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MATHEMATICAL FOUNDATIONS OF ADVANCED INFORMATICS: SPECIFICS OF TEACHING THE COURSE WITH UTILIZATION OF THE SYSTEM GRAN AND UTILIZATION FOR PARTICIPANT-DIRECTED LEARNING

Abstract: the article considers the issue of computer training of mathematics teachers in the context of using computer mathematics systems as a teaching tool. The analysis of propaedeutic tasks of the course "Mathematical foundations of Advanced Informatics" (MFAI) is carried out. The system of tasks is considered and situational tasks are analyzed with the use of rules-guidelines and methodical instructions for solving problems with the use of the GRAN system. An updated syllabus of the training course is offered taking into account modern requirements. The analysis of the logical-structural scheme of the training course "Mathematical foundations of Advanced Informatics" (MFAI) is considered. The main advantages of using the GRAN system are demonstrated. The results of the conducted pedagogical experiment using the computer mathematics system GRAN are offered. Criteria, indicators and methods of diagnosing the effectiveness of student learning are substantiated. The classification of problems is carried out and examples of didactic materials for teaching the course using computer mathematics systems GRAN are offered.

Key words: research studying, mathematical foundations of advanced informatics, GRAN, system of dynamic mathematics, computer-oriented learning environments.

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МАТЕМАТИЧНІ ОСНОВИ ІНФОРМАТИКИ: ОСОБЛИВОСТІ НАВЧАННЯ КУРСУ З ВИКОРИСТАННЯ СИСТЕМИ КОМП'ЮТЕРНОЇ МАТЕМАТИКИ GRAN

Анотація: у дослідженні розглядається проблематика інформатичної підготовки вчителів математики в контексті використання систем комп'ютерної математики як засобу навчання. Здійснено аналіз пропедевтичних завдань курсу "Математичні основи інформатики". Розглядається система задач і аналізуються ситуаційні задачі з використанням правил-орієнтирів та методичних вказівок до розв'язування завдань з використанням системи GRAN. Пропонується оновлений силабус навчального курсу із врахуванням сучасних вимог. Розглядається аналіз логічно-структурної схеми навчального курсу "Математичні основи інформатики". Демонструються основні переваги використання системи GRAN. Пропонуються результати проведеного педагогічного експерименту з використанням системи комп'ютерної математики GRAN. Обґрунтовуються критерії, показники та методи діагностики ефективності навчання студентів. Здійснено класифікацію задач і пропонуються приклади дидактичних матеріалів для навчання з використанням систем комп'ютерної математики GRAN.

Ключові слова: дослідницьке навчання, математичні основи інформатики, GRAN, система динамічної математики, комп'ютерно-орієнтована методична система навчання, математичне моделювання.

Елена Грибюк

МАТЕМАТИЧЕСКИЕ ОСНОВЫ ИНФОРМАТИКИ: ОСОБЕННОСТИ ОБУЧЕНИЯ С ИСПОЛЬЗОВАНИЕМ СИСТЕМЫ КОМПЬЮТЕРНОЙ МАТЕМАТИКИ GRAN

Аннотация: в статье рассматривается проблематика качественной подготовки учителей математики в контексте использования систем компьютерной математики как средства обучения. Осуществлен анализ пропедевтических задач курса "Математические основы информатики". Рассматривается система задач и анализируются исследовательские задачи с использованием правил–ориентиров и методических указаний к решению задач. Предлагается обновленная программа (профиль) учебного курса с учетом современных требований. Рассматривается анализ структурной схемы учебного курса "Математические основы информатики". Демонстрируются основные преимущества использования системы компьютерной математики GRAN. Предлагаются результаты проведенного педагогического эксперимента с использованием системы компьютерной математики GRAN. Обосновываются критерии, показатели и методы диагностики эффективности обучения студентов. Приводится классификация задач, рассматриваются примеры дидактических материалов для обучения с использованием систем компьютерной математики GRAN.

Ключевые слова: исследовательское обучение, математические основы информатики, GRAN, система динамической математики, компьютерно−ориентированная методическая система обучения, математическое моделирование.

Олена Гриб'юк

Розширена анотація для ознайомлення з цією темою:

"Математичні основи інформатики: особливості навчання курсу з використання системи комп'ютерної математики gran "

Постановка проблеми у загальному вигляді *та її зв'язок із важливими науковими чи практичними завданнями. У дослідженні розглядається проблематика інформатичної підготовки вчителів математики в контексті використання систем комп'ютерної математики як засобу навчання. Підготовка майбутніх фахівців є однією з головних проблем вищих навчальних закладів. В процесі підготовки вчителя математики необхідно враховувати особливості та умови навчання, а також формування особистості професіонала. Аналізуючи характер і зміст роботи вчителів математики в умовах інформаційного суспільства, необхідно враховувати мету та зміст освіти, організаційні форми, методи та підходи професійної підготовки майбутніх учителів математики .*

Аналіз останніх досліджень та публікацій. *Аналіз навчальної та навчально-педагогічної літератури демонструє напруженість досліджень з точки зору включення інформаційно-комунікаційних технологій, зокрема комп'ютерно-орієнтованих систем освіти. Освітні дослідження в цій галузі розпочав В. Биков, В. Глушков, В. Давидов [1], А. Єршов [2], Н. Моісєєв [7], М. Жалдак [12].*

Формулювання цілей статті *(основні завдання). Мета визначити зміст та вплив викладання курсу МФАІ, що сприяє формуванню системи інформатичної компетентності майбутніх фахівців.*

Виклад основного матеріалу дослідження *з повним обґрунтуванням отриманих наукових результатів. Здійснено аналіз пропедевтичних завдань курсу "Математичні основи інформатики". Ґрунтовно представлено можливості педагогічного проектування з педагогічно виваженим використанням комп'ютерно орієнтованих методичних систем навчання предметів математичного циклу в контексті неперервності освіти. Основна мета математичної освіти полягає також в розвитку вміння математично, логічно та усвідомлено досліджувати явища навколишнього світу. Реалізації такої ідеї сприятиме розв'язування на, тому використання вчителем дослідницьких задач є не тільки бажаним, але навіть обов'язковим елементом навчального процесу. Педагогічно виважене використання компонентів комп'ютерно орієнтованих методичних систем дослідницького навчання в дослідженні розглядається в декількох напрямах: уточнення термінологічного апарату та механізмів роботи інструментів із врахуванням системи понять і тверджень шкільного курсу математики; розширення спектру предметів математичного циклу і системи дослідницьких задач, розрахунково-*

графічних робіт з педагогічно виваженим і методично вмотивованим використанням систем комп'ютерної математики; розширення можливостей експорту та імпорту навчального матеріалу в рамках дослідницького навчання; підвищення доступності комп'ютерно орієнтованих методичних систем дослідницького навчання в умовах різного рівня технічного забезпечення. Переваги і недоліки комп'ютерного моделювання розглядаються в контексті навчальної і методичної діяльності, для підтримки якої вони призначені. Розглядається система задач і аналізуються ситуаційні задачі з використанням правил-орієнтирів та методичних вказівок до розв'язування завдань з використанням системи GRAN. Пропонується оновлений силабус навчального курсу із врахуванням сучасних вимог. Розглядається аналіз логічно-структурної схеми навчального курсу "Математичні основи інформатики". Демонструються основні переваги використання системи GRAN. Пропонуються результати проведеного педагогічного експерименту з використанням системи комп'ютерної математики GRAN. Обґрунтовуються критерії, показники та методи діагностики ефективності навчання студентів. Здійснено класифікацію задач і пропонуються приклади дидактичних матеріалів для навчання курсу з використанням систем комп'ютерної маремати-ки GRAN.

***Висновки з даного дослідження та перспективи подальших розвідок** у даному напрямку**.** Розроблено та апробовано навчальний курс "Математичні основи передової інформатики" (МЗВІ) для майбутніх учителів математики з педагогічним використанням незалежних компонентів комп'юте-рноорієнтованого середовища навчання. Побудовано логічну та структурну схеми навчального курсу МФАІ. Розроблено методичні рекомендації до навчального курсу МЗС. Проаналізовано використання системи GRAN у процесі вирішення пропедевтичних завдань для узгодження знань майбутніх учителів математики для успішного засвоєння курсу МФАІ.*

Problem setting. Preparation of future professionals is one of the main challenges of higher educational establishments. During the process of preparation of mathematics teacher, it is necessary to take into account the peculiarities and conditions of education, as well as formation of personality of the professional. While analyzing the character and content of mathematics teachers' work in the conditions of information society, it is necessary to take into account the goal and the content of education, the organizational forms, methods, and approaches of professional preparation of the future teachers of mathematics. Contemporary school needs specialists that can contribute to the development of independent and responsible personality of individual and development of his/her creative personality. Today, it is impossible to solve problems that appear in the process of construction and organization of educational process through the traditional methods, basing only on personal experience; it is necessary to take into account the social and cultural needs of the younger generation, as well as innovational and international experience. The modern programs in mathematics present the irrelevant mathematical knowledge that causes the absence of interdisciplinary relations. Educational syllabus need to be cleared of the secondary materials, and to be reviewed from the perspective of competent approach to education, reorientation of content towards the ideological function of natural sciences, profilization of mathematical disciplines to the practical orientation. It is especially important for the level of standards in the higher educational establishments. Students, the future teachers of mathematics, are not prepared enough for education. According to the Law of Ukraine on higher education, it is necessary to incorporate the leveling course "Mathematical foundations of Advanced Informatics" (MFAI) that contains the propedeutic chapter "Theory of research problems solution" that consists of the situational tasks, including the tasks with parameters that are solved using the system of the computer mathematics and that became the indicator of success. The course MFAI covers the mathematics methods that are also taught in the courses of informatics, mathematical modeling, mathematical informatics, etc. The direction of scientific research of theoretical foundations of informatics is the mathematical models and means that are used for modeling and research of information processes in different fields of activities of the human. This course teaches the main models, methods, and algorithms of problem solution that appear in the sphere of intellectualization of information systems. Also the problems of utilization of information (in particular mathematical) models and information technologies for their research are considered

Recent research and publications analysis. The analysis of educational and educational-pedagogic literature demonstrates the intensity of

the research in terms of incorporation of information and communication technologies, in particular the computer-oriented systems of education. The educational research in this sphere was started by V. Glushkov, V. Davydov [1], V. Bykov, A. Ershov [2], N. Moiseev [7], M. Zhaldak [12].

The pedagogic programs Derive, Gran1, Gran-2D, Gran-3D, GeoGebra, DG, Maple, Mathematika, MathLab, Maxima and others were developed and utilized. Some of them are oriented at specialists of relatively high qualification in the field of mathematics, while others are focused on the pupils or students that only started to learn the school course of mathematics. In the process of teaching the school course of mathematics in the secondary school, it is relevant to utilize the programs GRAN (Gran1, Gran-2D, Gran-3D) and Derive. The above-mentioned programs are easy to use and have the interface similar to the interface of the other similar, general programs. Thanks to the visual programming of dynamic applets GRAN, it is easier to solve the project problems, to complete the analytical research, to research functions and to make their graphs, as well as to automate the mathematical calculations.

The problem of creation and incorporation of computer-oriented methodical systems was researched by M. Zhaldak [12], E. Polat [8], S. Rakov and others. The problems of preparation of future teachers of mathematics in Ukraine on the contemporary stage were researched by the following famous mathematicians, pedagogues and methodists: O. Skafa, S. Rakov and others. The questions of incorporation of informational and communication technologies into the school education were researched by the following Ukrainian researchers: M. Zhaldak [12], A. Ershov [2], Yu. Mashbits [6], V. Monahov [7] and others. However, the question concerning the creation of methodological and didactic provision of the system of dynamic mathematics GRAN in the process of teaching the mathematical foundations of informatics, the choice of relevant heuristic, research and practical problems that bring the incorporation of STEM–education, creation of variative models and the utilization of the proposed system for the development of informatics and research competencies of the future teachers of mathematics is not studied and researched enough [5].

Current research demonstrates the lack of informatics preparation of mathematics teachers with the utilization of the system of computer mathematics as the method of education. The requirement of the students to have the relevant skills in the sphere of informational and communicational technologies is the part of the curriculum of the majority of subject disciplines [3].

In the syllabus of different disciplines, such competency is regarded as competency that is aimed to support teaching the mathematical foundations of advanced informatics; competency towards the future employment; and competency that motivates to study during the whole life. According to each of the above-mentioned points, the focus and the importance of the syllabus can be totally different, depending on the subject discipline. Talking about the future teachers of mathematics, the competency in informatics can be reflected in the ability to solve the problems through using the systems of computer mathematics, building the spatial figures with the utilization of the relevant software, as well as other skills.

Analyzing the syllabus of preparation of bachelors of such educational establishments as: National Pedagogical Dragomanov University, Rivne State University of Humanities, V.Karazin Kharkiv National University and others, and evaluating the logical and structural scheme of educational course MFAI, it is possible to make a conclusion that incorporation of such course is appropriate in the first semester of the second year to the certain educational disciplines of the fundamental, natural and scientific preparation. The structure of syllabus in Ukrainian higher educational establishments consists of normative and variation parts, each of which consists of three cycles of preparation: humanitarian, social-economic, mathematical, natural-scientific, professional and practical. At the same time, in the normative part of National Pedagogical Dragomanov University's syllabus, there is a cycle of psychological-pedagogical preparation that covers 5% of syllabus (table 1).

Table 1

Peculiarities of preparation of educational plans for the teachers of mathematics at National Pedagogical Dragomanov University

Cycles of educational disciplines	Normative educational disciplines	Selective educational disciplines
	Hours/ credits	*Hours/credits*

Continuation of table		
Humanitarian and social-economic preparation	900/30	360/12
Humanitarian and social-economic preparation	450/15	
Fundamental natural-scientific preparation	3360/112	450/15
Professional and practical preparation	690/23	990/33
Total	5400/180	1800/60

Total educational time, according to the baccalaureate program during 4 years of education is 240 ECTS credits, while the distribution of total educational time according to the cycle of preparation (in % from the total time of preparation) is the following: humanitarian and social-economic preparation – 20 ± 4%; natural and scientific preparation – 25 ± 4%; professional and practical preparation – 55 ± 4%. At the same time, there are also other recommendations concerning the distribution of the total educational time according to the cycles of preparation (table 2).

Table 2

Distribution of the total educational time according to the cycles of preparation

Cycle of preparation	Higher educational establishments		
	National Pedagogical Dragomanov University	Rivne State University of Humanities	V.N.Karazin Kharkiv National University
Humanitarian and social-economic	15±4%	10±4%	10±4%
Mathematical and natural-scientific	50±4%	40±4%	50±4%
Professional and practical	30±4%	50±4%	40±4%

From the obtained results, it is possible to make a conclusion that it is necessary to intensify the studies of information disciplines in the process of preparation of the future teachers of mathematics. To research the teaching specifics of the course "Mathematical foundations of Advanced Informatics" with utilization of the system of dynamic mathematics GRAN, for its further utilization in the higher educational establishments in the process of teaching the future teachers of mathematics [5].

Paper objective. For achieving the goal of the research, the complex of the following interrelated methods was utilized: analysis of psychological and pedagogical literature, educational plans and programs; learning the practical experience of utilization of the computer mathematical system in the higher and secondary educational establishments; selection of systems of computer mathematics that are relevant for teaching the course "Mathematical foundations of Advanced Informatics"; psychological and pedagogical survey of students through the method of questionnaires, primary control of knowledge in the process of test completion; presentation and fixing of the new material with the utilization of the developed methodology in the experimental group and with the utilization of the alternative (conventional) methodology in the test; test for both groups; analysis of test results, formulation of conclusions.

Paper main body. In this research, the process of teaching the mathematical foundations of informatics to the future teachers of mathematics is founded on the developed syllabus, the profile of which is presented below (table 3). It is made through the utilization of methodological recommendations and independent components of computer-oriented environment [5].

Table 3

Mathematical foundations of Advanced Informatics: profile of the educational course program MFAI

Type of diploma and scope of the program	Single degree, 150 credits ECTS.
Higher educational establishment	National Pedagogical Dragomanov University

A. Goal of the program
Provide education in the field of mathematics with substantial employment opportunities in terms of

Continuation of table

preparation of the students that have particular interest to certain fields of mathematical foundations of informatics for the further education.

Continuation of table

B. Characteristics of the program		
1	*Subject area, direction*	Mathematics
2	*Focus of the program: general/ special*	General education in the field of mathematics.
3	*Orientation of the program*	The program is based on the well-known applied results with the utilization of current state of mathematics, and is oriented on the actual specializations (mathematics, informatics, physics), in terms of which the further professional and educational career is possible.
C. Employment and continuation of education		
1	*Employment*	Jobs in the state structures, small enterprises and institutions of technological and informatics sector (research, quality assurance, commerce), teaching positions, posts in the financial institutions.
2	*Continuation of education*	Mathematics master programs, inter-disciplinary programs close to mathematics (theory of possibility, discrete mathematics), master programs in engineering and informatics disciplines.
D	Style and methodic of teaching	
1	*Approaches to teaching and education*	Lections, laboratory works, seminars, practical classes in small groups, individual independent work, consultations with professors, preparation of bachelor work, project-research approach, mixed studies, group work.
2	*Evaluation system*	Written and oral examinations, laboratory reports, oral presentations, ongoing monitoring, defense of the bachelor work.
E. Competency programs		
1	*General*	1. Analysis and synthesis. The ability to analyze and synthesize, basing on the logical arguments and verified facts [4]. 2. Flexibility of thinking. The acquisition of flexible thinking, openness towards the utilization of mathematical knowledge, formation and development of the competencies in the wide diapason of employment possibilities in the everyday life [10]. 3. Group work. The ability to complete the research in group, similar skills that demonstrate the ability to take into account the strict discipline requirements, planning and time management. 4. Communication skills. The ability to communicate effectively and to present difficult complex data in the short oral and written form, utilizing informational and communicational technologies and relevant terminological apparatus [13]. 5. Popularization skills. The ability to communicate with non-specialists, certain teaching skills. 6. Ethics. Following ethical principles from the professional perspective and from the perspective of understanding of potential impact of mathematical achievements in the context of fast innovational processes [11].
2	*Professional*	1. Deep knowledge and understanding. The ability to analyze the research problems of natural and technological origin, from the perspective of the fundamental mathematical principles and knowledge, as well as basing on the relevant mathematical methods [5]. 2. Evaluation skills. The ability to complete the evaluation of the order of magnitude and to find out the relevant solutions with a clear definition of assumptions and the use of the special and marginal cases [6]. 3. Mathematics skills. The ability to understand and to wisely utilize the mathematical and numerical methods that are often used in the process of solution of the research problems [13]. 4. Experimental skills. The ability to make experiments independently, and also to describe, analyze and critically assess the experimental data [8]. 5. Problem solution. The ability to solve wide range of problems and tasks through the understanding of their foundations and utilization of theoretical as well as practical methods [8]. 6. Calculation skills. The ability to use the appropriate software (systems of computer mathematics) for completing the mathematical research [5]. 7. Cognitive skills in the field of mathematics. The ability to describe the wide range of natural objects and processes; this ability should be based on the deep knowledge and understanding of mathematical theories and themes [5]. 8. Learning skills. The ability of self-education of new fields and domains, utilizing the obtained mathematical knowledge [6].
F. Educational program results		
	1. The ability to demonstrate the skills and understanding of mathematical foundations of informatics in the theory of probability, discrete mathematics, methods of optimization, quantity methods, algorithm theory and mathematical logics. The level of knowledge of these mathematical foundations of informatics should be foundational, to be the reflection of the level that is necessary for work in the traditional areas of	

Continuation of table

application, but not too high to make the research on the current scientific front.

2. The ability to demonstrate the knowledge and understanding of the branches of mathematics that are related to the basic level of foundations of informatics: differential and integral calculus, algebra, functional analysis of real and complex variables, vectors and matrices, vector calculus, differential equations in ordinary and partial derivatives, statistics, Fourier methods. The ability to use these instruments in the theory of solution of the research questions.
3. The ability to demonstrate experimental skills in mathematics (the knowledge of experimental methods and conducting of experiments) to test the hypothesis and to research the phenomena using mathematical laws; the ability to ask correct questions, the knowledge of standard equipment, planning of the experiment conducting, gathering and analysis of the data, including the detailed analysis of the mistakes and critical evaluation of the obtained results.

Continuation of table

4. The ability to demonstrate the knowledge and understanding of the following elements of mathematical foundations of informatics on the foundational level: derivative functions, elements of the theory of numbers, sums and recurrences, and binary operations; to perceive and understand the role of models and theories in the development of mathematics and in the creation of the flexible thinking.
5. The ability to demonstrate the knowledge and understanding of mathematical foundations of informatics (theory of research problems solution, special numbers, discrete probability, asymptotics, binominal coefficients) on the foundational level, according to the current development of mathematics.
6. The ability to utilize the knowledge and understanding of the elements of the applied mathematics and related disciplines of informatics on operational level to develop the understanding of interdisciplinary connections between the fundamental sciences.
7. Foundational knowledge and understanding of the special branches, according to the choice of the student for the future specialization and learning of the interdisciplinary approaches.
8. The ability to solve the research problems utilizing the appropriate software and the systems of computer mathematics, as well as the knowledge concerning the analysis and the representation of the results.
9. Learning the skills of independent individual work and work in group (the work on the project, including the leadership skills), the ability to achieve the result during the limited time and with attention to the professional integrity and impossibility of plagiarism.

The goal of educational course "Mathematical foundations of Advanced Informatics" (MFAI) is to familiarize students with the fundamental notions, main definitions and mathematical methods of informatics - the fundamental natural science that studies the processes of transmission and processing of the data. In the process of teaching of this course, the students get acquainted with theoretical material, learn the laws, and choose the methods of data procession, build mathematical models of informational systems for the concrete technical, social and physical systems, learn lineal, optimization models, discrete optimization tasks and algorithm theory [5].

The main goals of educational course MFAI are the formation of knowledge and skills, necessary for the rational work with the software tools of general purpose in the future professional work; formation of system vision about mathematical foundations of informatics, formation of skills needed for the solution of the research and practical problems; development of the ability to project and research activities of the future specialists and their independent work.

The content of the course MFAI consists of five content modules: M1. Theme 1. Solution theory of research problems (Tower of Hanoi problem. Problem of cutting pizza). M1. Theme 2. Sums and recurrences (Transformation of sums. General methods of summation). M1. Theme 3. Binary operations (Logics of predicates). M2. Theme 4**.** Elements of number theory (Prime numbers. Factorial numbers). M3. Theme 5**.**Special numbers (Euler numbers. Bernulli numbers. Fibonacci numbers). M4. Theme 6. Discrete probability (Expected value and dispersion. Hashing). M4. Theme 7. Asymptotics (Two asymptotic examples. Euler's summation formula). M5. Theme 8. Generator of the functions (Solving the recurrent relations. Dirchlet generating function). M5. Theme 9. Binominal coefficients (Generating function. Hyper-geometrical functions. Hyper–geometrical transformations).

While teaching the course MFAI, the methods of creation of real objects' mathematical models are utilized; axiomatic methods - establishing the truth/false allegations. During educational activities, the following forms of organization of educational activities of students are used: the project-research approach, mixed learning, group forms of work and relevant methods of education - system of dynamic mathematics GRAN and other systems of computer mathematics (if necessary), which all help to effectively learn mathematical foundations of informatics. The GRAN platform was created to

improve the service of storage, viewing, utilization and exchange of electronic open didactic materials, developed through utilization of GRAN. The users have the possibility to download own materials or to create them in the online mode. During the process of research, the prepedeutic problems were developed, classified and placed on the GRAN platform in order to improve the effectiveness of education of mathematical foundations of informatics with utilization of systems of computer mathematics GRAN. For each class of problems, the special orientation rule and relevant list of algorithms are presented.

The peculiarity of the educational course MFAI is orientation of goals, content, approaches and means of education on getting the knowledge, skills, and practical experience by the future teachers of mathematics with utilization of mathematical models (dynamic applets of GRAN) that are used by them in the different spheres of activities. Solution of practical tasks involves the functional components related to motivation, and formulation of the objectives of the study course, as well as the identification of the importance of practical side and practical potential of the abstract component of the training course by the future teachers of mathematics. The following number of educational activities related to the for practical activity teaching of components that are common is proposed: utilization of heuristic considerations, application of mathematical modeling as a foundation of educational course and as a method of solving practical tasks, development of mathematical skills necessary for solving practical tasks, the actions that are common for the professional and educational activity (planning skills, activity adjusting skills, independent work, creative activities, work with the computer programs); the actions related to modeling of mathematical situations.

It should be noted that not all the examples can be solved with the help of graphical methods. It is connected with some restrictions of computer graphics and computer mathematics in general. Determine the number of roots of the equation $\frac{1}{\sin\frac{1}{x}} = 2$ on the segment $[-1, 1]$.

Attempt to plot graph of the function $y = \frac{1}{\sin\frac{1}{x}}$ on given segment will be unsuccessful since the function has infinite number of points of discontinuity in the neighborhood of the point $x = 0$ (Fig. 1). That is why it is impossible to solve this problem graphically.

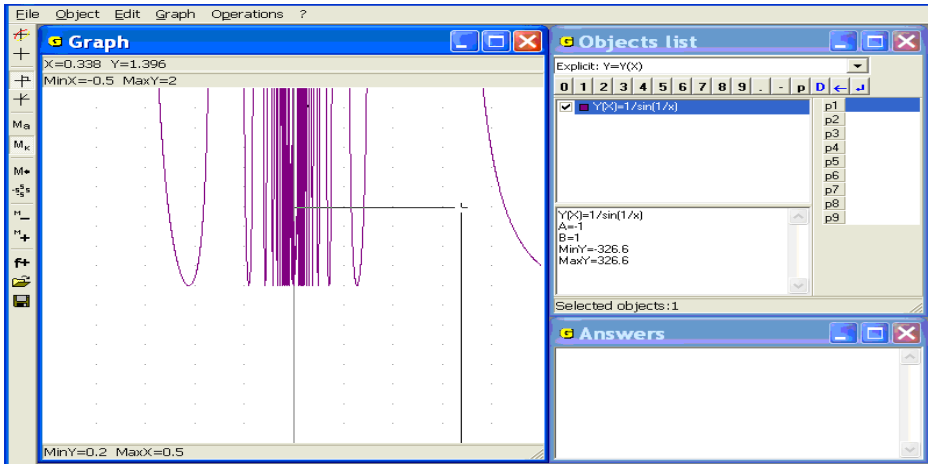

Fig. 1. The instruction card for mathematical experiments. Source: actual processing

These examples show that the use of software for solving mathematical problems should be well-grounded, and analysis of solutions of the problems and substantiation of their propriety stays very important.

The essence of the practicality of the course is about the realization of the interdisciplinary relations. The main method of realization of the practical orientation of the course is the method of mathematical modeling, while the most effective method is the practical (situational) problems the solution of which requires the deep knowledge of mathematics and other disciplines. The course MFAI has variative,

interdisciplinary character and is oriented on the students of physics and mathematics.

To test the effectiveness of the proposed course MFAI for the future teachers of mathematics with the utilization of the systems of computer mathematics, the pedagogical experiment was held. It included three stages and was held during 2014-2020. On the first stage of the research, the exploratory stage of the experiment was conducted in order to learn the state of the problem of the research and determine the ways of its solution. On this stage, the following goals were realized: regulatory-legal documents, scientific and methodological literature of the research problem, manuals and didactic materials related to the course MFAI were analyzed; the point of view of the teachers concerning the utilization of the systems of computer mathematics in educational process of the secondary school was discovered and evaluated; the level of formation of informatics competencies of the future teachers of mathematics was defined [5].

The results obtained at the first stage allowed to understand the current situation with teaching of the course MFAI for the future teachers of mathematics and to find out the controversies that lead to the necessity of solution of the problem of informatics competencies formation. On the second stage of the research, the search stage of the experiment was held. The main goals of it were the following: to develop the conceptual model of the process of teaching of the course MFAI for the future teachers of mathematics with utilization of the systems of computer mathematics and basing on it to describe the methodology of realization of such education; to define its structure and components, to find out the effective methods of teaching the course MFAI with utilization of the systems of computer mathematics in order to form informatics competencies; to develop the didactic materials for teaching the course MFAI with utilization of the systems of computer mathematics.

Basing on the results of the search stage of the experiment, the conceptual model of the process of teaching of the course MFAI was developed with utilization of systems of computer mathematics. Basing on this model, the didactic materials for teaching the future teachers were developed.

The goal of the formulation stage of the experiment was to test the applicability of the educational course MFAI for the future teachers of mathematics with utilization of the systems of computer mathematics. The goal of this stage was to incorporate the components of the developed methodology to the educational process of higher-educational establishments and to learn its impact on the formation of the informatics competencies of the future teachers of mathematics and to make the analysis of the results of its approbation. According to the developed methodology of teaching the course MFAI for the future teachers of mathematics with utilization of the systems of computer mathematics, the test of formation of each of its components was made (table 4).

Table 4

Criterions, indicators and methods of diagnostics of the effectiveness of the educational course MFAI

Criterions (C), indicators (I)	Levels	Method of diagnostics (MD), methods of evaluation of the results (MER)
C. Motivational – value based I. Level of development of the motivational sphere	Four internal negative; external negative; internal positive; external positive.	MD. Survey of the future teachers of mathematics, according to the methodology of COMSRL [5] MER. Quantitative analysis
C. Practice oriented I. Distribution of future mathematics teachers, according to the level of their educational achievements	Four beginner; average; sufficient; high.	MD. Completion of test MER. Quantitative and qualitative analysis, methods of statistical processing of data: λ-criteria of Kolmogorov and φ-angular transformation of Fischer.
C. Activity driven I. Distribution of future mathematics teachers, according to the level of their educational achievements.	Four beginner; average; sufficient; high.	MD. Completion of test MER. Quantitative and qualitative analysis, methods of statisticalprocessing of data: λ-criteria of Kolmogorov and φ-angular transformation of Fischer.
C. Reflexive I. State of development of reflection	Three: high; medium; low.	MD. Methodology of reflexive diagnostics of COMSRL [5] MER. Quantitative analysis

As a method of determining the effectiveness of motivational-value component of the methodology of teaching of the course MFAI for the future teachers of mathematics

with utilization of the systems of computer mathematics, the computer survey (methodology of motivational diagnostic of COMSRL) was utilized. The obtained results demonstrate the priority of external negative motivation (63%) and internal positive motivation (37%). After conducting the experimental study, the repeated survey demonstrated that the indicator of internal positive motivation (47%) increased, while the indicator of external negative motivation (53%) did not change.

For testing the effectiveness of practical, orientation and activity components of the methodology, the same tests were utilized both for control groups and experimental groups. The probability of the obtained data of experimental study was tested through the statistical criterions: λ-criteria of Kolmogorov and φ-angular transformation of Fischer.Test 1 was held to make sure that the future teachers of mathematics of experimental and control groups have the same level of informatics competency on the date of the formational experiment. According to the results of test #1, the following distribution of points was obtained, taking into account the levels of achievements according to the criterions of evaluation [5]. Basing on the obtained data, the calculation of the λ-criteria of Kolmogorov was made to compare the empirical distribution of points in the experimental and control groups. For finishing the experimental study in the control and experimental groups, test #2 was held. The main goal of the test was to compare the results in the both groups for the statistical confirmation of the fact that after experimental study the future teachers of mathematics of experimental groups have higher level of knowledge and skills, in relation to solving the situational and research tasks, compared to the control group.

According to the result of the test, the maximum difference between the accumulated empirical relative frequencies in distribution of points in the experimental and control groups is 0,121. It is accumulated in the position "average level of educational achievements (4-6 points)". Accordingly, it is possible to assume that "the effect is present", if the educational achievements of the future teachers of mathematics correspond to sufficient and high levels (7-12 points), and that "there is no effect", if the educational achievements of the future teachers of mathematics correspond to the low and average levels (1-6 points). The reliability of the experimental data was checked through the φ - angular transformation of Fischer together with λ-criteria of Kolmogorov. The conclusion was that the level of people that wrote the test on the sufficient and high levels in the experimental group is higher compared to the control group.

Effectiveness of the reflexive component of methodology of teaching of the course MFAI for the future teachers of mathematics with utilization of the systems of computer mathematics, the indicator of which is the level of development of reflection, was tested through the methodology of COMSRL. Before conducting the experiment, the majority of students were characterized by the average (32%) and low (53%) levels of reflection development, however after conducting the experimental study the amount of future teachers of mathematics with average level of reflection development increased to 39%, while the amount of students with low level of reflection development decreased to 44%.

Therefore, the experimental research that was held confirmed the effectiveness of teaching the course MFAI for the future teachers of mathematics with utilization of the systems of computer mathematics GRAN [5].

Conclusions of the research. The educational course "Mathematical foundations of Advanced Informatics" (MFAI) for the future teachers of mathematics with pedagogical utilization of independent components of computer-oriented environment of education was developed and tested. The logical and structural schemes of educational course MFAI were constructed. The methodological recommendations to the educational course MFAI were developed. The utilization of GRAN system in the process of solving propedeutic tasks to align the knowledge of the future teachers of mathematics for the successful mastering of the course MFAI was analyzed. In the process of teaching the course MFAI the system of informatics competencies of the future professionals is formed.

Therefore, the teaching of mathematical foundations of informatics is pedagogically balanced and methodically motivated solution. Through utilization of the system of dynamic mathematics GRAN in the educational process the educational process as well as the main methods, notions and mathematical foundations of informatics are improved, while solving the educational problems of the majority of subjects that are studied by the future teachers of mathematics and the activation of educational and cognitive activity of the students is promoted, if the technology is integrated into the computer-oriented environment of education.

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