Надано рішення задачі про дифракцію хвилювання малої амплітуди, яке набігає під довільним кутом на нерухоме судно в умовах мілководдя. Потенціал швидкостей дифрагованого хвильового руху визначений методом зрощуваних асимптотичних розкладань. Виконані розрахунки амплітуд хвилювання у заданих точках навколо судна, наведені приклади хвильових полів

Ключові слова: дифракція лінійних хвиль на судні, метод зрощуваних асимптотичних розвинень, мілководдя

Представлено решение задачи о дифракции волнения малой амплитуды, набегающего под произвольным углом на неподвижное судно в условиях мелководья. Потенциал скоростей дифрагированного волнового движения определен методом сращиваемых асимптотических разложений. Выполнены расчеты амплитуд волнения в заданных точках вокруг судна, приведены примеры волновых полей

Ключевые слова: дифракция линейных волн на судне, метод сращиваемых асимптотических разложений, мелководье

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DETERMINING THE CHARACTERISTICS OF DIFFRACTED WAVES OF SMALL AMPLITUDE AROUND A VESSEL IN SHALLOW WATER

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1. Introduction

In most of the ports of the Black and Azov Seas, large-capacity tankers and bulk carriers, as well as some passenger vessels, have to be additionally loaded, unloaded or fuelled outside of the ports protected by water basins.

The water areas of ports in Ukraine are shallow-water. The ships that are docked on outer anchorage are under the influence of the rough sea and, at the same time, they prevent the propagation of waves. A wave field around the ship at the port roads is essentially three-dimensional; it represents a superposition of incoming and diffracted waves. The height of waves in specific points of the field depends on the location of points around the ship.

The complexity of evaluating the interaction between a ship and such waves is associated with the non-stationarity of the region, which is filled with fluid, and the non-linearity of boundary condition at the free surface of the fluid. The solution of the diffraction problem in a general form is complicated. Solving the particular problems requires refinement and specialization of boundary conditions.

With respect to ships, a diffraction problem is tackled rarely, since overcoming the difficulties associated with the practical evaluation of characteristics of a wave field around the body of a vessel requires special processing of the computing aspects of the obtained theoretical results. At the same time, for determining the hydrodynamic forces acting on a ship in pitching, as shown in articles [1, 2], a solution of a simpler task of radiation will suffice.

Characteristics of the waves around the vessel, which is anchored at the open port roads, are to be considered when planning the operation of auxiliary ships. Auxiliary vessels are tugs, bunkering vessels, pilot and harbor boats, as well as oil/garbage collector boats and boom crafts. The work of these vessels is related to safe navigation (transfer of people and cargo from one ship to another) and environmental safety (elimination of oil and petroleum spills).

The height of waves imposes restrictions on the operation of auxiliary ships. Determining the transformation of waves on outer anchorage is necessary for their safe operation, which is why this is a relevant task.

2. Literature review and problem statement

In recent years, most of the existing solutions of diffraction problem have focused on assessing the impact of waves on the stationary or movable offshore structures.

Article [3] examined diffraction and refraction of waves in a fluid with defined depth. The problem is solved by the finite element method using discrete non-local (DNL) boundary conditions. The examined objects are a channel of rectangular cross section, circular cylindrical island with a parabolic bottom around it. The characteristics of wave fields are presented.

A diffraction problem is important, in particular, for the equipment that employs the energy of ocean waves to generate electricity – Oscillating Water Column (OWC). Paper [4] presented a first order analytical solution of the problem on diffraction of ocean waves on the hollow-body vertical cylinder in the ocean with defined depth. In article [5], authors defined a wave field for the same object, created by a swinging cylinder, and solved a combined problem on diffraction-radiation.

Article [6] investigated numerical simulation (CFD) of the interaction between waves and a vertical cylinder. The Reynolds equation of the averaged turbulent fluid flow is solved (Reynolds Averaged Navier-Stokes, RANS). They modeled regular and irregular waves of small and finite amplitude (second-order Stokes) in the numerical testing pool. The results of determining the wave fields and wave forces are presented.

Paper [7] is devoted to the experimental study of influence of waves on a floating cylinder. In a small-sized wave flume, ocean waves are simulated. They examine their interaction with the floating cylinder, docked to the shore. The characteristics of waves and motion of a floating body are presented.

Article [8] describes numerical modeling of the interaction between nonlinear waves and a system of two vertical circular cylinders. One of them is rigidly fixed to the bottom, the second floats. Authors identified characteristics of wave forces and moments, as well as displacement of the floating cylinder. In connection with the operation of tension leg platforms (TLPs), the problems are solved on the impact of waves on the groups of vertical cylinders.

Paper [9] described the solution of potential problems of first and second orders by the method of finite elements. Vertical motions of vertical circular cylinders are examined. Wave fields and wave loads are determined for a single cylinder and the groups of two and four cylinders.

Article [10] demonstrates interaction between nonlinear waves and a vertical cylinder and the group of four cylinders. Authors employed the method of finite differences (FDM), methods of finite elements (Weakly Nonlinear and Weakly Dispersive FEM, Fully Nonlinear and Weakly Dispersive FEM). They defined the elevation of a free surface and coefficients of hydrodynamic forces at propagation and collision of separate waves. Results of the calculations are presented, as well as a comparison to experimental data.

Worth noting here is a complete analytic solution, presented in paper [11], of second order nonlinear diffraction problem for the two-dimensional stationary rectangular cylinders at the free surface of the fluid with defined depth. Authors determined the magnitudes of vertical and horizontal forces of first and second orders. The likelihood of solution is confirmed by a comparison with experiments and calculations of other authors.

Article [12] presented a solution of the problem on diffraction of monochromatic and bichromatic waves on a stationary horizontal cylindrical body that crosses free surface. The depth of the fluid is infinite, the incident waves are lateral or scant. Diffraction potential is used to determine the forces acting on the floating oil tank (a body with semi-elliptical waterlines in the bow part, rectangular frames in the middle part and prismatic stern).

An analysis of the above research results reveals that wave fields are defined around the objects of simplified forms.

Paper [13] considers a two-dimensional nonlinear potential problem on the fluctuations of cramped contour in the fluid of limited depth. Boundary conditions on the contour and the free surface of fluid are non-linear. Nonlinear forces are determined with an accuracy to the second order. Calculations are performed for different cramped contours. The influence of change in the relative depth on the value of nonlinear forces is investigated.

Article [14] addresses a three-dimensional potential problem on the fluctuations of a vessel in the liquid wtih limited depth and its solution by a numerical method. The influence of change in the depth on the values of attached masses and damping coefficients is examined; results of calculations of the given magnitudes for different types of vessels are presented.

Paper [15] defined the potential of radiation and diffraction potential in the pitching of a vessel moving in the significantly shallow water. The method used is the matched asymptotic expansion method (MAEM). The potentials of radiation for longitudinally-horizontal, vertical, and pitching fluctuations were determined at motion in the quiet water. The expression for the components of diffraction potential at vertical pitching is presented.

Article [16], by employing the improved matching method, defined the wave profiles around the ship that moves in deep water. The region, which is filled with fluid, is divided into a near-field and a far-field, which applies a radiation condition while the boundary condition at free surface is considered to be linear. The vessel is replaced with a system of features. Characteristics of these features and the potential of speeds of the near field are determined by a double technique. The first is the use of conditions on free surface and on the body in the near-field. The second is using the continuity of speed potentials and its derivative by a normal in the transition through the interface (matching surface).

For ships, a diffraction problem is solved when determining the wave loads in deep water or for determining the hydrodynamic forces in pitching in the shallows; wave profiles around the ship are not investigated.

The waves around the ship significantly differ from the waves at a significant distance from the ship. A wave field is transformed around the large tonnage vessels at the outer anchorage. This must be considered when assessing the safety of operation of auxillary (relatively small) vessels.

Thus, the study of wave diffraction processes at diffraction of regular waves of small amplitude that is incident under an arbitrary angle on the hull of a large tonnage ship, which floats without a run on the outer anchorage, is a relevant issue. Considering that under conditions of limited depth of the anchorage the occurrence of long waves that can cause noticeable pitching of a large tonnage ship is highly unlikely, the latter is considered stationary.

A choice of linear wave theory for solving the diffraction problem is predetermined by the following considerations.

First, the waves that come from deepwater areas of the open sea is transformed in a complex way in the shallows – the waves transform from three-dimensional into the two-dimensional ones. Big waves are reduced, while small waves increase in length and height [17].

Second, in the case of moderate waves, calculations of seaworthiness of a vessel using the linear theory yield fully acceptable results [2].

3. The aim and tasks of the study

The aim of present work is to determine the characteristics of a wave field that occurs around the ship, which floats idling at outer anchorage and that is exposed to incident waves of small amplitude. A presence of these characteristics provides for the enhancement of safety of operation of auxiliary vessels at outer anchorage. These data should also be used when carrying out search-and-rescue operations and elimination of oil spills in open water area.

Taking into account the accepted assumptions used to determine the characteristics of a wave field around the ship, the following tasks are to be solved:

– to determine the potential of speeds of perturbed fluid motion caused by the diffraction of waves of small amplitude that are incident under an arbitrary angle to a stationary vessel under conditions of shallow water;

– to define in the specified points around the ship the amplitudes of waves caused by the incident waves and their diffraction on the vessel as an obstacle in wave propagation.

4. Materials and methods of examining the diffraction of regular waves of small amplitude on a stationary ship in shallow waters

4. 1. Statement of diffraction problem

Let us consider the interaction between a stationary vessel of length L floating in shallow water of depth H with the wave of small amplitude, which is incident under an arbitrary angle. Denote length of the wave λ, wave amplitude as a, wave propagation speed as c. The fluid is considered perfect, heavy, incompressible, its traffic – potential.

We shall introduce a rectangular Oxyz coordinate system, associated with the vessel. The Oxz plane coincides with the non-perturbed free surface of the fluid. The Ox axis is directed towards the bow; Oy – towards the right board; Oz is vertically upward. The location of the ship relative to the incident wave is defined by the course angle β between the Ox asis and a vector of wave's speed.

Perturbed fluid motion is described by the potential of velocities $\Phi^E(x,y,z,t)$. The region of its determining E is limited by the bottom of water reservoir D, wetted surface of a ship S and free surface of the fluid Σ (part of the plane z=0 outside the waterline of a stationary ship). Given the linearity of the problem, we shall represent the potential $\Phi^E(x,y,z,t)$ in the form of a sum:

$$\Phi^E(x,y,z,t)=\Phi^*(x,y,z,t)+\Phi^d(x,y,z,t), \tag{1}$$

where $\Phi^*(x,y,z,t)$ is the potential of speeds of the system of regular incident waves that the vessel is exposed to; $\Phi^d(x,y,z,t)$ is the potential of velocities of wave motion of the fluid, caused by the diffraction of incident wave, on the hull of the ship.

The potential of incident waves' speeds in the coordinate system related to a ship is written in the following form [1]:

$$\Phi^*=\frac{ag}{\sigma}\frac{\mathrm{ch}\left[\alpha_0(z-H)\right]}{\mathrm{ch}(\alpha_0 H)}\cdot\sin(\alpha_0 x\cos\beta+\alpha_0 y\sin\beta-\sigma t), \tag{2}$$

where g=9.81 m/s^2; σ is the frequency of the incident wave; α_0 is the wavenumber (form frequency), which in shallow waters is defined as the only real positive root of transcendental equation:

$$\alpha g\cdot \mathrm{th}(\alpha H)=\sigma^2. \tag{3}$$

Let us represent formula (2) in the form:

$$\Phi^*(x,y,z,t)=\Phi_c^*(x,y,z)\cos(\sigma t)+\Phi_s^*(x,y,z)\sin(\sigma t); \tag{4}$$

$$\left.\begin{matrix}\Phi_c^*(x,y.z)\\ \Phi_s^*(x,y.z)\end{matrix}\right\}= \\ =\pm\frac{ag}{\sigma}\frac{\mathrm{ch}\left[\alpha_0(z-H)\right]}{\mathrm{ch}(\alpha_0 H)}\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(\alpha_0 x\cos\beta+\alpha_0 y\sin\beta). \tag{5}$$

Potential $\Phi^d(x,y,z,t)$ of speeds of diffracted wave motion is a solution of the following boundary problem:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi^d(x,y,z,t)=0,\quad (x,y,z)\in E; \tag{6}$$

$$\left(\frac{\partial^2}{\partial t^2}-g\frac{\partial}{\partial z}\right)\Phi^d(x,y,0,t)=0,\quad (x,y)\in\Sigma; \tag{7}$$

$$\frac{\partial\Phi^d(x,y,z,t)}{\partial N}=-\frac{\partial\Phi^*(x,y,z,t)}{\partial N},\quad (x,y,x)\in S; \tag{8}$$

$$\frac{\partial\Phi^d(x,y,H,t)}{\partial z}=0,\quad (x,y)\in\left]-\infty;\infty\right[; \tag{9}$$

$$\lim_{r\to\infty}\left[\overline{\mathrm{grad}}\Phi^d(x,y,z,t)\right]=0,\quad (x,y,z)\in E,\quad r=\sqrt{x^2+y^2}, \tag{10}$$

where N is the outward normal to the surface S.

Similarly (4) $\Phi^d(x,y,z,t)$ is represented in the form of a sum:

$$\Phi^d(x,y,z,t)= \\ =\Phi_c^d(x,y,z)\cos(\sigma t)+\Phi_s^d(x,y,z)\sin(\sigma t). \tag{11}$$

Further on, index d in the expressions of diffraction potentials is omitted to simplify the record.

Article [18] stated a general hydrodynamic problem on the potential of perturbed fluid velocities at motion of a ship in waves in a restricted fairway. A stepwise linearization of the given problem was carried out. Solution of the corresponding linear problems by the MAEM method is described.

The above boundary problems differ from the corresponding problems on the potential of radiation [18] only by the form of boundary condition on the vessel hull. Therefore, to determine the potential of velocities of the diffracted motion of fluid, we used the method similar to the one employed in [18].

Consider a structure of the normal derivative on the ship hull. We assume that a ship is an extended body, whose transverse dimensions are small compared to its length while the longitudinal distances at which noticeable changes in the form of the hull occur, are finite. Given this, we shall refine the type of a normal derivative from the potential on the wetted surface S. It follows from (8) and (11):

$$\frac{\partial\Phi(x,y,z,t)}{\partial N}= \\ =\frac{\partial\Phi_c(x,y,z)}{\partial N}\cos(\sigma t)+\frac{\partial\Phi_s(x,y,z)}{\partial N}\sin(\sigma t). \quad (12)$$

According to [18], in the problems on radiation, the expressions for boundary conditions on the ship hull in the case of its vertical fluctuations contain a multiplier cos(N,z), and in the case of cross-horizontal fluctuations – cos(N,y). By analogy, we shall group the constituents of normal derivatives of the potentials that contain these multipliers. According to formulas (5) and (12):

$$\frac{\partial\Phi_c(x,y,z)}{\partial N}=B_C^{EV}+B_C^{OD}; \quad (13)$$

$$\frac{\partial\Phi_s(x,y,z)}{\partial N}=B_S^{EV}+B_S^{OD}; \quad (14)$$

$$\left.\begin{matrix}B_C^{EV}\\B_S^{EV}\end{matrix}\right\}=\mp\frac{ag\alpha_0\cos(N,z)}{\sigma\,\mathrm{ch}(\alpha_0 H)}\times \\ \times\mathrm{sh}\left[\alpha_0(z-H)\right]\begin{cases}\sin\alpha_0(x\cos\beta+y\sin\beta),\\ \cos\alpha_0(x\cos\beta+y\sin\beta);\end{cases} \quad (15)$$

$$\left.\begin{matrix}B_C^{OD}\\B_S^{OD}\end{matrix}\right\}=-\frac{ag\alpha_0\cos(N,y)\sin\beta}{\sigma\,\mathrm{ch}(\alpha_0 H)}\times \\ \times\mathrm{ch}\left[\alpha_0(z-H)\right]\begin{cases}\cos\alpha_0(x\cos\beta+y\sin\beta),\\ \sin\alpha_0(x\cos\beta+y\sin\beta).\end{cases} \quad (16)$$

The linearity of a boundary problem for the diffraction potential, as well as an analysis of the structure of the normal derivative from the potential on the wetted surface of a ship indicate that this potential is advisable to represent in the form of a sum:

$$\Phi(x,y,z,t)=\sum_{i=1}^{4}\Phi_i(x,y,z,t). \quad (17)$$

Consequently, the diffracted potentials $\Phi_i(x,y,z,t)$ are determined by the MAEM method, similar to [18].

4. 2. Determining the potentials of diffracted wave motion

According to the procedure for appying the MAEM method, we shall conditionally split the region filled with fluid into the following zones: external, where (y/L)=O(1), and internal, where (y/L)=O(ε), ε<<1.

Boundary transition ε→0 at y and z, fixed in the external zone, transforms the hull into a segment Δ={–L/2≤x≤L/2, y=z=0}. Region E is transformed into region $\bar{E}_0$ (a layer of fluid 0≤z≤H with a cut-out segment Δ). Free surface Σ turns into plane $\bar{\Sigma}_0$ (plane z=0 with a cut-out segment Δ).

In each zone, separate boundary problems are stated, consequently their solutions asymptotically converge on the boundaries of zones, forming an approximate solution of the problem, uniformly applicable over the entire region, which is filled with fluid.

4. 2. 1. Solving the boundary problems in external zone

As shown in [18], at ship fluctuations by the sinusoidal law, a function of radiation contains not only sine, but also cosine, component, and vice versa. That is why we shall represent potentials $\Phi_i(x,y,z,t)$ in the form of a sum:

$$\Phi_i(x,y,z,t)= \\ =\Phi_i^c(x,y,z)\cos(\sigma t)+\Phi_i^s(x,y,z)\sin(\sigma t). \quad (18)$$

In the external zone (that is region $\bar{E}_0$), we state the boundary problems for amplitude functions $\Phi_i^c(x,y,z)$ and $\Phi_i^s(x,y,z)$. The problems include conditions of harmony, the boundary conditions at the free surface of fluid $\bar{\Sigma}_0$, at the bottom of reservoir D, condition of attenuation of diffracted waves at an infinite distance from the ship. The potentials should also satisfy the fundamental radiations. Boundary conditions at the wetted surface of a ship are not formulated because this surface does not belong in the external zone. It is known only that at diffraction on the ship hull of longitudinal waves, the potential of perturbed velocities is continuous, and its normal derivative is discontinued while passing through the diametral plane of the ship. At diffraction of transverse waves, the normal derivative form the potential is continuous, while the potential is discontinued when passing through the diametral plane. Thus, the bondary conditions of the following form are formulated:

$$\frac{\partial}{\partial y}\Phi_1^c(x,\pm0,z)=\begin{cases}\pm f_c, & |x|\le L/2,\ 0\le z\le H;\\ 0, & |x|>L/2,\ 0\le z\le H;\end{cases} \quad (19)$$

$$\frac{\partial}{\partial y}\Phi_1^s(x,\pm0,z)=0,\ -\infty\le x\le\infty,\ 0\le z\le H; \quad (20)$$

$$\Phi_2^c(x,\pm0,z)=\begin{cases}\pm p_c, & |x|\le L/2,\ 0\le z\le H;\\ 0, & |x|>L/2,\ 0\le z\le H;\end{cases} \quad (21)$$

$$\Phi_2^s(x,\pm0,z)=0,\ \ -\infty\le x\le\infty,\ \ 0\le z\le H; \quad (22)$$

$$\frac{\partial}{\partial y}\Phi_3^c(x,\pm0,z)=0,\ \ -\infty\le x\le\infty,\ \ 0\le z\le H; \quad (23)$$

$$\frac{\partial}{\partial y}\Phi_3^s(x,\pm0,z)=\begin{cases}\pm f_s, & |x|\le L/2,\ 0\le z\le H;\\ 0, & |x|>L/2,\ 0\le z\le H;\end{cases} \quad (24)$$

$$\Phi_4^c(x,\pm0,z)=0,\ \ -\infty\le x\le\infty,\ \ 0\le z\le H; \quad (25)$$

$$\Phi_4^s(x,\pm0,z)=\begin{cases}\pm p_s, & |x|\le L/2,\ 0\le z\le H;\\ 0, & |x|>L/2,\ 0\le z\le H.\end{cases} \quad (26)$$

Note also that

$$(H/L)=O(\varepsilon^{\alpha}).$$

In this case, solutions of the boundary problems are applicable both for the moderate shallow waters (boundary transition α→0 is executed, then:

$(H/L)=O(1)$

and for the considerable shallow waters (then $\alpha\to 1$ and $(H/L)=O(\varepsilon)$).

We shall employ the Fourier method to the boundary problems for potentials $\Phi_i^c(x,y,z)$ and $\Phi_i^s(x,y,z)$ in each of the regions y>0 and y<0. Decompose the potentials segment [0; H] along the full orthogonal system of functions [18]:

$$\left.\begin{aligned} &Z_0(z)=N_0^{-\frac{1}{2}}\cdot \mathrm{ch}\left[\alpha_0(z-H)\right];\\ &Z_m(z)=N_m^{-\frac{1}{2}}\cdot \cos\left[\alpha_m(z-H)\right]; \end{aligned}\right\} \tag{27}$$

$$N_0=\frac{1}{2}\left[1+\frac{\mathrm{sh}(2\alpha_0 H)}{2\alpha_0 H}\right];$$

$$N_m=\frac{1}{2}\left[1+\frac{\sin(2\alpha_m H)}{2\alpha_m H}\right],\ m=1,2,\ldots \tag{28}$$

$\alpha_1<\alpha_2<\ldots$ is the sequence of real positive roots of transcendental equation:

$$\alpha g \mathrm{Tg}(\alpha H)=-\sigma^2. \tag{29}$$

Note that the system of functions (27) is the same that was applied in solving the problems in [4, 5, 11] taking into account the transformations of coordinates.

By using a condition of harmony, the boundary conditions in rectangle 0≤z≤H, –L/2≤x≤L/2, as well as the principle of radiation, we shall receive a solution of the problems in the external zone. Then:

– for components of the potentials even along y:

$$\left.\begin{matrix}\Phi_1^c\\ \Phi_3^s\end{matrix}\right\}=\frac{1}{2H}Z_0(z)\int\limits_{-\frac{L}{2}}^{\frac{L}{2}}N_0(\alpha_0 R)\int\limits_0^H\left\{\begin{matrix}f_c(\xi,\varsigma)\\ f_s(\xi,\varsigma)\end{matrix}\right\}Z_0(\varsigma)d\varsigma d\xi-$$

$$-\frac{1}{\pi H}\sum_{m=1}^{\infty}Z_m(z)\int\limits_{-\frac{L}{2}}^{\frac{L}{2}}K_0(\alpha_m R)\int\limits_0^H\left\{\begin{matrix}f_c(\xi,\varsigma)\\ f_s(\xi,\varsigma)\end{matrix}\right\}Z_m(\varsigma)d\varsigma d\xi; \tag{30}$$

$$\left.\begin{matrix}\Phi_1^s\\ \Phi_3^c\end{matrix}\right\}=\mp\frac{1}{2H}Z_0(z)\int\limits_{-\frac{L}{2}}^{\frac{L}{2}}J_0(\alpha_0 R)\int\limits_0^H\left\{\begin{matrix}f_c(\xi,\varsigma)\\ f_s(\xi,\varsigma)\end{matrix}\right\}Z_0(\varsigma)d\varsigma d\xi; \tag{31}$$

– for components of the potentials odd along y:

$$\left.\begin{matrix}\Phi_2^c\\ \Phi_4^s\end{matrix}\right\}=\frac{1}{2H}Z_0(z)\frac{\partial}{\partial y}\int\limits_{-\frac{L}{2}}^{\frac{L}{2}}N_0(\alpha_0 R)\int\limits_0^H\left\{\begin{matrix}p_c(\xi,\varsigma)\\ p_s(\xi,\varsigma)\end{matrix}\right\}Z_0(\varsigma)d\varsigma d\xi-$$

$$-\frac{1}{\pi H}\frac{\partial}{\partial y}\sum_{m=1}^{\infty}Z_m(z)\int\limits_{-\frac{L}{2}}^{\frac{L}{2}}K_0(\alpha_m R)\int\limits_0^H\left\{\begin{matrix}p_c(\xi,\varsigma)\\ p_s(\xi,\varsigma)\end{matrix}\right\}Z_m(\varsigma)d\varsigma d\xi; \tag{32}$$

$$\left.\begin{matrix}\Phi_2^s\\ \Phi_4^c\end{matrix}\right\}=\mp\frac{1}{2H}Z_0(z)\frac{\partial}{\partial y}\int\limits_{-\frac{L}{2}}^{\frac{L}{2}}J_0(\alpha_0 R)\int\limits_0^H\left\{\begin{matrix}p_c(\xi,\varsigma)\\ p_s(\xi,\varsigma)\end{matrix}\right\}Z_0(\varsigma)d\varsigma d\xi, \tag{33}$$

where $R=\sqrt{(x-\xi)^2+y^2}$, ξ, ζ are the variables of integration along the length of a ship and depth, respectively; J_0, N_0, K_0 are the Bessel, Neumann, and Macdonald functions of zero order of real argument, respectively.

4. 2. 2. Solving the boundary problems in internal zone

Let us perform a boundary transition ε→0 when y and z are fixed in the external zone. Introduce the "stretched" coordinates Y=y/ε, Z=z/ε. With accuracy to low O(ε), the motion in the internal zone can be considered as two-dimensional. The potential of speeds of the diffracted wave movement – a harmonic function Y and Z – should satisfy the following differential system:

$$\left(\frac{\partial^2}{\partial Y^2}+\frac{\partial^2}{\partial Z^2}\right)\Phi(Y,Z)=0,\ \ (Y,Z)\in E(x); \tag{34}$$

$$\left(\frac{\partial}{\partial Z}+k_1\right)\Phi(Y,0)=0,\ \ k_1=\frac{\varepsilon\sigma^2}{g},\ \ |Y|>\frac{1}{2}b(x); \tag{35}$$

$$\frac{\partial\Phi(Y,H)}{\partial Z}=0,\ \ (Y)\in\left]-\infty;\infty\right[; \tag{36}$$

$$\frac{\partial\Phi(Y,Z)}{\partial N}=B(Y,Z),\ \ (Y,Z)\in L(x), \tag{37}$$

where region E(x) is the band 0≤Z≤h with a cut-out in the form of a frame contour L(x), the coordinates of points of which are (Y, Z); b(x) is the width of the frame along a waterline; h is the depth of water.

The magnitudes b(x) and h are taken in a linear scale of the internal zone.

The given boundary problem does not state boundary conditions at an infinite distance from the ship Y→∞, that is, on the outer border of the internal zone. These conditions are set at the stage of converging the solutions of boundary problems of the internal and external zones at their border. Therefore, the solution can be found with accuracy to some arbitrary additive function. The merging technique accepted in the present paper, according to [18], allows us to find a solution without determining this function, as for merging the solutions we apply the asymptotics of speed potential at the outer border of the internal zone.

The asymptotics of speed potentials Φ(Y, Z) are written in the form:

– for the components even along Y:

$$\Phi^{ev}(Y,Z)\sim\frac{V(\lambda_0)}{2}Q(\lambda_0)\mathrm{ch}\left[\lambda_0(Z-h)\right]\sin(\lambda_0 Y); \tag{38}$$

– for the components odd along Y:

$$\Phi^{od}(Y,Z)\sim\pm\frac{1}{2}V(\lambda_0)P(\lambda_0)\mathrm{ch}\left[\lambda_0(Z-h)\right]\cos(\lambda_0 Y), \tag{39}$$

where

$$V(\lambda_0)=\frac{2\mathrm{ch}(\lambda_0 h)}{2\lambda_0 h+\mathrm{sh}2\lambda_0 h}. \tag{40}$$

P and Q are the real functions k, defined by formulas:

$$\left.\begin{matrix}P(k_1)\\ Q(k_1)\end{matrix}\right\}=$$

$$=2\int\limits_{-\frac{b}{2}}^{\frac{b}{2}}e^{-k_1\varsigma_0(Y)}\left\{\begin{matrix}\left[T_1(Y)\sin(k_1Y)+T_2(Y)\cos(k_1Y)\right]\\ \left[T_1(Y)\cos(k_1Y)-T_2(Y)\sin(k_1Y)\right]\end{matrix}\right\}dY; \tag{41}$$

where $\varsigma_0(Y)$ is the equation of contour L(x);

$$T_1 = k_1 A_+(Y) + B_+(Y)\sqrt{1+\left[\frac{d\varsigma_0(Y)}{dY}\right]^2}; \qquad (42)$$

$$T_2 = k_1 A_+(Y)\frac{d\varsigma_0(Y)}{dY}; \qquad (43)$$

$A_+(Y)$ is the value of speed potential on contour L(x);

$B_+(Y)$ is the value of a normal derivative from the potential on contour L(x).

The value of the normal derivative $B_+(Y)$ is defined by the boundary condition. Potential $A_+(Y)$ on the contour is unknown as it is actually a solution of the problem. In accordance with the practice of applying the Kochin functions in wave problems [18], instead of $A_+(Y)$, formulas (42) and (43) are substituted with the value of the potential at infinite frequency. This potential is the solution to the boundary problem in the internal zone, which consists of equations (34), (36), (37), and instead of condition (35), we accept:

$$\Phi(Y,0)=0,\ |Y|>\frac{1}{2}b(x). \qquad (44)$$

For the components of speed potentials odd along Y in accordance with boundary conditions (19), (20), (23), (24), not the potential itself is needed but its derivative along Y. We shall obtain from (38):

$$\frac{\partial}{\partial Y}\Phi^{EV}(Y,Z)\sim \pm$$
$$\pm\frac{\lambda_0 V(\lambda_0)}{2}Q(\lambda_0)\mathrm{ch}\left[\lambda_0(Z-h)\right]\cos(\lambda_0 Y). \qquad (45)$$

4. 2. 3. Merging the solutions. Formulas for a speed potential

In order to merge the solutions, we use a method of boundary merging – "the inner boundary of the external boundary equals the outer boundary of the internal boundary" [19].

In chapter 4. 2. 1, the solution of problem on the diffraction of oblique waves on the hull of a stationary elongated ship in the external zone was derived on the assumption that in band $-\infty\le x\le\infty$, $0\le z\le H$ the conditions (19)–(26) are satisfied. The value of functions $f_{c,s}$ and $p_{c,s}$ will be determined from the solution of problem in the internal zone. Let us return to (38) and (45) to the external variables $y=\varepsilon Y$ and $z=\varepsilon Z$. In this case, $\lambda_0 h$ proceeds to $\alpha_0 H$, $V(\lambda_0)$ – to $V(\alpha_0 H)$, while the potentials and their derivatives at $y=\pm 0$ will take the form:

– for the components even along y:

$$\frac{\partial}{\partial Y}\Phi^{EV}(\pm 0,z)=\pm\frac{\alpha_0 Q(\alpha_0)}{2}\mathrm{ch}\left[\alpha_0(z-H)\right]V(\alpha_0 H); \qquad (46)$$

– for the components odd along y :

$$\Phi^{OD}(\pm 0,z)=\mp\frac{P(\alpha_0)}{2}\mathrm{ch}\left[\alpha_0(z-H)\right]V(\alpha_0 H). \qquad (47)$$

Therefore,

$$\left.\begin{matrix} f_c \\ f_s \end{matrix}\right\}=\left\{\begin{matrix} Q_c(x,\alpha_0) \\ Q_s(x,\alpha_0) \end{matrix}\right\}\frac{\alpha_0}{2}\mathrm{ch}\left[\alpha_0(z-H)\right]V(\alpha_0 H); \qquad (48)$$

$$\left.\begin{matrix} p_c \\ p_s \end{matrix}\right\}=-\frac{1}{2}\left\{\begin{matrix} P_c(x,\alpha_0) \\ P_s(x,\alpha_0) \end{matrix}\right\}\mathrm{ch}\left[\alpha_0(z-H)\right]V(\alpha_0 H). \qquad (49)$$

The resulting formulas for the components of speed potential of the diffracted wave motion are derived at substituting the functions $f_{c,s}$ and $p_{c,s}$ from (47) and (48) into (30)–(33).

It should be noted that functions $Q_{c,s}$ and $P_{c,s}$, which are icluded in (48) and (49), do not depend on the vertical coordinate. Consider internal integrals in (30)–(33). For the components of potentials that contain functions $Z_0(\varsigma)$, upon substitutions and transformations, we obtain:

$$\int_0^H \frac{V(\alpha_0 H)}{2}\mathrm{ch}\left[\alpha_0(\varsigma-H)\right]Z_0(\varsigma)d\varsigma=\frac{V(\alpha_0 H)}{2}N_0^{-\frac{1}{2}}I_0, \qquad (50)$$

where it is written to simplify the record

$$I_0=\int_0^H \mathrm{ch}^2\left[\alpha_0(\varsigma-H)\right]d\varsigma=\frac{2\alpha_0 H+\mathrm{sh}(2\alpha_0 H)}{4\alpha_0}. \qquad (51)$$

For the components of potentials that contain function $Z_m(\varsigma)$, we receive:

$$\int_0^H \frac{V(\alpha_0 H)}{2}\mathrm{ch}\left[\alpha_0(\varsigma-H)\right]Z_m(\varsigma)d\varsigma=\frac{V(\alpha_0 H)}{2}N_m^{-\frac{1}{2}}I_m, \qquad (52)$$

where it is written to simplify the record

$$I_m=\int_0^H \mathrm{ch}\left[\alpha_0(\varsigma-H)\right]\cos\left[\alpha_m(\varsigma-H)\right]d\varsigma=0, \qquad (53)$$

because the functions of system (27) are orthogonal. Therefore, other terms in formulas (31) and (32) are converted to zero.

When computing functions $Q_{c,s}$ and $P_{c,s}$, we use the corresponding formulas for normal derivatives of potentials at the wetted surface of a vessel, in particular: B_C^{EV} for Q_c; B_S^{EV} – for Q_s; B_C^{OD} – for P_c; B_S^{OD} – for P_s.

Note also that for the components of potentials odd along y in the resulting formulas, instead of Bessel function $J_0(\alpha_0 R)$ and Neumann function $N_0(\alpha_0 R)$ of zero order of real argument, we use first order Bessel function $J_1(\alpha_0 R)$ and Neumann function $N_1(\alpha_0 R)$ [20], respectively.

Solution of the problem on the cosine component of the potential is given in article [21].

4. 4. Determining the characteristics of a wave field around a ship

A theoretical solution of the diffraction problem, derived above, is used for determining the amplitudes of wave in the assigned points around the hull of a ship, which floats idling in the shallow waters.

The equation of a wave profile is written in the following form:

$$z_B=\frac{1}{g}\frac{d}{dt}\left[\Phi^E(x,y,0,t)\right]. \qquad (54)$$

Considering (1), (2), (4), (11), (17), formula (54) takes the form:

$$z_B=r_c\cos(\sigma t)+r_s\sin(\sigma t), \qquad (55)$$

where the amplitude functions are equal to, respectively:

$$r_c = a\cos(\Theta) + \varsigma_1^s + \varsigma_2^s + \varsigma_3^s + \varsigma_4^s;$$

$$r_s = a\sin(\Theta) - \varsigma_1^c - \varsigma_2^c - \varsigma_3^c - \varsigma_4^c. \qquad (56)$$

The components of diffracted waves $\varsigma_i^{c,s}$, i=1, 2, 3, 4, taking into account the substitution z=0 and function evenness $\mathrm{ch}(\alpha_0 H)$ are determined by formulas:

$$\varsigma_1^c = \frac{\sigma\alpha_0}{2g} E(\alpha_0 H) \int_{-\frac{L}{2}}^{\frac{L}{2}} Q_c(\xi,\alpha_0) N_0(\alpha_0 R) d\xi; \qquad (57)$$

$$\varsigma_2^c = y\frac{\sigma\alpha_0}{2g} E(\alpha_0 H) \int_{-\frac{L}{2}}^{\frac{L}{2}} P_c(\xi,\alpha_0) \frac{N_1(\alpha_0 R)}{R} d\xi; \qquad (58)$$

$$\varsigma_3^c = \frac{\sigma\alpha_0}{2g} E(\alpha_0 H) \int_{-\frac{L}{2}}^{\frac{L}{2}} Q_s(\xi,\alpha_0) J_0(\alpha_0 R) d\xi; \qquad (59)$$

$$\varsigma_4^c = y\frac{\sigma\alpha_0}{2g} E(\alpha_0 H) \int_{-\frac{L}{2}}^{\frac{L}{2}} P_s(\xi,\alpha_0) \frac{J_1(\alpha_0 R)}{R} d\xi; \qquad (60)$$

$$\varsigma_1^s = -\frac{\sigma\alpha_0}{2g} E(\alpha_0 H) \int_{-\frac{L}{2}}^{\frac{L}{2}} Q_c(\xi,\alpha_0) J_0(\alpha_0 R) d\xi; \qquad (61)$$

$$\varsigma_2^s = -y\frac{\sigma\alpha_0}{2g} E(\alpha_0 H) \int_{-\frac{L}{2}}^{\frac{L}{2}} P_c(\xi,\alpha_0) \frac{J_1(\alpha_0 R)}{R} d\xi; \qquad (62)$$

$$\varsigma_3^s = \frac{\sigma\alpha_0}{2g} E(\alpha_0 H) \int_{-\frac{L}{2}}^{\frac{L}{2}} Q_s(\xi,\alpha_0) N_0(\alpha_0 R) d\xi; \qquad (63)$$

$$\varsigma_4^s = y\frac{\sigma\alpha_0}{2g} E(\alpha_0 H) \int_{-\frac{L}{2}}^{\frac{L}{2}} P_s(\xi,\alpha_0) \frac{N_1(\alpha_0 R)}{R} d\xi, \qquad (64)$$

where it is written to simplify the record

$$E(\alpha_0 H) = \frac{V(\alpha_0 H)}{2} \mathrm{ch}(-\alpha_0 H) = \frac{1}{\dfrac{2\alpha_0 H}{\mathrm{ch}^2(\alpha_0 H)} + 2\mathrm{th}(\alpha_0 H)}. \qquad (65)$$

The amplitude of waves is determined by formula:

$$r = \sqrt{r_c^{\,2} + r_s^{\,2}}. \qquad (66)$$

When performing calculations, each frame contour L(x) is given in the form of sets of points. The points divide the contour into fairly small elements, each of which is considered as a straight-line segment.

Functions $Q_{c,s}$ and $P_{c,s}$ are computed for each frame contour L(x), that is for fixed x. Calculation formulas are transformed considering the replacement of variables in (41) at the stage of merging the solutions and trnasition from the integration to the summation by points.

It should be noted that the employed calculation techniques are not related to any special form of frame contrours.

5. Results of examining a wave field around a ship

Using the formulas given above, we calculated wave amplitudes in the assigned points of observation around a ship. These points form a grid, coordinates of the points are within |x|≤L, |y|≤L points inside the contour of the waterline of a ship are excluded. A step by abscissas and ordinates of the grid is 0.0125L. Such an arragement of points is chosen according to the results of numerical experiments; it provides a satisfactory accuracy of subsequent calculations at the smallest volume of source data.

In the grid's nodes, we determine the cosine and sine components of r_c ans r_s (56), which are consequently employed for the calculation of wave amplitudes (66).

As the object of research we chose a wave field around a bulk carrier of "Zoya Kosmodemyanskaya" type. The hull of this ship is a typical representative of the class of large tonnage bulkers and tankers. Its features are a tuber-like bow, transom stern, a long cylindrical insert with vertical sides, a flat bottom. Principal dimensions of the vessel: length L=201.6 m, width B=19.8 m, draught T=11.73 m.

Calculations of waves' amplitudes were performed for the following characteristics:

– relative water depth H/T=1.1, 1.3, 1.5, 2.0;

– course angles of incident waves β=90°, 120°, 135°, 150°, 180°;

– relative wavelength λ/L=0.5, 0.6, 0.7, 0.8, 0.9, 1.0.

Using results of the calculations for all combinations of water depth, length of waves and course angles of waves, we obtained relative wave amplitudes $\bar{r} = r/a$. The points of observation are around the hull of the ship in a square field |x|≤L, |y|≤L. We built lines of equal amplitudes at step $\Delta\bar{r} = 0.1$.

As an example, Fig. 1, 2 show the wave fields around the hull of the ship.

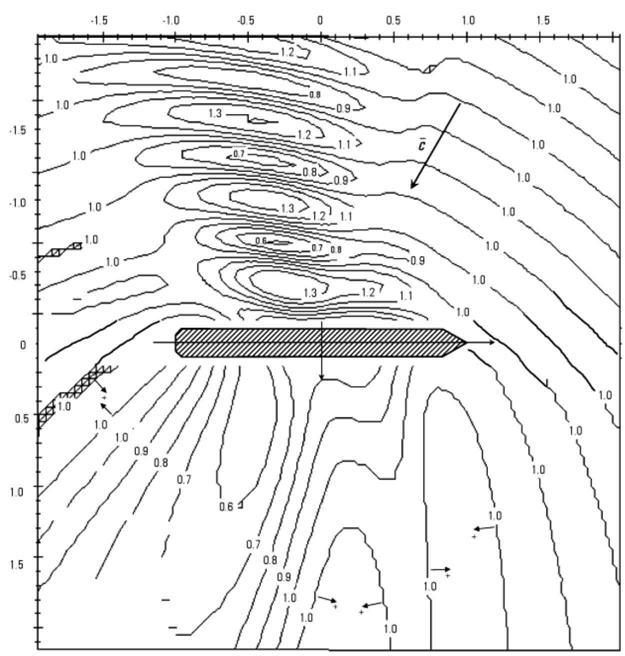

Fig. 1. Wave field around the hull of the vessel H/T=1.1, λ/L=0, β=120°

The distributions of relative wave amplitudes are shown around the hull of the ship in a square box |x|/L≤1, |y|/L≤1 at relative depth of water area H/T=1.1, relative length of waves λ/L=0.5, and course angles of incident waave β=120° and β=150°, respectively.

The following patterns are established:

1. At transverse and oblique wave near the ship from the side of incidence, the standing waves appear. Wave antinodes are at a distance from each other that roughly equals a half the wavelength.

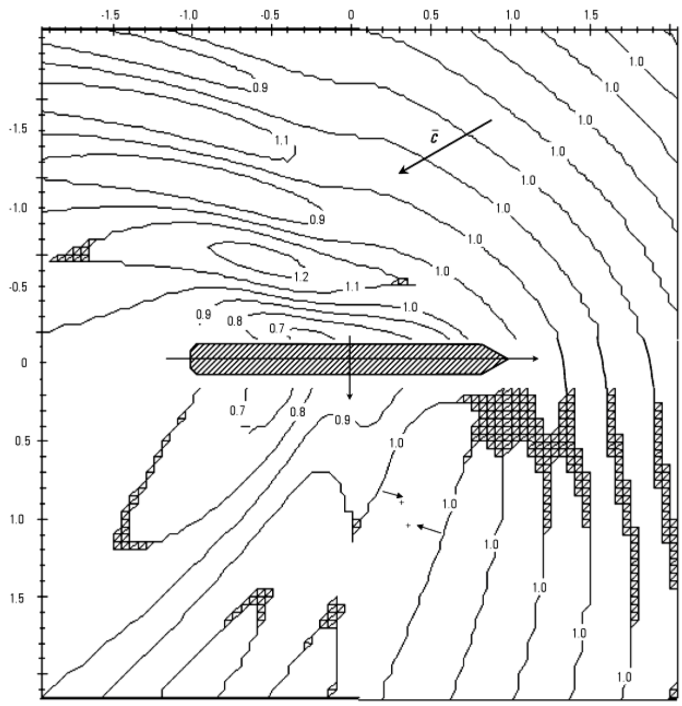

Fig. 2. Wave field around the hull of the vessel H/T=1.1, λ/L=0, β=150°

2. The arrangement of antinodes depends on the course angle of waves, in particular:

– at β=90°, the antinodes are located in the region of a middle frame, perpendicular to the incident waves' velocity vector;

– at oblique angles (β=120°, 135°, 150°), the antinodes are shifted to the stern, the angles between the antinodes and an incident waves' velocity vector are blunt; the shorter the incident wave the larger the shift of antinodes towards the stern.

3. Increase in the relative wave ordinates $\bar{r}=r/a$ depends on the course angle of waves and wavelengths. The largest values $\bar{r}=1.45$ are at β=90° and λ/L=0.5. The longer the waves and the larger the course angle of waves, the less wave ordinates are. Thus, at β=90° and λ/L=1.0 $\bar{r}=1.12$.

4. Among the antinodes, there are zones of reduction in wave coordinates – we observe value $\bar{r}\approx 0.8\div 0.9$. It should be noted that $\bar{r}<1.0$ does not mean reducing the level of water, but the moderation of fluctuations in a given area.

5. At the waves incident from the bow (β=180°), an increase in the wave ordinates of the order $\bar{r}=1.3$ is observed before the ship.

6. The magnitudes of reduction in the wave ordinates in a region of shadow are approximately $\bar{r}=0.4$ at λ/L=0.5, $\bar{r}=0.6$ at λ/L=0.8 and $\bar{r}=0.8$ at λ/L=1.0.

6. Discussion of results of determining a wave field around a ship

An analysis of the calculated wave fields reveals that at transverse and oblique waves near a ship, there occur the standing waves from the side of incidence. In this case:

– the slope of waves in this region increases by about twice compared with the region of hydrodynamic shadow or the region at a considerable distance from the hull of the vessel;

– the arrangement of antinodes in the examined range of lengths and course angles of waves changes little at depth of the water area;

– the depth influences the magnitude of growth and reduction in the wave ordinates, in particular: the lower the depth, the larger the magnitude $\bar{r}$ and the bigger the difference between the maximum and minimum relative amplitudes is.

It was also established that in the region of a hydrodynamic shadow, the standing waves are missing. In this case:

– location and length of the region of shadow is mainly determined by the magnitude of course angle of waves and the wave length and almost does not depend on the depth of water area;

– the shorter the incident waves the less the depth of water area, the more pronounced is the weakening of fluctuations in the region of a hydrodynamic shadow, but the zone of maximum attenuation is also narrower;

– the magnitude of waves' attenuation in the region of a shadow mostly depends on the wavelengths and course angle of waves, and to a lesser extent on the depth.

The longer the incident waves, the smaller are the changes in the wave field caused by a vessel.

The results obtained are preliminary, as they were received using the linear theory. Their refinement is implied at the next stage of study that will address solution of the problem on the diffraction of oblique waves of ultimate amplitude on the hull of a stationary elongated vessel under conditions of shallow water.

7. Conclusions

1. The present work reports a solution by the MAEM method of the problem on diffraction of oblique waves of small amplitude on a vessel under conditions of significant shallow water. By applying this method, we for the first time obtained the formulas for the potential of speeds of diffracted wave motion of fluid in the assigned points near the hull of the ship. The vessel is considered to be elongated and motionless. A shape of the frame contours of the vessel may take U-shaped, V-shaped, tuber-shaped form. The methodogy of solution and the results obtained represent a further development of the MAEM method for the study and solution of boundary problems on the hydrodynamics of a ship.

2. We obtained the equation of a wave profile for a combination of incident waves and the diffracted waves. The calculations are performed of the amplitudes of waves in the preset points of observation around the ship. The variable parameters are the depth of water area, wave length and a course angle of waves. We analyzed the transformation of the incident waves on the hull of the ship and around it. Examples are given of the distributions of relative wave amplitudes around the hull of the ship.

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