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(14-b, Metrologichna Str., Kyiv 03680, Ukraine)**INDUCED VACUUM CURRENT  
AND MAGNETIC FLUX IN QUANTUM  
SCALAR MATTER IN THE BACKGROUND  
OF A VORTEX DEFECT WITH THE NEUMANN  
BOUNDARY CONDITION**

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*A topological defect in the form of the Abrikosov–Nielsen–Olesen vortex in the space of an arbitrary dimension is considered as a gauge-flux-carrying tube that is impenetrable for quantum matter. The charged scalar matter field is quantized in the vortex background with the perfectly rigid (Neumann) boundary condition imposed at the side surface of the vortex. We show that a current circulating around the vortex is induced in the vacuum, if the Compton wavelength of the matter field exceeds the transverse size of the vortex considerably. The vacuum current is periodic in the value of the gauge flux of the vortex, providing a quantum-field-theoretical manifestation of the Aharonov–Bohm effect. The vacuum current leads to the appearance of an induced vacuum magnetic flux that, for some values of the tube thickness, exceeds the vacuum magnetic flux induced by a singular vortex filament. The results are compared to those obtained earlier in the case of the perfectly reflecting (Dirichlet) boundary condition imposed at the side surface of the vortex. It is shown that the absolute value of the induced vacuum current and the induced vacuum magnetic flux in the case of the Neumann boundary condition is greater than in the case of the Dirichlet boundary condition.*

*Keywords:* vacuum polarization, Aharonov–Bohm effect, vortex defect.

**1. Introduction**

There are many theoretical models in field theory which contain the phenomenon of spontaneous breakdown of symmetries, see, e.g., [1]. This phenomenon gives rise to topological defects of various kinds, see, e.g., [2, 3]. In this paper, we will consider a linear topological defect known as the Abrikosov–Nielsen–Olesen (ANO) vortex in condensed matter physics [4, 5] or a cosmic string in cosmology [2, 6, 7]. This topological object is formed, when the first homotopy group of the group space of the broken

symmetry group is nontrivial. Abrikosov vortices are real physical objects in the type-II superconductors [8, 9]. Cosmic strings are currently hypothetical objects, and their possible manifestations such as gravitational waves, high-energy cosmic rays, and gamma-ray bursts are actively searched in the Universe [10–12].

In the classical theory of the ANO vortex, a spin-0 (Higgs) field condenses, and a spin-1 field corresponds to the spontaneously broken gauge group; they are coupled in the minimal way with constant  $\tilde{e}_H$ . The transverse size of the vortex is of the order of the correlation length or the Compton wavelength,  $\hbar(m_{HC})^{-1}$ , where  $m_H$  is the mass of the condensate

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field. The physical requirements of single-valuedness of the condensate field and finiteness of the vortex energy result in the following dependence of the vortex flux on  $\tilde{e}_H$ :  $\Phi = \oint d\mathbf{x} \mathbf{A}(\mathbf{x}) = 2\pi\hbar c \tilde{e}_H^{-1}$ , where  $\mathbf{A}(\mathbf{x})$  is the vector potential of the gauge field, and the integral is over a path enclosing the vortex once. The quantized matter field is coupled minimally to the gauge field with constant  $\tilde{e}$ . So, the quantum effects in the background of the ANO vortex depend on the value of  $\tilde{e}\Phi$ .

Since the phase with broken symmetry exists only outside the vortex, the quantum matter field cannot penetrate inside the vortex. We assume further that the interaction between the ANO vortex and the quantized matter field is mediated by the vector potential of the vortex-forming spin-1 field only, and the direct coupling between the vortex-forming spin-0 field and the quantized matter field can be neglected. If so, the ANO vortex does not affect the surrounding matter in the framework of classical theory, and such an influence is of the purely quantum nature. The effect is a quantum-field-theoretical manifestation of the famous Aharonov–Bohm effect [13], see review [14]. It is characterized by the periodic dependence on the value of the vortex flux,  $\Phi$ , with the period equal to the London flux quantum,  $2\pi\hbar c \tilde{e}^{-1}$ . A particular case of  $\tilde{e}_H = 2\tilde{e}$  ( $\Phi = \pi\hbar c \tilde{e}^{-1}$ , half of the London flux quantum) is implemented in ordinary superconductors, see, e.g., [8]. Cases of fractional values of the London flux quantum are physically meaningful as well, and can be implemented in chiral superfluids, liquid crystals, and quantum liquids, see [15, 16].

The physical condition of non-penetration of the matter field inside the vortex means the absence of the matter field current through the side surface of the vortex, namely  $j_r|_{r_0} = 0$ , where  $r$  is a radial coordinate which is perpendicular to the side surface, and  $r_0$  is the radius of the vortex. Hence, the ANO vortex can be considered as a magnetic tube of the finite transverse size. In the case of the scalar matter field, the condition of non-penetrability can be satisfied with the use of a family of boundary conditions of the Robin type

$$(\cos\theta\psi + \sin\theta r\partial_r\psi)|_{r_0} = 0, \quad (1)$$

where  $\theta$  is some arbitrary parameter. Among all possible values of the parameter  $\theta$ , two values are prominent. The case of  $\theta = 0$  corresponds to the perfectly

reflecting (Dirichlet) boundary condition  $\psi|_{r_0} = 0$ . The case of  $\theta = \pi/2$  corresponds to the perfectly rigid (Neumann) boundary condition  $r\partial_r\psi|_{r_0} = 0$ .

In the present paper, we shall study the current which is induced in the vacuum of the quantized charged scalar matter field by the ANO vortex with nonvanishing transverse size with the perfectly rigid (Neumann) boundary condition on its side surface ( $\theta = \pi/2$ ). This current creates a magnetic field in the vacuum, and the total induced vacuum magnetic flux will be studied in detail in what follows. In the case of the ANO vortex of zero transverse size, this problem was solved previously, see [17–19] and references therein. For the case of the finite transverse size of the ANO vortex, the induced vacuum current, magnetic flux, energy and the Casimir force were considered in the [20–24] for the case of the perfectly reflecting (Dirichlet) boundary condition.

It should be noted that, for the quantized fermion matter field, the condition of non-penetration of the matter field inside the finite transverse size ANO vortex has a form different from (1), and it can be parametrized with the help of one parameter in the case of two-dimensional space and four parameters in the case of three-dimensional space. The induced vacuum current and magnetic flux in these cases were considered for all values of parameters in [25–27].

## 2. Induced Vacuum Current and Total Magnetic Flux

We start with the Lagrangian for a complex scalar field  $\psi$  in the  $(d+1)$ -dimensional space-time

$$\mathcal{L} = (\nabla_\mu\psi)^*(\nabla^\mu\psi) - m^2\psi^*\psi, \quad (2)$$

where  $\nabla_\mu$  is the covariant derivative, and  $m$  is the mass of the scalar field. The vacuum current is conventionally defined as

$$\mathbf{j}(\mathbf{x}) = -i \sum_{\lambda} (2E_{\lambda})^{-1} \times \{\psi_{\lambda}^*(\mathbf{x})[\nabla\psi_{\lambda}(\mathbf{x})] - [\nabla\psi_{\lambda}(\mathbf{x})]^*\psi_{\lambda}(\mathbf{x})\}, \quad (3)$$

where  $\lambda$  is the set of parameters (quantum numbers) specifying the state, wave functions  $\psi_{\lambda}(\mathbf{x})$  form a complete set of solutions to the stationary Klein–Fock–Gordon equation

$$(-\nabla^2 + m^2)\psi_{\lambda}(\mathbf{x}) = E_{\lambda}^2\psi_{\lambda}(\mathbf{x}), \quad (4)$$

$E_\lambda = E_{-\lambda} > 0$  is the energy of the state; symbol  $\sum_\lambda$  denotes summation over discrete and integration (with a certain measure) over continuous values of  $\lambda$ .

In the present paper we are considering a static background in the form of the cylindrically symmetric gauge flux tube of the finite transverse size. The coordinate system is chosen in such a way that the tube is along the  $z$  axis. The tube in a 3-dimensional space is obviously generalized to the  $(d-2)$ -tube in a  $d$ -dimensional space by adding extra  $d-3$  dimensions as longitudinal ones. The covariant derivative is  $\nabla_0 = \partial_0$ ,  $\nabla = \partial - i\tilde{e}\mathbf{V}$  with  $\tilde{e}$  being the coupling constant of the dimension  $m^{(3-d)/2}$  and the vector potential possessing only one nonvanishing component given by

$$V_\varphi = \Phi/2\pi, \quad (5)$$

outside the tube; here,  $\Phi$  is the value of the gauge flux inside the  $(d-2)$ -tube, and  $\varphi$  is the angle in polar  $(r, \varphi)$  coordinates on a plane which is transverse to the tube. The Neumann boundary condition at the side surface of the tube ( $r = r_0$ ) is imposed on the scalar field:

$$\partial_r \psi_\lambda|_{r=r_0} = 0, \quad (6)$$

i.e., the surface of the flux tube is a perfectly rigid boundary for the matter field.

The solution to (4) and (6) outside the impenetrable tube of radius  $r_0$  takes the form

$$\psi_{kn\mathbf{p}}(\mathbf{x}) = (2\pi)^{(1-d)/2} e^{i\mathbf{p}\mathbf{x}_{d-2}} e^{in\varphi} \Omega_{|n-\tilde{e}\Phi/2\pi|}(kr, kr_0), \quad (7)$$

where

$$\Omega_\rho(u, v) = \frac{Y'_\rho(v)J_\rho(u) - J'_\rho(v)Y_\rho(u)}{[J_\rho^2(v) + Y_\rho^2(v)]^{1/2}}, \quad (8)$$

and  $0 < k < \infty$ ,  $-\infty < p^j < \infty$  ( $j = \overline{1, d-2}$ ),  $n \in \mathbb{Z}$  ( $\mathbb{Z}$  is the set of integer numbers),  $J_\rho(u)$  and  $Y_\rho(u)$  are the Bessel functions of order  $\rho$  of the first and second kinds, the prime near the function means the derivative with respect to the function argument. Solutions (7) obey orthonormalization condition

$$\begin{aligned} & \int_{r>r_0} d^d\mathbf{x} \psi_{kn\mathbf{p}}^*(\mathbf{x}) \psi_{k'n'\mathbf{p}'}(\mathbf{x}) = \\ & = \frac{\delta(k-k')}{k} \delta_{n,n'} \delta^{d-2}(\mathbf{p}-\mathbf{p}'). \end{aligned} \quad (9)$$

Using (3) and (7), we get  $j_r = \mathbf{j}_{d-2} = 0$  and

$$\begin{aligned} j_\varphi(r) & \equiv x^1 j^2(\mathbf{x}) - x^2 j^1(\mathbf{x}) = (2\pi)^{1-d} \int d^{d-2}p \times \\ & \times \int_0^\infty dk k (\mathbf{p}^2 + k^2 + m^2)^{-1/2} S(kr, kr_0), \end{aligned} \quad (10)$$

where

$$S(u, v) = \sum_{n \in \mathbb{Z}} \left( n - \frac{\tilde{e}\Phi}{2\pi} \right) \Omega_{|n-\tilde{e}\Phi/2\pi|}^2(u, v). \quad (11)$$

Due to the infinite range of the summation, the last expression is periodic in the flux  $\Phi$  with a period equal to  $2\pi\tilde{e}^{-1}$ , i.e., it depends on the quantity

$$F = \frac{\tilde{e}\Phi}{2\pi} - \left[ \left[ \frac{\tilde{e}\Phi}{2\pi} \right] \right], \quad (12)$$

where  $[[u]]$  is the integer part of the quantity  $u$  (i.e., the integer which is less than or equal to  $u$ ).

Let us rewrite (11) in the form

$$S(u, v) = S_0(u) + S_1(u, v), \quad (13)$$

where

$$\begin{aligned} S_0(u) & = \\ & = \sum_{n=0}^\infty [(n+1-F)J_{n+1-F}^2(u) - (n+F)J_{n+F}^2(u)] \end{aligned} \quad (14)$$

and

$$\begin{aligned} S_1(u, v) & = \sum_{n=0}^\infty [(n+1-F)\Lambda_{n+1-F}(u, v) - \\ & - (n+F)\Lambda_{n+F}(u, v)], \end{aligned} \quad (15)$$

where

$$\begin{aligned} \Lambda_\rho(u, v) & = \\ & = \frac{J_\rho'^2(v) [Y_\rho^2(u) - J_\rho^2(u)] - 2J_\rho'(v)J_\rho(u)Y_\rho'(v)Y_\rho(u)}{Y_\rho'^2(v) + J_\rho'^2(v)}. \end{aligned} \quad (16)$$

The vacuum current  $j_\varphi$  circulating around the  $(d-2)$ -tube leads to the appearance of a vacuum magnetic field with strength  $B^{3\dots d}$  directed along the  $(d-2)$ -tube; this is a consequence of the Maxwell equation

$$r\partial_r B_{(I)}^{3\dots d}(r) = -ej_\varphi(r), \quad (17)$$

where the coupling constant  $e$  differs in general from  $\tilde{e}$ . The total flux of the induced vacuum magnetic field across a plane which is orthogonal to the  $(d-2)$ -tube is defined as

$$\Phi_d^{(I)} = 2\pi \int_{r_0}^{\infty} dr r B_{(I)}^{3,\dots,d}(r) \quad (18)$$

and is given by the expression

$$\Phi_d^{(I)} = e\pi \int_{r_0}^{\infty} dr r j_{\varphi}(r) \left(1 - \frac{r_0^2}{r^2}\right). \quad (19)$$

Inserting  $j_{\varphi}(r)$  (10) and changing the order of integration over  $r$  and  $\mathbf{p}$ , we get, see [20],

$$\Phi_d^{(I)} = em^{d-3} \frac{(4\pi)^{(2-d)/2}}{2\Gamma(d/2)} \int_0^{\infty} \frac{du}{\sqrt{1+u^{2/(d-2)}}} \times \mathcal{D}(mr_0 \sqrt{1+u^{2/(d-2)}}), \quad (20)$$

where  $\Gamma(v)$  is the Euler gamma-function and

$$\mathcal{D}(y) = \int_y^{\infty} dx \left(1 - \frac{y^2}{x^2}\right) \int_0^{\infty} \frac{dz z}{\sqrt{z^2+x^2}} S\left(z, z\frac{y}{x}\right). \quad (21)$$

It should be noted that the function  $\mathcal{D}(y)$  (21) is immediately related to the total induced vacuum magnetic flux in the  $d=2$  case:

$$\Phi_2^{(I)} = \frac{e}{2m} \mathcal{D}(mr_0). \quad (22)$$

Since  $S_1(u, 0) = 0$ , one can obtain

$$\begin{aligned} \mathcal{D}(0) &= \int_0^{\infty} dx \int_0^{\infty} \frac{dz z}{\sqrt{z^2+x^2}} S_0(z) = \\ &= \frac{1}{3} F(1-F) \left(F - \frac{1}{2}\right), \end{aligned} \quad (23)$$

and the total induced vacuum magnetic flux in the  $d=2$  case is finite in the limit of a singular (i.e., infinitely thin) vortex filament,  $r_0 \rightarrow 0$  [17]:

$$\lim_{r_0 \rightarrow 0} \Phi_2^{(I)} = \frac{e}{6m} F(1-F) \left(F - \frac{1}{2}\right). \quad (24)$$

However, in the  $d \geq 3$  cases, the total induced magnetic flux becomes infinite in the limit of the infinitely thin vortex filament [18]. In this case, the consideration of the vacuum polarization by the vortex filament of the finite transverse size is especially actual.

### 3. Numerical Analysis of the Induced Vacuum Characteristics

As one can see from the previous section, in order to find the induced vacuum magnetic flux in the  $d$ -dimensional space, we first need to find it in the 2-dimensional space. Unfortunately, this task in the case of a vortex tube of finite transverse size can be solved only by numerical methods.

With this aim, we rewrite expression (10) in the  $d=2$  case in the dimensionless form

$$rj_{\varphi}(r)|_{d=2} = \frac{1}{2\pi} \int_0^{\infty} dz z \left[z^2 + \left(\frac{mr_0}{\lambda}\right)^2\right]^{-1/2} S(z, \lambda z), \quad (25)$$

where  $\lambda = r_0/r$  ( $\lambda \in [0, 1]$ ).

In the limit of a singular filament ( $r_0 = 0$ ), expression (25) contains only  $S_0$  (14). The summation in (14) can be performed analytically and (25) is reduced to the following form, see [20]:

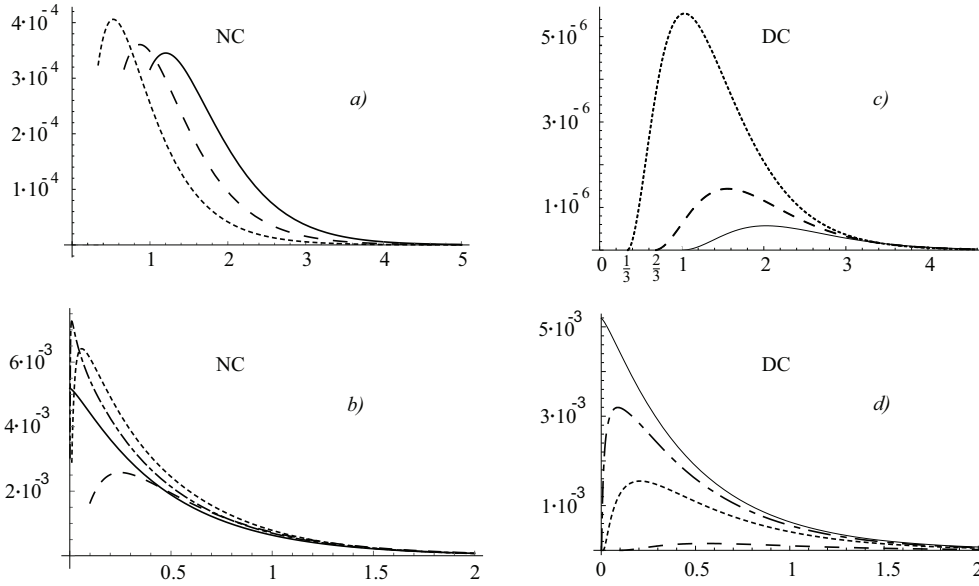
$$\begin{aligned} rj_{\varphi}^{\text{sing}}(r)|_{d=2} &= \frac{\sin(F\pi)}{\pi^3} \int_{mr}^{\infty} dw \frac{w^2}{\sqrt{w^2 - (mr)^2}} \times \\ &\times \{w[K_{1-F}^2(w) - K_F^2(w)] + (2F-1)K_F(w)K_{1-F}(w)\}, \end{aligned} \quad (26)$$

where  $K_{\rho}(u)$  is the Macdonald function of order  $\rho$ . The total induced vacuum magnetic flux in this case, see (24), attains the maximal absolute value equal to  $|e|/(72\sqrt{3}m)$  at  $F = F_{\pm}$ , where

$$F_{\pm} = \frac{1}{2} \left(1 \pm \frac{1}{\sqrt{3}}\right). \quad (27)$$

Next, we will numerically compute the induced vacuum current in the  $d=2$  case at  $F = F_{+}$ , when the integral in (25) is likely to be most distinct from zero (note that the current at  $F = F_{-}$  equals to that at  $F = F_{+}$  with the opposite sign). So, for a vortex tube of nonvanishing radius, we have to compute values of dimensionless quantity  $rj_{\varphi}$  at different values of  $\lambda$ . To do this, we perform the high-precision numerical integration in (25) with the help of a technique developed earlier in [20–24]. The results of computation can be approximated by an interpolation function in the form

$$rj_{\varphi}(r)|_{d=2} = \left[\frac{e^{-2x}}{\sqrt{x}}\right] \left[\frac{P_n(x-x_0)}{x^n}\right] \frac{Q_k(x^2)}{R_k(x^2)}, \quad x > x_0, \quad (28)$$



**Fig. 1.** The dimensionless induced vacuum current ( $rj_\varphi$ ) as a function of the dimensionless distance from the axis of the tube ( $x$ ) for different values of the dimensionless tube radius ( $x_0$ ) for the case of the Neumann (NC) and the Dirichlet (DC) boundary conditions: (a), (c) solid line corresponds to  $rj_\varphi \times 10^2$  for  $x_0 = 1$ , dashed line corresponds to  $rj_\varphi \times 10$  for  $x_0 = 2/3$ , and dotted line corresponds to  $rj_\varphi$  for  $x_0 = 1/3$ ; (b), (d) solid line corresponds to the cases of a singular filament ( $x_0 = 0$ ), dashed line corresponds to  $x_0 = 10^{-1}$ , dotted line corresponds to  $x_0 = 10^{-2}$ , and dash-dotted line corresponds to  $x_0 = 10^{-3}$ . Variable  $x$  is along the abscissa axis

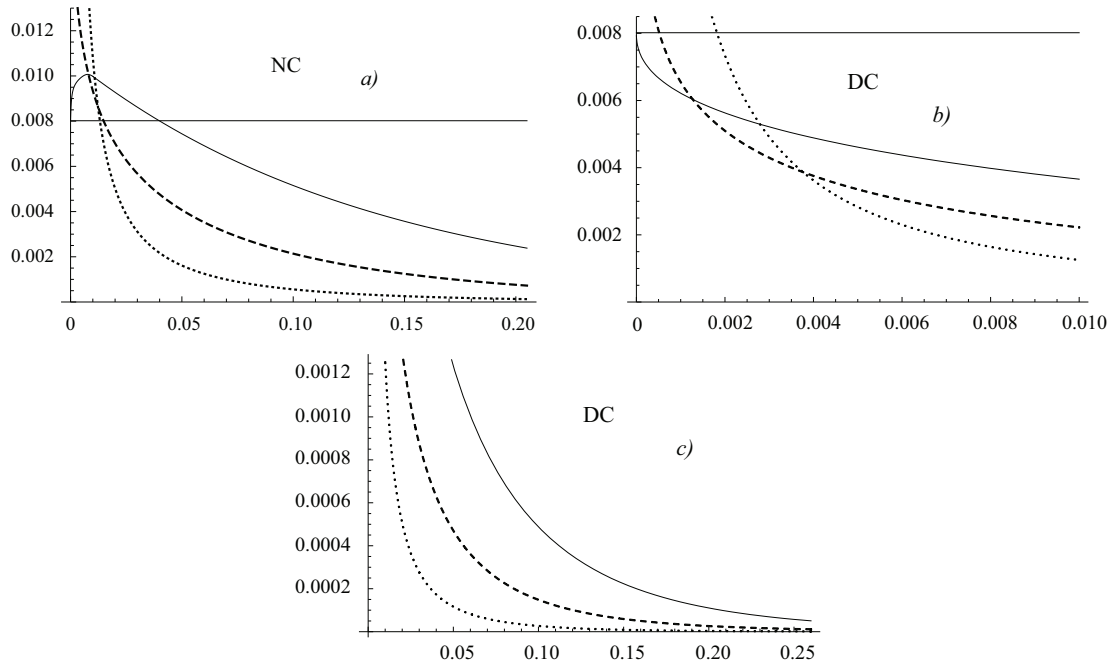
where  $x = mr$ ,  $x_0 = mr_0$  and  $P_j(y)$ ,  $Q_j(y)$ ,  $R_j(y)$  are polynomials in  $y$  of the  $j$ -th order with the  $x_0$ -dependent coefficients. It turns out that, for the interpolation of data, the most suitable choice of function (28) contains the polynomials with indices  $n = 9$  and  $k = 4$ . The first factor in square brackets describes the large-distance behavior in the case of a zero-radius tube (filament), the second factor in square brackets is an asymptotics at small distances from the side surface of the tube, the last factor describes the behavior at intermediate distances. Since the vortex tube is impenetrable,  $rj_\varphi(r)$  (28) vanishes at  $x \leq x_0$ .

The results in the  $d = 2$  case for the induced vacuum current in the case of the Neumann boundary condition are presented on Fig. 1, a and Fig. 1, b. For comparison, we also present the results for the induced vacuum current in the case of the Dirichlet boundary condition [20], see Fig. 1, c and Fig. 1, d. As one can see, the current is negligible for the tube of large radius, i.e., of order of the Compton wavelength and greater,  $r_0 \geq m^{-1}$ , see Fig. 1, a and Fig. 1, c. One can note that the induced vacuum current in the case of the Neumann boundary condition has a much weaker dependence on the tube thickness

$x_0$  (Fig. 1, a) as compared to that in the case of the Dirichlet boundary condition (Fig. 1, c). The current in the case of  $r_0 \ll m^{-1}$  is comparable with the current in the case of a singular filament, see Fig. 1, b and Fig. 1, d. It should be noted that, in the case of the Dirichlet boundary condition, the induced vacuum current is always less in value than in the case of a singular filament, see Fig. 1, d. It is not true for the case of the Neumann boundary condition, see Fig. 1, b. It should be noted that the value of the current in the case of the Neumann boundary condition is greater than the value of the current in the case of the Dirichlet boundary condition at any value of the tube thickness.

Using (19), (21), (22), and (28), we compute numerically the total induced vacuum magnetic flux in the  $d = 2$  case for different values of the tube thickness (parameter  $x_0 = mr_0$ ). The results of the computation in the dimensionless form can be approximated by an interpolation function in the form

$$\ln \frac{m\Phi_2^{(I)}}{e} = \ln \mathcal{D}(x_0) = M_1(x_0)\Theta(X - x_0) + \left[ a + bx_0^1 + dx_0^5 + \sqrt{L_2(x_0)} \right] \Theta(x_0 - X), \quad (29)$$



**Fig. 2.** The dimensionless induced vacuum magnetic flux in spaces of different dimensionality as a function of the dimensionless tube radius ( $x_0$ ) for the case of the Neumann (NC) and the Dirichlet (DC) boundary conditions:  $e^{-1}m\Phi_2^{(I)}$  – solid line,  $e^{-1}\Phi_3^{(I)}$  – dashed line,  $(em)^{-1}\Phi_4^{(I)}$  – dotted line. The case of  $x_0 = 0$  and  $d = 2$  is presented by a horizontal solid line

**The dimensionless induced vacuum magnetic flux in the cases of dimension  $d = 2, 3, 4$  for tubes of different radii and for the case of the Neumann (NC) and Dirichlet (DC) boundary condition**

$x_0$	1	2/3	1/3	$10^{-1}$	$10^{-2}$	$10^{-3}$
$\frac{m}{e}\Phi_2^{(I)}, \text{NC}$	$7.09 \times 10^{-6}$	$7.79 \times 10^{-5}$	$9.05 \times 10^{-4}$	$5.34 \times 10^{-3}$	$9.92 \times 10^{-3}$	0.01
$\frac{m}{e}\Phi_2^{(I)}, \text{DC}$	$2.36 \times 10^{-8}$	$5.76 \times 10^{-7}$	$2.07 \times 10^{-5}$	$4.88 \times 10^{-4}$	$3.66 \times 10^{-3}$	$6.22 \times 10^{-3}$
$\frac{1}{e}\Phi_3^{(I)}, \text{NC}$	$10^{-6}$	$1.41 \times 10^{-5}$	$2.26 \times 10^{-4}$	$2.14 \times 10^{-3}$	$9.34 \times 10^{-3}$	$1.64 \times 10^{-2}$
$\frac{1}{e}\Phi_3^{(I)}, \text{DC}$	$3.07 \times 10^{-9}$	$8.83 \times 10^{-8}$	$4.13 \times 10^{-6}$	$1.48 \times 10^{-4}$	$2.22 \times 10^{-3}$	$6.55 \times 10^{-3}$
$\frac{1}{me}\Phi_4^{(I)}, \text{NC}$	$7.5 \times 10^{-8}$	$1.3 \times 10^{-6}$	$3.02 \times 10^{-5}$	$5.58 \times 10^{-4}$	$1.08 \times 10^{-2}$	$1.15 \times 10^{-1}$
$\frac{1}{me}\Phi_4^{(I)}, \text{DC}$	$2.06 \times 10^{-10}$	$7.06 \times 10^{-9}$	$4.43 \times 10^{-7}$	$2.68 \times 10^{-5}$	$1.25 \times 10^{-3}$	$1.32 \times 10^{-2}$

where  $M_j(y)$ ,  $L_j(y)$  are polynomials in  $y$  of the  $j$ -th order,  $\Theta(y)$  is the Heaviside step function and  $X = 10^{-2}$ . Using (20) and (29), we compute numerically the total induced magnetic flux in the  $d = 3, 4$  cases, i.e.  $\Phi_3^{(I)}$  and  $\Phi_4^{(I)}$ . The results for the  $d = 2, 3, 4$  cases are presented in Fig. 2, a and in Table. For comparison, we also present the results for the induced flux in the case of the Dirichlet boundary condition [20], see Fig. 2, b and Fig. 2, c.

As one can see, in the case of the Neumann boundary condition (for  $d = 2$ ), there is a region of the tube thickness ( $0 < x_0 < 0.4$ ), where the absolute value of the induced flux is greater than in the case of a singular filament, see Fig. 2, a. Whereas the absolute value of the flux induced by a singular filament is always greater than the absolute value of the flux induced by a tube of the nonvanishing radius with the Dirichlet boundary condition, see Fig. 2, b and Fig. 2, c. One

can see also that the induced flux from a tube with the Neumann boundary condition is greater than the induced flux from a tube with the Dirichlet boundary condition at any value of the tube thickness.

In the case of higher space dimensions, the behavior of the induced flux in the case of the Neumann and Dirichlet boundary conditions is similar, see Fig. 2. For the case of a space with a higher dimension ( $d = 4$ ) at large radii of the tube, the induced flux is a more strongly decreasing function. But, at smaller radii of the tube, the induced flux is a more strongly increasing function compared to that in the case of the space with a smaller dimension ( $d = 3$ ). While the induced flux in the unphysical case of a singular filament is infinite for  $d > 2$ , the induced flux in the physical case of a tube of nonvanishing radius is finite. As one see from Table, the absolute value of the flux induced by a tube with the Neumann boundary condition is always greater than the absolute value of the flux induced by a tube with the Dirichlet boundary condition for any dimension of the space.

#### 4. Summary

In the present paper, we have considered the current and the total magnetic flux which are induced in the vacuum of the quantized charged scalar matter field by a topological defect in the form of the ANO vortex in a space of arbitrary dimension. We assume that the ANO vortex is impenetrable for the quantum matter. The perfectly rigid (Neumann) boundary condition is imposed on the matter field at the side surface of the vortex. The same problem was considered previously for the vortex with the perfectly reflecting (Dirichlet) boundary condition on its side surface [20]. So, we compare the results obtained for the Neumann and Dirichlet boundary conditions at the side surface of the vortex.

In both cases of the above-mentioned boundary conditions, the induced current is circulating around the vortex, and it is vanishingly small in the case of the vortex transverse size being of the order of or exceeding the Compton wavelength of the matter field ( $x_0 \gtrsim 1$ ), see Fig. 1, Fig. 2 and Table for the  $d = 2, 3, 4$  dimensions of the space. This confirms the previously obtained conclusion [21–24] that the vacuum polarization effects are essential only for matter fields with masses which are much smaller than the scale of the spontaneous symmetry breaking (mass of the Higgs field forming the topological defect).

In both cases of the above-mentioned boundary conditions, the induced vacuum current decreases exponentially at large distances from the vortex. The current and the induced vacuum magnetic flux are odd in the value of the vortex flux,  $\Phi$ , and periodic in this value with the period equal to the London flux quantum,  $2\pi\tilde{e}^{-1}$ . They vanish at  $F = 0, 1/2, 1$  and are of opposite signs in the intervals  $0 < F < 1/2$  and  $1/2 < F < 1$ , with their absolute values being symmetric with respect to the point  $F = 1/2$ . In the case of a space of dimension  $d > 2$ , the account for the finite transverse size of the vortex eliminates an unphysical divergence for the total induced vacuum flux, which takes place in the case of a singular vortex filament.

The visible difference between the cases of the Neumann and Dirichlet boundary conditions lies in the magnitude of the vacuum polarization effects. The absolute value of the induced vacuum current and the induced vacuum magnetic flux in the case of the Neumann boundary condition is greater than that in the case of the Dirichlet boundary condition. In particular, as one can see from Fig. 1,  $d$  and Fig. 2,  $b$  for the dimension of the space  $d = 2$ , the vacuum effects in the case of the Dirichlet boundary condition are always smaller than in the case of a singular vortex filament. However, as one can see from Fig. 1,  $b$ , for the case of the Neumann boundary condition, the absolute value of the induced vacuum current can exceed the absolute value of the induced vacuum current in the case of a singular vortex filament. Moreover, there is a region of the vortex thickness values ( $0 < x_0 < 0.4$ ), see Fig. 2,  $a$ , where the absolute value of the induced vacuum magnetic flux is greater than that in the case of a singular vortex filament for the  $d = 2$  space.

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ІНДУКОВАНИ ВАКУУМНИЙ СТРУМ  
ТА МАГНІТНИЙ ПОТІК У КВАНТОВАНІЙ  
СКАЛЯРНІЙ МАТЕРІЇ В ПРИСУТНОСТІ  
ВИХРОВОГО ДЕФЕКТУ З ГРАНИЧНОЮ  
УМОВОЮ ТИПУ НЕЙМАНА

Топологічний дефект у вигляді вихору Абрикосова–Нільсена–Олесена у просторі довільної вимірності розглядається як трубка, що містить потік калібрувального поля та є не проникливою для поля матерії. Квантоване заряджене скалярне поле матерії квантується з урахуванням наявності вихору, на поверхні якого накладено умову типу Неймана. Показано, що навколо вихору індукується вакуумний струм за умови, що комптонівська довжина хвилі поля матерії значно перевищує поперечний розмір вихору. Вакуумний струм є періодичною функцією від потоку калібрувального поля вихору, що є квантово-польовим проявом ефекту Ааронова–Бома. Вакуумний струм викликає появу індукованого вакуумного магнітного потоку, який (за деяких значень товщини трубки) перевищує вакуумний магнітний потік, що індукується сингулярним вихором. Отримані результати були порівняні з результатами, отриманими для випадку, коли на поверхню трубки накладалися граничні умови типу Діріхле. Було показано, що у випадку граничної умови типу Неймана абсолютні значення індукованого вакуумного струму та індукованого вакуумного магнітного потоку перевищують значення відповідних величин у випадку, коли на трубку накладаються граничні умови типу Діріхле.

*Ключові слова:* вакуумна поляризація, ефект Ааронова–Бома, вихровий дефект.