Symmetry conditions for studying torsion stress-induced gradient piezogyration

Kvasnyuk O., Zapeka B., Vasylkiv Yu., Kostyrko M. and Vlokh R.

Institute of Physical Optics, 23 Dragomanov St., 79005 Lviv, Ukraine,
e-mail: vlokh@ifo.lviv.ua

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Abstract. We have analyzed a gradient piezogyration effect induced by torsion stresses in optical materials belonging to different point groups of symmetry. It has been shown that the effect manifests itself as a rotation of light polarization plane only in the tetragonal and cubic crystals and textures described by the point symmetry groups $\infty/\infty/\mathrm{mmm}$, $\infty\mathrm{mm}$, ∞/m, 4/m, 4/mmm, m3 and m3m, provided that light propagates along the optic axis in the tetragonal crystals, along one of the crystallographic axes in the cubic crystals, or along the infinity fold axis in the textures and the crystals are twisted around these axes.

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As shown in our recent studies [1–3], a torsion stress-induced optical activity is the effect caused by spatial derivatives of mechanical stresses rather than by the stresses themselves. It is described by the tensorial relation

$$\Delta g_{ln} = \beta_{lnkmv} \partial\sigma_{km} / \partial X_v , \quad (1)$$

where Δg_{ln} is the induced increment of the gyration tensor, $\partial\sigma_{km} / \partial X_v$ the coordinate derivative of the stress tensor, and β_{lnkmv} a fifth-rank axial tensor. Notice that the β_{lnkmv} tensor has nonzero components even in centrosymmetric media. As a result, generation of spatially inhomogeneous distribution of mechanical stress tensor components in such media would lead to appearance of optical rotation. It is this property that distinguishes the gradient piezogyration effect from a commonly known piezogyration [4–7] described by the formula

$$\Delta g_{ln} = \vartheta_{lnkm} \sigma_{km} , \quad (2)$$

with ϑ_{lnkm} being the fourth-rank axial piezogyration tensor equal to zero for any centrosymmetric media.

Comparing Eqs. (1) and (2), one can see that the gradient piezogyration appears alone, without a usual accompanying piezogyration effect, in centrosymmetric materials since the piezogyration is symmetry forbidden here. It has been shown in Refs. [1–3] that it would be convenient to study the gradient piezogyration under crystal torsion, because the torsion moments can be successfully controllable unlike, e.g., residual internal inhomogeneous stresses.

From the other hand, it is known that the shear stress tensor components σ_{31} and σ_{32} induced by the torque moment M_Z are described by the relation [8]

$$\sigma_{km} = \sigma_{\mu} = \frac{2M_Z}{\pi R^4}\left(X\delta_{4\mu} - Y\delta_{5\mu} \right). \quad (3)$$

Here Z axis coincides with the direction of torsion, X and Y are orthogonal to Z, $M_Z = \int_S (r \times P) dS$ denotes the torsion torque, $\delta_{\nu\mu}$ the Kronecker delta, R the cylinder radius, S the square of the cylinder basis, and P the mechanical load. The above shear stress tensor components are equal to zero along the torsion axis, while their coordinate derivatives are not. Moreover, the latter peculiarity implies that piezooptically induced birefringence is equal to zero at the very torsion axis (see, e.g., [9]). We remind that the piezooptic effect is described by the relation [10]

$$\Delta B_{ij} = B_{ij} - B_{ij}^0 = \pi_{ijkm}\sigma_{km}, \quad (4)$$

where π_{ijkm} is the fourth-rank polar piezooptic tensor, and B_{ij} and B_{ij}^0 stand for the impermeability tensors of stressed and free samples, respectively (with $B_{ij} = (1/n^2)_{ij}$ and n denoting the refractive index). Propagation of light beam along the direction of optic axis in optically uniaxial crystals, which remains to be isotropic after the torsion moment is applied, enables studying the optical rotation using a direct method for measuring polarization plane rotation.

Until now the torsion-induced optical activity has been studied only in the centrosymmetric $NaBi(MoO_4)_2$ crystals [1–3] belonging to the point symmetry group 4/m. While choosing other optical materials appropriate for the studies of the effect mentioned, it is necessary to formulate the requirements to the properties of these materials. Let us remind that the optical birefringence does not appear along the direction of optic axis in $NaBi(MoO_4)_2$ when the torsion moment M_Z is applied and the shear stress components σ_{31} and σ_{32} are in action [3]. This fact allows one to scan over laser beam propagating in the Z direction inside the cross section XY of a twisted sample and measure the XY distribution of the optical rotation [3].

Hence, the conditions needed for studying experimentally the torsion-induced optical activity may be formulated as follows:

A material should be isotropic or optically uniaxial in order to provide propagation of optical beam along the isotropic direction.

For avoiding interference of the traditional piezogyration effect, a material should be centrosymmetric or, at least, the axial fourth-rank tensor should lack nonzero components ϑ_{3331} and ϑ_{3332}, which might have led to appearance of piezogyration optical rotation along the Z direction. Notice also that we will not consider here the crystals possessing a natural optical activity, in order to avoid a cumbersome case of mutual overlapping of the piezogyration, the natural gyration and the gradient piezogyration.

The piezooptic birefringence should not be induced in all of the cross sections perpendicular to the direction of light beam propagation.

The components β_{33321} and/or β_{33312} of the gradient piezogyration tensor should not be equal to zero.

According to the first and second conditions, we should consider the crystals of the symmetry groups 3m, $\bar{3}$, $\bar{3}$m, 4mm, $\bar{4}$, $\bar{4}$2m, 4/m, 4/mmm, 6mm, $\bar{6}$, $\bar{6}$2m, 6/m, 6/mmm, m3, $\bar{4}$3m, and m3m. One can also consider the textures with the symmetry groups ∞mm and ∞/m, and the isotropic media described by the group $\infty/\infty/\text{mmm}$. These symmetry classes can be divided into a number of piezooptic subgroups, i.e. the subgroups for which the matrix of the piezooptic tensor has the same form.

The isotropic media ($\infty/\infty/\text{mmm}$) and the cubic crystals ($\bar{4}3\text{m}$ and m3m) belong to the subgroup denoted hereafter as **A**. The matrix of piezooptic coefficients for these classes is as follows:

$$\begin{vmatrix} \pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_{44} \end{vmatrix}, \qquad (5)$$

where the relationship $\pi_{44} = \pi_{11} - \pi_{12}$ holds true for the class $\infty/\infty/\text{mmm}$. When the torsion-induced stress components σ_{31} and σ_{32} act, the *XY* cross section of the optical indicatrix undergoes no changes, so that these symmetry groups satisfy the third condition, too.

The subgroup **B** includes the only point symmetry group m3. The piezooptic matrix for this class is similar to that given by Eq. (5), though $\pi_{13} = \pi_{32} = \pi_{21}$. Nonetheless, the shear stresses mentioned above will not induce any birefringence in the *XY* cross section.

The next subgroup, **C**, embraces the crystals and the textures with the symmetries 4mm, $\bar{4}2\text{m}$, 4/mmm, 6mm, $\bar{6}2\text{m}$, 6/mmm, ∞mm and ∞/m, with the piezooptic matrix

$$\begin{vmatrix} \pi_{11} & \pi_{12} & \pi_{13} & 0 & 0 & 0 \\ \pi_{12} & \pi_{11} & \pi_{13} & 0 & 0 & 0 \\ \pi_{31} & \pi_{31} & \pi_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_{66} \end{vmatrix}. \qquad (6)$$

Upon inspecting this matrix one can see that the stress components σ_{31} and σ_{32} do not induce the birefringence along the *Z* direction.

The same is also true of the point symmetry groups $\bar{4}$ and 4/m, which form the subgroup **D** and include some additional piezooptic coefficients, when compared with Eq. (6) ($\pi_{54} = -\pi_{45}$, $\pi_{26} = -\pi_{16}$, and $\pi_{62} = -\pi_{61}$). The next subgroup, **E**, involves the symmetry classes $\bar{6}$ and 6/m, for which the additional components $\pi_{54} = -\pi_{45}$, $\pi_{16} = -\pi_{26} = 2\pi_{62}$ and $\pi_{62} = -\pi_{61}$ occur. Crystals of all these symmetry groups remain optically isotropic under the torsion moment M_Z applied.

From the other hand, the crystals belonging to the point symmetry groups 3m and $\bar{3}\text{m}$, which constitute the next subgroup, **F**, do not remain optically isotropic in all of the *XY* cross sections when the torsion moment M_Z is applied (see [9]). In these conditions, the linear birefringence does not appear only when the light beam propagates exactly along the torsion axis, whereas the *XY* distribution of the birefringence has a conical shape. Thus, the latter crystals should not be preferred if one tries to detect the torsion-induced optical activity.

The last subgroup, **G**, consist of only one point symmetry group $\bar{3}$. Here the piezooptic matrix is given by

$$\begin{vmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} & -\pi_{25} & 2\pi_{62} \\ \pi_{12} & \pi_{11} & \pi_{13} & -\pi_{14} & \pi_{25} & -2\pi_{62} \\ \pi_{31} & \pi_{31} & \pi_{33} & 0 & 0 & 0 \\ \pi_{41} & -\pi_{41} & 0 & \pi_{44} & \pi_{45} & 2\pi_{52} \\ -\pi_{52} & \pi_{52} & 0 & -\pi_{45} & \pi_{44} & 2\pi_{41} \\ -\pi_{62} & \pi_{62} & 0 & \pi_{25} & \pi_{14} & \pi_{66} \end{vmatrix}, \quad (7)$$

with $\pi_{66} = \pi_{11} - \pi_{12}$. Now let us consider the *XY* cross section of the optical indicatrix perturbed by the stresses σ_{31} and σ_{32} appearing due to application of the torque moment M_Z to the crystals belonging to the subgroup **G**:

$$(B_1 + \pi_{14}\sigma_{32} - \pi_{25}\sigma_{31})X^2 + (B_1 - \pi_{14}\sigma_{32} + \pi_{25}\sigma_{31})Y^2 + 2(\pi_{25}\sigma_{32} + \pi_{14}\sigma_{31})XY = 1. \quad (8)$$

The spatial distribution of the induced birefringence in the *XY* plane may be written as [11]

$$\Delta n_{12} = n_o^3\sqrt{(\pi_{14}^2 + \pi_{25}^2)(\sigma_4^2 + \sigma_5^2)} = 2n_o^3\frac{M_z}{\pi R^4}\sqrt{(\pi_{14}^2 + \pi_{25}^2)(X^2 + Y^2)}, \quad (9)$$

whereas the angle of optical indicatrix rotation around the *Z* axis may be presented as

$$\tan 2\zeta_Z = \frac{\pi_{25}\sigma_4 + \pi_{14}\sigma_5}{\pi_{14}\sigma_4 - \pi_{25}\sigma_5} = \frac{\pi_{25}X + \pi_{14}Y}{\pi_{14}X - \pi_{25}Y}. \quad (10)$$

Introducing the polar coordinate system ($X = \rho\cos\varphi,\ Y = \rho\sin\varphi$), we rewrite Eqs. (9) and (10) as

$$\Delta n_{12} = 2n_o^3\frac{M_z}{\pi R^4}\sqrt{(\pi_{14}^2 + \pi_{25}^2)(X^2 + Y^2)} = 2n_o^3\frac{M_z}{\pi R^4}\rho\sqrt{\pi_{14}^2 + \pi_{25}^2} \quad (11)$$

and

$$\tan 2\zeta_Z = \frac{\pi_{25}X + \pi_{14}Y}{\pi_{14}X - \pi_{25}Y} = \frac{\pi_{25}\cos\varphi + \pi_{14}\sin\varphi}{\pi_{14}\cos\varphi - \pi_{25}\sin\varphi}. \quad (12)$$

As seen from Eq. (11), the birefringence does not appear only along the torsion axis, while the *XY* coordinate distribution of the birefringence has a conical shape. As a consequence, the subgroup **G** should also be regarded as unsuitable for studying the torsion-induced optical rotation.

Now let us consider the implications of the fourth condition for the subgroups **A**, **B**, **C**, **D**, and **E**. As far as the subgroup **A** is concerned, the fifth-rank axial tensor components β_{33321} and β_{33312} are equal to zero for the crystals of the class $\bar{4}3\text{m}$, and the condition $\beta_{33321} = -\beta_{33312} \neq 0$ takes place for the materials of the class $\infty/\infty/\text{mmm}$ and m3m. For the subgroup **B** (i.e., the point symmetry group m3) we have $\beta_{33321} \neq \beta_{33312} \neq 0$. Consider now the subgroup **C** embracing the classes 4mm, $\bar{4}2\text{m}$, 4/mmm, 6mm, $\bar{6}2\text{m}$, 6/mmm, ∞mm, and ∞/m. We have the relation $\beta_{33321} = -\beta_{33312} \neq 0$ for the classes 4mm and 4/mmm ∞mm and ∞/m and $\beta_{33321} = \beta_{33312} = 0$ for the classes $\bar{4}2\text{m}$, 6mm, $\bar{6}2\text{m}$, 6/mmm. Concerning the subgroup **D**, one has $\beta_{33321} = -\beta_{33312} \neq 0$ for the class 4/m and $\beta_{33321} = \beta_{33312} = 0$ for the group $\bar{4}$. Finally, the equality $\beta_{33321} = \beta_{33312} = 0$ is valid for the subgroup **E** consisting of the classes 6/m and $\bar{6}$.

In conclusion, the studies of polarization plane rotation due to the torsion stress-induced gradient piezogyration have to be performed with the materials belonging to the point symmetry groups 4/m, 4/mmm and m3m, ∞mm, ∞/m and $\infty/\infty/mmm$ for which $\beta_{33321} = -\beta_{33312} \neq 0$, and the symmetry group m3, for which $\beta_{33321} \neq \beta_{33312} \neq 0$. Here the fifth-rank axial tensor components under interest remain nonzero. These groups of symmetry refer to the tetragonal and cubic systems and include the inversion centre among their symmetry operations.

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Анотація. *У роботі проаналізовано ефект градієнтної п'єзогірації, індукований напруженнями кручення в кристалах різних груп симетрії. Показано, що цей ефект виявлятиметься в повороті площини поляризації світла лише в кристалах кубічної і тетрагональної сингоній та текстурах, що належать до груп симетрії* $\infty/\infty/mmm$, ∞mm, ∞/m, *m3m, m3, 4/mmm і 4/m за умов поширення світла вздовж кристалографічних осей у кубічних кристалах, оптичної осі в тетрагональних кристалах та осі безмежного порядку в текстурах і кручення навколо цих осей.*