_1: iFRA settlement occurs
$$V_{T_1} = (L(T_1, T_1, T_2) - iL(t, T_1, T_2)).$$
We fund this payoff (lending it or borrowing depending on sign of payoff) using $L(T_1, T_1, T_2)$ rate.
- T_2: FRA settlement occurs
$$V_{T_2} = \underbrace{(1 + L(t, T_1, T_2))(L(t, T_1, T_2) - L(T_1, T_1, T_2))}_{\text{FRA payoff}} + \\ + \underbrace{(L(T_1, T_1, T_2) - iL(t, T_1, T_2))}_{\text{iFRA payoff}}(1 + L(T_1, T_1, T_2)).$$
Using the fact that $iL(t, T_1, T_2) \le L(t, T_1, T_2)$, rewrite the portfolio value as $V_{T_2} \ge (L(t, T_1, T_2) - L(T_1, T_1, T_2))^2 \ge 0$.

We managed to construct an arbitrage portfolio which is in contradiction with our assumption of no-arbitrage. Hence, $iL(t, T_1, T_2) > L(t, T_1, T_2)$. □

Remark. It is worth noting that when $t = T_1$ we get $iL(t, T_1, T_2) = L(t, T_1, T_2)$ because the reference rate $L(T_1, T_1, T_2)$ is fixed immediately at t and should be equal to the forward rate.

We proved Statement 3.1 without assuming anything about the stochastic model. Therefore, it holds regardless of the model which we use to simulate financial processes.

4. Calculation of the in-arrears forward LIBOR rate

We use the Vasicek model as a model for instantaneous interest spot-rate

$$dr(t) = (\theta - ar(t))dt + \sigma dW(t).$$

Solving this SDE explicitly [2], one can see that

$$r(t) \sim N\left(r(0)e^{-at} + \theta\frac{1-e^{-at}}{a}, \sigma^2\frac{1-e^{-2at}}{2a}\right).$$

Zero-coupon bond with maturity T has the following value under this model

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)},$$

where

$$B(t,T) = \frac{1-e^{-a(T-t)}}{a},$$

$$A(t,T) = \exp\left((B(t,T)-(T-t))\left(\frac{\theta}{a} - \frac{\sigma^2}{2a^2}\right) - \frac{\sigma^2 B(t,T)^2}{4a}\right).$$

Consider iFRA. Forward rate is the expected value of the future rate under appropriate forward measure [10]:

$$iL(t,T_1,T_2) = \mathbb{E}_{Q_{T_1}}[L(T_1,T_1,T_2)|\mathcal{F}_t]. \tag{1}$$

Process $L(t,T_1,T_2)$ is not a martingale under Q_{T_1} as it is under Q_{T_2}. Therefore, we will change measure using the Radon–Nykodym theorem [9]

$$\frac{dQ_{T_1}}{dQ_{T_2}} = \frac{P(0,T_2)}{P(0,T_1)}(1+(T_2-T_1)L(T_1,T_1,T_2)).$$

Changing measure in (1), we get

$$iL(t,T_1,T_2) = \frac{P(t,T_2)}{P(t,T_1)}L(t,T_1,T_2) + (T_2-T_1)\frac{P(t,T_2)}{P(t,T_1)}\mathbb{E}_{Q_{T_2}}[L(T_1,T_1,T_2)^2|\mathcal{F}_t]. \tag{2}$$

To calculate $iL(t,T_1,T_2)$, we should find

$$\mathbb{E}_{Q_{T_2}}[L(T_1,T_1,T_2)^2|\mathcal{F}_t] = \frac{1}{(T_2-T_1)^2}\mathbb{E}_{Q_{T_2}}\left[\left(\frac{P(T_1,T_1)}{P(T_1,T_2)} - 1\right)^2\middle|\mathcal{F}_t\right]. \tag{3}$$

We need to know the distributions of the following functions: $\left(\frac{P(T_1,T_1)}{P(T_1,T_2)}\right)^2$ and $\frac{P(T_1,T_1)}{P(T_1,T_2)}$.

Using Ito's lemma [10], we find $P(t,T_i)$ under the risk-neutral measure

$$dP(t,T_1) = r(t)P(t,T_1)dt + \sigma B(t,T_1)P(t,T_1)dW(t),$$
$$dP(t,T_2) = r(t)P(t,T_2)dt + \sigma B(t,T_2)P(t,T_2)dW(t).$$

We denote $\sigma B(t,T_i) = \zeta^{T_i}(t)$. We write the dynamics of the ratio of bonds with maturity T_1 and T_2 respectively under the risk-neutral measure [10]:

$$d\left(\frac{P(t,T_1)}{P(t,T_2)}\right) = \frac{P(t,T_1)}{P(t,T_2)}\big(\zeta^{T_1}(t) - \zeta^{T_2}(t)\big)\big(dW(t) - \zeta^{T_2}(t)dt\big).$$

Now, change the risk-neutral measure to the forward Q_{T_2}-measure using the Girsanov theorem [10]

$$W^{T_2}(t) = W(t) - \int_0^t \zeta^{T_2}(s)ds.$$

Then

$$d\left(\frac{P(t,T_1)}{P(t,T_2)}\right) = \frac{P(t,T_1)}{P(t,T_2)}(\zeta^{T_1}(t) - \zeta^{T_2}(t))dW^{T_2}(t).$$

We solve the following stochastic differential equation

$$d\left(\ln\left(\frac{P(t,T_1)}{P(t,T_2)}\right)\right) = \frac{1}{\frac{P(t,T_1)}{P(t,T_2)}} d\left(\frac{P(t,T_1)}{P(t,T_2)}\right) - \frac{1}{2}\frac{1}{\left(\frac{P(t,T_1)}{P(t,T_2)}\right)^2} d\left(\frac{P(t,T_1)}{P(t,T_2)}\right)^2 =$$

$$= (\zeta^{T_1}(t) - \zeta^{T_2}(t))dW^{T_2}(t) - \frac{1}{2}(\zeta^{T_1}(t) - \zeta^{T_2}(t))^2 dt.$$

$$\frac{P(T_1,T_1)}{P(T_1,T_2)} = \frac{P(t,T_1)}{P(t,T_2)} e^{\int_t^{T_1}(\zeta^{T_1}(t)-\zeta^{T_2}(t))dW^{T_2}(t) - \frac{1}{2}\int_t^{T_1}(\zeta^{T_1}(t)-\zeta^{T_2}(t))^2 dt}.$$

Next, we find the expectation of this lognormal process

$$\mathbb{E}_{Q_{T_2}}\left[\frac{P(T_1,T_1)}{P(T_1,T_2)}\middle|\mathcal{F}_t\right] = \frac{P(t,T_1)}{P(t,T_2)}. \tag{4}$$

So, $\frac{P(t,T_1)}{P(t,T_2)}$ is a martingale under Q_{T_2}-measure.

Now, we do the same for the $\left(\frac{P(t,T_1)}{P(t,T_2)}\right)^2$ under Q_{T_2}-measure.

$$d\left(\ln\left(\frac{P(t,T_1)}{P(t,T_2)}\right)^2\right) = 2\cdot d\left(\ln\left(\frac{P(t,T_1)}{P(t,T_2)}\right)\right),$$

$$d\left(\ln\left(\frac{P(t,T_1)}{P(t,T_2)}\right)^2\right) = 2(\zeta^{T_1}(t) - \zeta^{T_2}(t))dW^{T_2}(t) - (\zeta^{T_1}(t) - \zeta^{T_2}(t))^2 dt.$$

It follows that

$$\left(\frac{P(T_1,T_1)}{P(T_1,T_2)}\right)^2 = \left(\frac{P(t,T_1)}{P(t,T_2)}\right)^2 e^{2\int_t^{T_1}(\zeta^{T_1}(t)-\zeta^{T_2}(t))dW^{T_2}(t) - \int_t^{T_1}(\zeta^{T_1}(t)-\zeta^{T_2}(t))^2 dt}.$$

$$\mathbb{E}_{Q_{T_2}}\left[\left(\frac{P(T_1,T_1)}{P(T_1,T_2)}\right)^2\middle|\mathcal{F}_t\right] = \left(\frac{P(t,T_1)}{P(t,T_2)}\right)^2 e^I, \tag{5}$$

where $I = \int_t^{T_1}(\zeta^{T_1}(t) - \zeta^{T_2}(t))^2 dt$,

$$I = \frac{\sigma^2}{a^2}\left(\frac{1}{2a} - \frac{1}{2a}e^{-2a(T_1-t)} - \frac{1}{a}e^{-a(T_2-T_1)} + \frac{1}{2a}e^{-2a(T_2-T_1)} + \right.$$
$$\left. + \frac{1}{a}e^{-a(T_1+T_2)+2at} - \frac{1}{2a}e^{-2a(T_2-t)}\right).$$

Substituting (4) and (5) in (3), we rewrite (2) distinguishing $L(t,T_1,T_2)$

$$iL(t,T_1,T_2) = L(t,T_1,T_2) + L(t,T_1,T_2)\frac{P(t,T_2) - P(t,T_1)}{P(t,T_1)} +$$
$$+ \frac{P(t,T_2)}{(T_2-T_1)P(t,T_1)}\left[\left(\frac{P(t,T_1)}{P(t,T_2)}\right)^2 e^I - 2\left(\frac{P(t,T_1)}{P(t,T_2)}\right) + 1\right]. \tag{6}$$

It can be seen that CA has an upper bound, and we can find it for $T_1 \to \infty$. Let $T_2 = T_1 + \tau$, then

$$\lim_{T_1\to\infty} CA = \frac{1}{\tau}e^{(\frac{\theta}{a} - \frac{\sigma^2}{2a})\tau}(e^{\frac{\sigma^2}{2a^3}(1-2e^{-a\tau}+e^{-2a\tau})} - 1).$$

Here are some values of the convexity adjustment for different parameters as a function of beginning of accruing period T_1, speed of mean reversion θ and volatility σ.

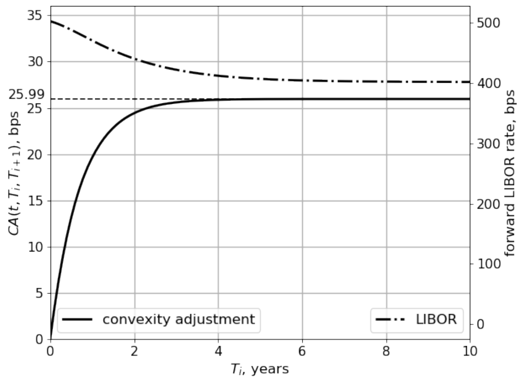

FIGURE 1. Figure of adjustment as a function of maturity with $t = 0$, $\sigma = 10\,\%$, $T_{i+1} - T_i = 0.5$, $\theta = 0.035$, $r(t) = 5\,\%$ and forward LIBOR rate

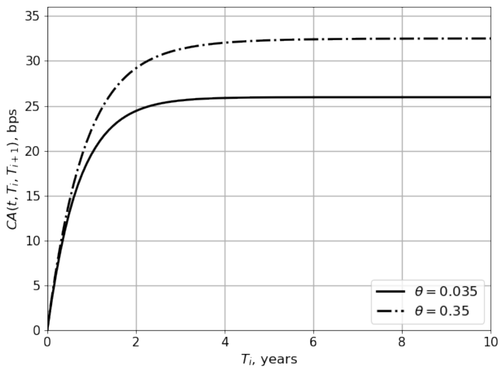

FIGURE 2. $t = 0$, $\sigma = 10\,\%$, $T_{i+1} - T_i = 0.5$, $a = 0.7$, $r(t) = 5\,\%$

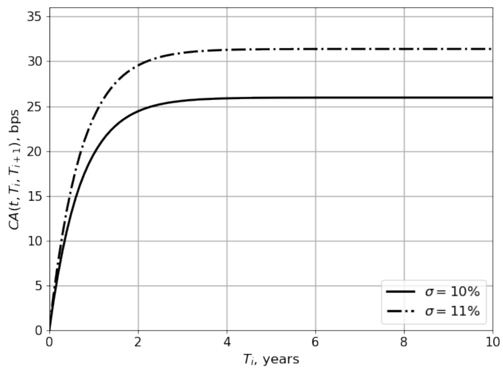

FIGURE 3. $t = 0$, $\theta = 0.035$, $T_{i+1} - T_i = 0.5$, $a = 0.7$, $r(t) = 5\,\%$

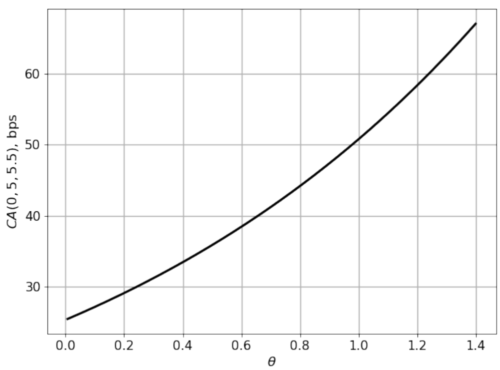

FIGURE 4. $t = 0$, $T_1 = 5$ years, $T_2 - T_1 = 0.5$, $r(t) = 5\,\%$

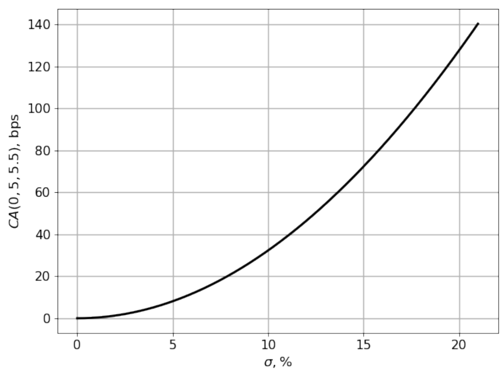

FIGURE 5. $t = 0$, $T_1 = 5$ years, $T_2 - T_1 = 0.5$, $r(t) = 5\,\%$

5. PROPERTIES

Now, we study the sign of the first and the second derivative of the CA with respect to mean-reversion. Rewrite it from (6) as follows:

$$CA(t, T_1, T_2) = \frac{1}{T_2 - T_1} \frac{P(t, T_1)}{P(t, T_2)} (e^I - 1).$$

Then the first derivative is

$$\frac{\partial CA(t,T_1,T_2)}{\partial \theta} = \frac{e^I - 1}{T_2 - T_1}\frac{\partial}{\partial \theta}\left(\frac{P(t,T_1)}{P(t,T_2)}\right), \tag{7}$$

$$\frac{\partial}{\partial \theta}\left(\frac{P(t,T_1)}{P(t,T_2)}\right) = \frac{1}{a}\frac{P(t,T_1)}{P(t,T_2)}(B(t,T_1) - B(t,T_2) + T_2 - T_1), \tag{8}$$

$$B(t,T_1) \geq B(t,T_2),\ I \geq 0.$$

So,

$$\frac{\partial CA(t,T_1,T_2)}{\partial \theta} \geq 0. \tag{9}$$

It is clear from the (7) and (8) that

$$\frac{\partial^2 CA(t,T_1,T_2)}{\partial \theta^2} \geq 0. \tag{10}$$

Finally, we can summarise the properties of the convexity adjustment:

- Fig. 1 confirms our Statement 3.1. that CA is non-negative.
- CA has a limit when $T_1 \to \infty$.
- From (9) and (10) we could note that CA is increasing and convex function of mean-reversion.

6. Options

As a part of our study of an in-arrears contract we also consider in-arrears options on interest rate – caplet and floorlet.

Definition 6.1. An in-arrears caplet (floorlet) is a European-style call (put) option on interest rate which is fixed at T_1. Buyer of this option at $t \leq T_1$ with maturity T_1, strike K and principal amount N is offered with the following rights at time T_1:

- **pay (receive)** $(T_2 - T_1)K \cdot N$ currency units,
- **receive (pay)** $(T_2 - T_1)L(T_1,T_1,T_2) \cdot N$ currency units.

Now, we find prices of this options.

$$\mathrm{Cpl}(t,T_1,T_2,K) = (T_2 - T_1)P(t,T_1)\mathbb{E}_{Q_{T_1}}[(L(T_1,T_1,T_2) - K)^+|\mathcal{F}_t],$$
$$\mathrm{Fl}(t,T_1,T_2,K) = (T_2 - T_1)P(t,T_1)\mathbb{E}_{Q_{T_1}}[(K - L(T_1,T_1,T_2))^+|\mathcal{F}_t].$$

We apply the same change of measure technique as in Section 3.

$$\mathrm{Cpl}(t,T_1,T_2,K) = P(t,T_2)\mathbb{E}_{Q_{T_2}}\left[\frac{P(T_1,T_1)}{P(T_1,T_2)}\left(\frac{P(T_1,T_1)}{P(T_1,T_2)} - 1 - (T_2 - T_1)K\right)^+\middle|\mathcal{F}_t\right]. \tag{11}$$

We split the mathematical expectation in (11) into 2 parts:

$$\mathbb{E}_{Q_{T_2}}\left[\left(\frac{P(T_1,T_1)}{P(T_1,T_2)}\right)^2 \mathbb{1}_{\frac{P(T_1,T_1)}{P(T_1,T_2)} > 1 + (T_2 - T_1)K}\middle|\mathcal{F}_t\right] =$$
$$= \left(\frac{P(t,T_1)}{P(t,T_2)}\right)^2 \frac{1}{\sqrt{2\pi}}\int_l^\infty \exp\left(\frac{-x^2}{2}\right)\exp\left(2x\sqrt{I} - I\right)dx =$$
$$= \left(\frac{P(t,T_1)}{P(t,T_2)}\right)^2 e^I(1 - N(l - 2\sqrt{I})),$$

and

$$\mathbb{E}_{Q_{T_2}}\left[\frac{P(T_1,T_1)}{P(T_1,T_2)}\mathbb{1}_{\frac{P(T_1,T_1)}{P(T_1,T_2)}>1+(T_2-T_1)K}\middle|\mathcal{F}_t\right] =$$
$$= \frac{P(t,T_1)}{P(t,T_2)}\frac{1}{\sqrt{2\pi}}\int_l^{\infty}\exp\left(\frac{-x^2}{2}\right)\exp\left(x\sqrt{I}-\frac{1}{2}I\right)dx =$$
$$= \frac{P(t,T_1)}{P(t,T_2)}(1-N(l-\sqrt{I})),$$

where

$$l = \frac{\ln\left(\frac{1+(T_2-T_1)K}{\frac{P(t,T_1)}{P(t,T_2)}}\right)+\frac{1}{2}I}{\sqrt{I}},$$

and

$$I = \int_t^{T_1}(\zeta^{T_1}(t)-\zeta^{T_2}(t))^2dt.$$

So, the caplet price is

$$\mathrm{Cpl}(t,T_1,T_2,K) = \frac{P(t,T_1)^2}{P(t,T_2)}e^I N(2\sqrt{I}-l) - (1+(T_2-T_1)K)P(t,T_1)N(\sqrt{I}-l).$$

We will find the floorlet price using put-call parity of European options.

$$\mathrm{Fl}(t,T_1,T_2,K) = \mathrm{Cpl}(t,T_1,T_2,K) - (T_2-T_1)P(t,T_1)(iL(t,T_1,T_2)-K).$$

Here, we compare vanilla and in-arrears options on the figures.

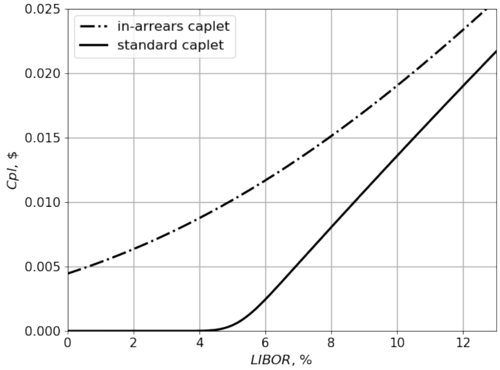

FIGURE 6. $K = 5\,\%$, $t = 0$, $T_1 = 0.5$ year, $T_2 = 1$ year, $\sigma = 10\,\%$, $\theta = 0.035$, $r(t) = 5\,\%$

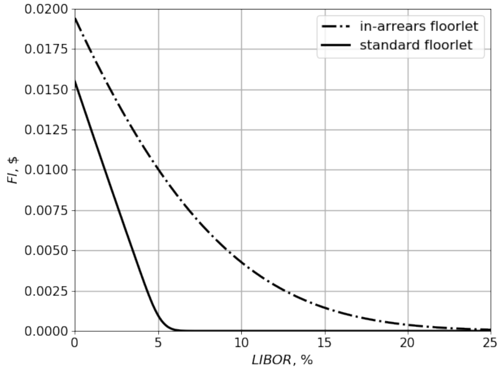

FIGURE 7. $K = 5\,\%$, $t = 0$, $T_1 = 0.5$ year, $T_2 = 1$ year, $\sigma = 10\,\%$, $\theta = 0.035$, $r(t) = 5\,\%$

As in the case of iFRA, it is clear from Fig. 6 and Fig. 7 that prices of in-arrears options are greater than of vanilla options due to the adjustment.

7. Conclusion

We derived the formula for calculation of the forward rate in iFRA. It was proved that the convexity adjustment to the plain vanilla forward rate can not be negative. We also studied in-arrears option contracts and found that their prices are greater than those of vanilla options.

Further development of this work assumes usage of two-factor interest rate model which captures more realistic forward curve. In addition, there may be cases when contract is settled at any time, not only T_2 or T_1.

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РОЗРАХУНОК ПОПРАВКИ ДО ФОРВАРДНОЇ СТАВКИ В МОДЕЛІ ВАСІЧЕКА ДЛЯ ФОРВАРДНИХ IN-ARREARS КОНТРАКТІВ НА ВІДСОТКОВУ СТАВКУ LIBOR

М. О. МАЛИХ, І. С. ПОСТЄВОЙ

Анотація. У роботі ми розрахували поправку до форвардної ставки в моделі Васічека для in-arrears форвардних контрактів. З умов безарбітражності ринку показано, що така поправка повинна бути невід'ємною. Також знайдено аналітичну поправку для in-arrears опціонів на відсоткову ставку.

РАСЧЕТ ПОПРАВКИ К ФОРВАРДНОЙ СТАВКЕ В МОДЕЛИ ВАСИЧЕКА ДЛЯ ФОРВАРДНЫХ IN-ARREARS КОНТРАКТОВ НА ПРОЦЕНТНУЮ СТАВКУ LIBOR

Н. О. МАЛЫХ, И. С. ПОСТЕВОЙ

Аннотация. В данной работе мы рассчитали поправку к форвардной ставке в модели Васичека для in-arrears форвардных контрактов. Из условий безарбитражности рынка показано, что такая поправка должна быть неотрицательной. Также была найдена аналитическая поправка для in-arrears опционов на процентную ставку.