

Розв'язано задачу оптимального керування групи паралельно працюючих газоперекачувальних агрегатів при мінімізації загальних витрат на експлуатацію нагнітачів за умови обмеження частоти обертання нагнітача, температури газу на виході із нагнітача та температури продуктів згорання на виході турбіни низького тиску. Враховано нечіткість продуктивності нагнітача, спричинена великою похибкою вимірювання

Ключові слова: генетичні алгоритми, нечітка величина, процес компримування, вхідний конфузор, оптимальне керування

Решена задача оптимального управління группы паралельно работающих газоперекачивающих агрегатов при минимизации общих затрат на эксплуатацию нагнетателей при условии ограничения частоты вращения нагнетателя, температуры газа на выходе из нагнетателя и температуры продуктов сгорания на выходе турбины низкого давления. Учтена нечеткость производительности нагнетателя, вызванная большой погрешностью измерения

Ключевые слова: генетические алгоритмы, нечеткая величина, процесс компримирования, входящий конфузор, оптимальное управление

SOLUTION OF THE OPTIMIZATION PROBLEM ON THE CONTROL OVER OPERATION OF GAS PUMPING UNITS UNDER FUZZY CONDITIONS

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1. Introduction

A network of gas pipelines that run through the territory of Ukraine ensures continuous supply of gas both to enterprises of the country and for export to the countries of Central and Eastern Europe. As present [1], the total length of gas pipelines reaches 34.8 thousand km. The estimated through-put capacity of gas transportation system when entering Ukraine is 288 billion m³ per year (800 million m³ per day).

Design capacity of the gas transportation system (GTS) of Ukraine in the direction of countries of Western Europe and Turkey is 142 billion cubic meters per year while the actual volume of transit in 2014 was 60 billion cubic meters [2].

Thus, the Ukrainian GTS has excessive capacity, which makes it a relevant task to choose the number of parallel working units at each compressor station and their rotation frequency, based on the selected criteria for performance efficiency of the groups of parallel units.

2. Literature review and problem statement

In papers [3–9], authors stated and solved problems of the optimal redistribution of flows between parallel working units. For the given cases, the authors apply different approaches to solving such problems. Article [3] sets the task

to minimize consumption of fuel gas for the selected section of a gas pipeline, which has a certain number of compressor stations. The problem is set for the assigned pressures at the inlet and outlet of each element of a gas pipeline on finding such controlling actions so that a cost criterion acquires the minimum value. In order to solve the set problem, the authors employed a deterministic model disregarding an interaction between the system and the external environment, as well as neglecting a constraint for the controlling actions.

Paper [4] addresses the role of technical condition of GPU at periodic maintenance that affects the total cost of gas compression, however, the authors fail to take into account the impact of the environment on the technical condition of GPU, moreover, they chose a deterministic model.

Characteristics of productivity and energy efficiency of parallel working GPU are covered in [5], but the model that is employed in the paper does not consider significant errors of instruments for measuring performance of compressors.

The task of selecting the optimal mode for the operation of compressors is tackled in article [6], where it is solved by the redistribution of gas flows through GPU provided the effective overall efficiency of a group of parallelly connected units has the maximum possible magnitude. This work did not consider any other circuits for connecting GPU except for the parallel one.

In order to increase performance efficiency of a compressor station, author of paper [7] points to the need to stabilize productivity of parallel working compressors using

PI-controllers. The author rightly indicates that the problem on stabilizing the performance of compressors is rather controversial, while achieving the lowest consumption of fuel gas requires a certain technique to distribute the load on working units, however, the model presented does not account for the fuzziness of technological parameters.

In article [8], authors developed an integrated algorithm of the optimal load on GPU for the optimal operation of CU. However, a decision on the minimization of costs is made by using a consistent computer-based brute-force method for the operational modes.

A slightly different approach to solving the task of optimal distribution of flows between the elements of a gas transportation system is proposed in paper [9]. In order to solve it, the authors apply methods of linear programming and the theory of networks taking into account constraints predetermined by the material balances of flows. Since the problem is determined, so the authors neglected such issues as the impact of the environment on the work of GTS, technical condition of separate units of the system, the effect of measurement errors on a decision-making process, etc.

3. The aim and objectives of the study

The aim of present work is to solve a problem on the optimal control over operation of parallel working gas pumping units under a condition that the total consumption of fuel gas is minimal, while satisfying the constraints for technological parameters and taking into account a significant error in the measurement of pressure drop on confuser.

To accomplish the aim, the following tasks have been set:

- to determine a membership function for the productivity of a compressor;
- to construct mathematical models and to formalize a problem on the optimal control over the process of natural gas compression;
- to determine rotation frequency of the rotors of centrifugal compressors of natural gas.

4. Formalization of a problem on the optimal control over the process of natural gas compression

The total costs of operating compressors at a compressor station with a gas-turbine drive can be calculated from the following formula:

$$J(\bar{n}) = c_g \sum_{i=1}^m G_i(n_i), \tag{1}$$

where $J(\bar{n})$ is the cost of work m of parallel operating units relative to the unit of time; c_g is the cost of volumetric unit of the fuel gas used to drive a gas-turbine unit; $G(n_i)$ is the fuel gas consumption, relative to normal conditions, which is used by the i -th GPU; n_i is the frequency of the rotor of the i -th compressor.

In line with the technological regime, it is necessary to limit gas temperature at the outlet from compressor T_{out} and the temperature of combustion products at the outlet of a low-pressure turbine – T_v . For the case of a pump-free work, it is necessary to limit lower frequency of the compressor's rotor [10]:

$$T_{out} \leq T_{out}^{(max)}, \tag{2}$$

$$T_v \leq T_v^{(max)}, \tag{3}$$

$$n_i^{(min)} \leq n_i \leq n_i^{(max)}, \tag{4}$$

where $T_{out}^{(max)}$, $T_v^{(max)}$ are the maximum permissible values for the magnitudes T_{out} and T_v ; $n_i^{(min)}$, $n_i^{(max)}$ are the lower and upper limit for the compressor's rotor frequency. The

Temperature of exhaust gases $T_v^{(i)}$ of the i -th GPU, gas temperature at the outlet from the i -th compressor $T_{out}^{(i)}$ are the functions of such technological factors as the rotor's frequency n_i , temperature T_{in} and gas pressure P_{in} at the inlet to compressor, a degree of pressure increase ϵ , and depend on the parameters of the environment – atmospheric pressure P_c and temperature T_c

$$T_v^{(i)} = f_v^{(i)}(n_i, P_{in}, T_{in}, \epsilon, P_c, T_c), \tag{5}$$

$$T_{out}^{(i)} = f_{out}^{(i)}(n_i, P_{in}, T_{in}, \epsilon, P_c, T_c). \tag{6}$$

When constraints (2)–(4) are satisfied, a requirement must be fulfilled to ensure the preset performance efficiency Q of the compressor station

$$Q = \sum_{i=1}^m Q_i, \tag{7}$$

where Q_i is the performance efficiency of the i -th compressor.

Productivity of the i -th compressor is a function of such parameters as n_i , P_{in} , T_{in} , ϵ , P_c and T_c

$$Q_i = f_q^{(i)}(n_i, P_{in}, T_{in}, \epsilon, P_c, T_c). \tag{8}$$

Dependences (5), (6) and (8) are given in paper [11] as empirical models obtained based on the results of observation of work of GPU. Approximation of the results of observation was carried out by a polynomial of degree r

$$y = \sum_{i=0}^{M-1} a_i \prod_{j=1}^k x_j^{s_{ij}}, \tag{9}$$

where M is the number of terms of the polynomial; a_i are the coefficients of the polynomial; s_{ij} are the degree of arguments, which must satisfy the constraint; k is the number of independent variables

$$\sum_{j=1}^k s_{ji} \leq r. \tag{10}$$

The number of terms M in polynomial (10) is determined from formula [12]

$$M = \frac{(r+k)!}{r!k!}. \tag{11}$$

Since dependences (5), (6), and (8) are the functions of technological parameters P_{in} , T_{in} , ϵ and environmental parameters P_c and T_c , which are known, then, by substituting them in the derived regression equations that take the form of polynomial (9), we shall obtain dependences $T_{out}^{(i)} = \Psi_1^{(i)}(n_i)$, $T_v^{(i)} = \Psi_2^{(i)}(n_i)$ and $Q_i = Q_i(n_i)$. These dependences will be the functions of only one variable n_i . Constraints (2) and (3)

assign the upper bound of temperature values $T_{out}^{(\max)}$ and $T_v^{(\max)}$, which leads to equations

$$\Psi_2^{(i)}(n_i) - T_{out}^{(\max)} = 0 \text{ and } \Psi_4^{(i)}(n_i) - T_v^{(\max)} = 0.$$

As a result, we obtain the equations whose positive roots will be magnitudes $n_i^{(out)}$ and $n_i^{(v)}$. Then

$$\tilde{n}_{i,\max} = \min(n_i^{(out)}, n_i^{(v)}, n_{i,\max}).$$

Thus, we shall solve the following problem on the optimal control over the work of GPU:

$$\min : J(\bar{n}) = c_g \sum_{i=1}^m G(n_i) \quad (12)$$

under constraints

$$Q = \sum_{i=1}^m Q_i(n_i), \quad (13)$$

$$n_i^{(\min)} \leq n_i \leq \tilde{n}_i^{(\max)}, \quad i = \overline{1, m}. \quad (14)$$

Using a shop automation, performance efficiency of the compressor was determined by measuring pressure drop ΔP_i at the inlet confuser with subsequent calculation using the following formula [13]:

$$Q_i = A_H^{(i)} \sqrt{\Delta P_i \rho_i}, \quad (15)$$

where $A_H^{(i)}$ is the coefficient of volumetric productivity of confuser of the i -th compressor; ρ_i is the gas density at the inlet to a compressor.

Calculation of volumetric productivity Q_i of the centrifugal compressor (CC) using formula (15) produces an error of tens of percent [14].

In order to obtain empirical models, by employing results of observations over the work of GPU, we applied a method of synthesis of empirical models based on genetic algorithms [15]. The essence of the method is that it is required to choose a polynomial of degree r , in which part of the coefficients is assigned with a value of zero while the rest of the coefficients are different from zero.

We shall construct an ordered structure of length M , where the i -th place is taken by unity or zero depending on whether parameter a_i , $i = \overline{0, M-1}$ in model (9) differs from zero, or is zero.

The problem of synthesis of an empirical model will be stated as follows: it is required to choose by evolutionary selection such a chromosome from the initial population of chromosomes, which provides the best value for the adaptation function (a minimum value of the selection criterion).

In order to choose a model from a set of all possible models, the set of experimental values is divided into two parts – training and testing. The training part of the total set is used to calculate parameters of the model, while the testing one is employed in order to select the best model out of a given set of models. Such a selection is implemented using the criteria of regularity, or shift [11, 15].

We shall use a method of synthesis of empirical models based on genetic algorithms. We shall obtain a model of dependence (8) in the form of a polynomial (9). Substituting numerical values of magnitudes $n_i, P_{in}, T_{in}, \epsilon, P_c$ and T_c , which

are measured by means of the shop automation, we shall obtain a polynomial in the form:

$$Q_i = \sum_k a_k^{(i)} n_i^k. \quad (16)$$

Given the fact that measurement of the productivity of the i -th compressor yields a significant error, then there is every reason to consider coefficients of model (16) fuzzy numbers. We shall assume that a membership function $\mu(a_k^{(i)})$ of the fuzzy magnitude $a_k^{(i)}$ takes a triangular shape (Fig. 1). In Fig. 1, the following denotations are adopted

$$z = a_k^{(i)}, \quad \mu(z) = \mu(a_k^{(i)}), \quad z_0 = a_{k,0}^{(i)},$$

where $a_{k,0}^{(i)}$ is the modal value of fuzzy magnitude $a_k^{(i)}$.

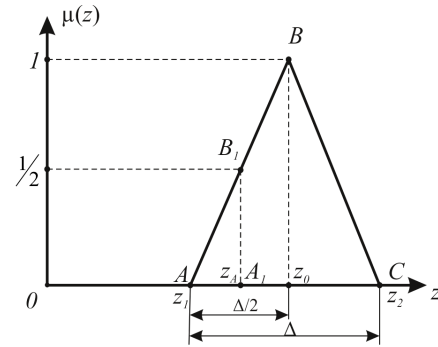


Fig. 1. Triangular membership function of a fuzzy magnitude

Find an analytical expression for the membership function $\mu(z)$. A section of straight line AB passes through the points with coordinates $A(z_1; 0)$ and $B(z_0; 1)$. Figure 1 shows that $z_1 = z_0 - \Delta/2$.

Therefore, the equation of the straight line that passes through points $A(z_0 - \Delta/2; 0)$ and $B(z_0; 1)$, will take the form

$$\mu(z) = q_1 + q_2 z,$$

$$z \in [z_0 - \Delta/2; z_0].$$

Considering coordinates of points A and B , through which the straight line passes, we find that

$$q_1 = -2 \frac{z_0}{\Delta} + 1, \quad q_2 = \frac{2}{\Delta}.$$

Similarly, we can find an equation of the straight line passing through points $B(z_0; 1)$ and $C(z_0 + \Delta/2; 0)$

$$\mu(z) = q_1 + q_2 z,$$

$$z \in [z_0; z_0 + \Delta/2],$$

where

$$q_1 = 2 \frac{z_0}{\Delta} + 1,$$

$$q_2 = -\frac{2}{\Delta}.$$

Thus, the membership function of a fuzzy magnitude z will be described by the following analytical expression:

$$\mu(z) = \begin{cases} -\frac{2}{\Delta}(z_0 - z) + 1, & z \in [z_0 - \Delta/2; z_0], \\ \frac{2}{\Delta}(z_0 - z) + 1, & z \in [z_0; z_0 + \Delta/2]. \end{cases} \quad (17)$$

A triangular membership function is inconvenient for practical use. This is explained by its piecewise-linear form, where there are no derivatives at points A, B, and C (Fig. 1).

We shall approximate a piecewise-linear membership function (17) by exponential function

$$\mu(z) = \exp\left(-\frac{(z - z_0)^2}{2\sigma^2}\right). \quad (18)$$

Parameter σ^2 , which is the parameter of fuzziness concentration, will be chosen such that the membership function (18) passes through point B_1 with coordinates $B_1(z_A; 1/2)$ (Fig. 1). Let us find abscissa z_A . To do this, we shall consider triangles ABD and AB_1A_1 . We obtain from the condition of similarity of these triangles

$$\frac{BD}{AD} = \frac{A_1B_1}{AA_1}.$$

Since

$$AD = \Delta/2, \quad BD = 1, \quad A_1B_1 = 1/2,$$

then

$$AA_1 = \frac{AD \cdot A_1B_1}{BD} \quad \text{or} \quad AA_1 = \frac{\Delta}{4}.$$

Considering

$$z_A = z_1 + AA_1 \quad \text{and} \quad z_1 = z_0 - \Delta/2,$$

we obtain

$$z_A = z_0 - \frac{\Delta}{4}.$$

Thus, we receive

$$\mu(z_A) = \exp\left(-\frac{(z_A - z_0)^2}{2\sigma^2}\right).$$

Hence

$$\sigma^2 = -\frac{(z_A - z_0)^2}{2 \ln \mu(z_A)}.$$

Considering the values of z_A and $\mu(z_A)$, we shall find

$$\sigma^2 = -\frac{\Delta^2}{32 \cdot \ln 2}.$$

Fig. 2 shows a triangular membership function (17) and approximation by dependence (18) at

$$\Delta = 0,4 \quad \text{and} \quad z_0 = 1,2.$$

Each coefficient of empirical model (17) will have its value of magnitude Δ , and, accordingly

$$\sigma_i^2 = \frac{\Delta_i^2}{32 \cdot \ln 2}. \quad (19)$$

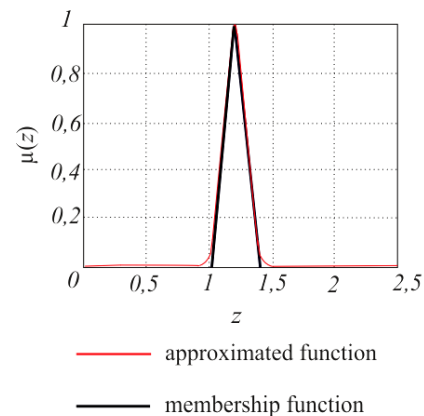


Fig. 2. Triangular membership function of fuzzy magnitude z and its approximation

Given that the coefficients of empirical model (16) are fuzzy magnitudes with a membership function (18), where the coefficient of concentration is determined from formula (19), we shall find $\mu(Q_i)$. In this case, one should bear in mind that when one performs operations of addition and when multiplying a Gauss number by a clear magnitude, we shall again obtain the Gauss number [8], which means that

$$\mu(Q_i) = \exp\left(-\frac{(Q_i - m_q^{(i)})^2}{2(\sigma_q^{(i)})^2}\right). \quad (20)$$

Parameters $m_q^{(i)}$ and $\sigma_q^{(i)}$ of the membership function (20) will be found by using the rules of arithmetic operations on fuzzy numbers of the (L-R)-type in the Gauss basis [16, 17].

Following the structure of model (16), in order to determine parameters $m_q^{(i)}$ and $\sigma_q^{(i)}$, it is necessary to perform such operations on fuzzy magnitudes as addition and the multiplication of a fuzzy number by a clear number.

Assume that $A_{LR} = \langle a_1, \alpha_1, \beta_1 \rangle$ and $B_{LR} = \langle a_2, \alpha_2, \beta_2 \rangle$ are fuzzy numbers of the (L-R)-type, where a_1, a_2 are the modal values; $\alpha_1, \alpha_2, \beta_1, \beta_2$ are the left and right coefficients of fuzziness. Then the parameters of a fuzzy number

$$C_{LR} = A_{LR} + B_{LR} = \langle a, \alpha, \beta \rangle$$

are calculated from formula:

$$a = a_1 + a_2, \quad \alpha = \alpha_1 + \alpha_2, \quad \beta = \beta_1 + \beta_2. \quad (21)$$

Find the sum of n fuzzy numbers

$$s = \sum_{i=1}^n v_i,$$

for each of which the membership function is determined from formula (18).

It is obvious that

$$v_i^{(LR)} = \langle a_{v,i}, \alpha_{v,i}, \beta_{v,i} \rangle.$$

We obtain

$$s_{1.2} = v_1 + v_2.$$

According to formula (21)

$$s_{1.2}^{(LR)} = \langle a_{1.2}, \alpha_{1.2}, \alpha_{1.2} \rangle,$$

where $a_{1.2} = a_{v1} + a_{v2}$, $\alpha_{1.2} = \alpha_{v1} + \alpha_{v2}$.

Because

$$s_{1.3} = v_1 + v_2 + v_3 = s_{1.2} + v_3,$$

then

$$s_{1.3}^{(LR)} = \langle a_{1.3}, \alpha_{1.3}, \alpha_{1.3} \rangle,$$

where $a_{1.3} = a_{1.2} + a_{v3}$, $\alpha_{1.3} = \alpha_{1.2} + \alpha_{v3}$.

Considering the values of $a_{1.2}$ and $\alpha_{1.2}$, we obtain

$$a_{1.3} = a_{v1} + a_{v2} + a_{v3}, \quad \alpha_{1.3} = \alpha_{v1} + \alpha_{v2} + \alpha_{v3}.$$

By repeating such an iterative process, we shall obtain in a general case:

$$s_{LR} = \langle a_s, \alpha_s, \alpha_s \rangle,$$

where

$$a_s = \sum_{i=1}^n a_{vi}, \quad \alpha_s = \sum_{i=1}^n \alpha_{vi}. \quad (22)$$

In the case when a fuzzy number is multiplied by a clear number, the membership function remains Gaussian [16].

Let a fuzzy number z be multiplied by clear number n_i^k . Then $v = zn_i^k$ or

$$z = \frac{v}{n_i^k}.$$

Substituting the value of z into formula (18), we shall obtain a membership function of the fuzzy magnitude v . We receive

$$\mu(v) = \exp\left(-\frac{(v - n_i^k z_0)^2}{2n_i^{2k} \sigma^2}\right).$$

Thus, $a_{vi} = n_i^k z_0$, $\alpha_{vi} = n_i^{2k} \sigma^2$.

Now we take into account that, in accordance with membership function (18), $z_0 = a_{k,0}^{(i)}$. With regard to formula (22), we obtain

$$m_q^{(i)} = \sum_k a_{k,0}^{(i)} n_i^k, \quad (23)$$

$$\left(\sigma_q^{(i)}\right)^2 = \sum_k n_i^{2k} \sigma_i^2. \quad (24)$$

For function (20), we shall set a γ -cut. As a result, we receive equation

$$\exp\left(-\frac{(Q_i - m_q^{(i)})^2}{2\left(\sigma_q^{(i)}\right)^2}\right) = \gamma,$$

that will be solved relative to variable Q_i . Consequently, we obtain

$$Q_i = m_q^{(i)} + \sigma_q^{(i)} \sqrt{\ln \frac{1}{\gamma^2}},$$

where $0 < \gamma \leq 1$.

The value of Δ_i will be computed from the following formula:

$$\Delta_i = \delta \alpha_{k,0}^{(i)},$$

where $0 \leq \delta \leq \Delta_m$, Δ_m is the magnitude, which is determined by the accuracy of measuring productivity of the compressor by a pressure difference on the confuser.

Substituting the value of σ_i^2 , which is calculated from formula (19), into equality (24), and taking into account the value of Δ_i , we shall obtain

$$Q_i = m_q^{(i)} + \frac{\delta}{4(\ln 4)^{1/2}} \left(\sum_k \left(n_i^k a_{k,0}^{(i)} \right)^2 \right)^{1/2} \sqrt{\ln \frac{1}{\gamma^2}}.$$

Let us introduce the following denotation:

$$K_\gamma = \frac{\delta}{4(\ln 4)^{1/2}} \cdot \sqrt{\ln \frac{1}{\gamma^2}}$$

and take into account the value of $m_q^{(i)}$, which is computed from formula (24). Then

$$\tilde{Q}_i(n_i) = \sum_k a_{k,0}^{(i)} n_i^k + K_\gamma \left(\sum_k \left(a_{k,0}^{(i)} n_i^k \right)^2 \right)^{1/2}. \quad (25)$$

Thus, we have obtained the following problem on the optimal control over a group of parallel working compressor units:

$$\min: J(\bar{n}) = c_g \sum_{i=1}^m G_i(n_i) \quad (26)$$

under constraints

$$Q = \sum_{i=1}^m \tilde{Q}_i(n_i), \quad (27)$$

$$\begin{aligned} n_i^{(\min)} &\leq n_i \leq \tilde{n}_i^{(\max)}, \\ i &= \overline{1, m}, \end{aligned} \quad (28)$$

where $\tilde{Q}_i(n_i)$ is calculated from formula (25).

Fig. 3 shows block diagram of an automated system of control over the process of natural gas compression for the i -th compressor.

The criterion of optimality (26) includes dependence $G(n_i)$, which is synthesized, based on the observations, in the form of a polynomial (9) on the basis of genetic algorithms. At the known values of P_m , T_m , ε , P_c and T_c , measured by the tools of the shop automation, the dependence $G_i(\bar{x})$, where

$$\bar{x} = (n_i, P_m, T_m, \varepsilon, P_c, T_c)^T,$$

will be a function of one variable n_i -controlling action.

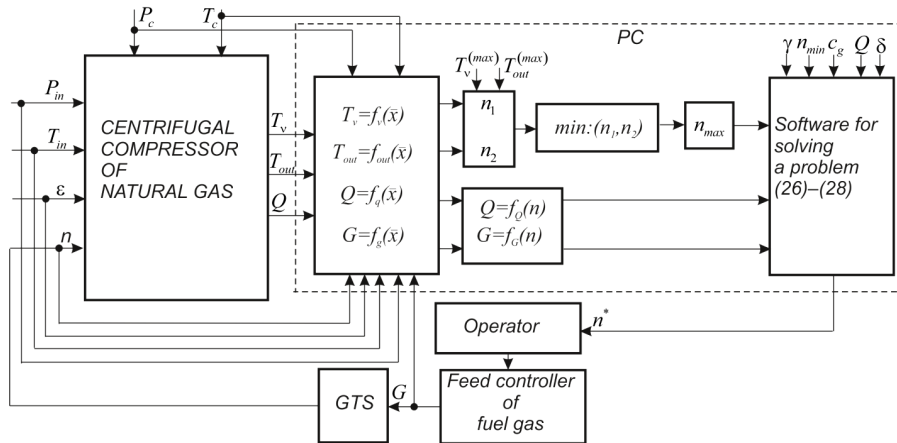


Fig. 3. Block diagram of an automated system of control over the process of natural gas compression

```
% fuel gas consumption
params = [P_in,T_in,e,P_a,T_a,G];
[corel,kG] = fun_GeneticPolyfit(n, params, r);

% Initial data of the problem
% Lower limit for the compressor's rotor frequency, rpm
n_min = 3000;
% Upper limit for the compressor's rotor frequency, rpm
n_max = 5000;
% Upper limit for gas temperature at the outlet from compressor, rpm
T_out_max = 36;
% Upper limit for the exhaust gas temperature
T_v_max = 460;
```

5. Solving a problem of the optimal control over gas pumping units

We shall solve the optimization problem under the following conditions:

- number of units working in a group: 2;
- lower limit for the compressor's rotor rotation frequency $n_1^{(min)}=2,800$ rpm, $n_2^{(min)}=3000$ rpm;
- upper limit for the compressor's rotor rotation frequency $n_1^{(max)}=4,800$ rpm, $n_2^{(max)}=5000$ rpm;
- gas temperature at the outlet from compressor $T_1^{out}=52$ °C, $T_2^{out}=54$ °C;
- temperature of exhaust gases $T_1^v=450$ °C, $T_2^v=460$ °C;
- preset volume of gas pumping - 210 000 n/m³.

In order to solve then optimization problem, we developed software based on the MatLab package. We utilized a built-in function of the nonlinear optimization "fmincon", as an objective function taken (26) at linear constraints (28), at non-linear constraints (27). A code snippet of the software is given below.

```
% Read data from a file of observations
[n,P_in,P_out,T_in,T_out,T_v,P_a,T_a,Q,G] = fun_ReadData();
```

```
% Calculation of the degree of pressure increase e
e = fun_CalculateEpsilon(P_a,P_in,P_out);
r = 3; % degree of a polynomial
```

```
% Calculation of coefficients of the empirical model of temperature at the outlet of compressor
params = [P_in,T_in,e,P_a,T_a,T_out];
[corel,kT_out] = fun_GeneticPolyfit(n, params, r);
```

```
% Calculation of coefficients of the empirical model of temperature of exhaust gases
params = [P_in,T_in,e,P_a,T_a,T_v];
[corel,kT_v] = fun_GeneticPolyfit(n, params, r);
```

```
% Calculation of coefficients of the empirical model of gases pressure drop on the confuser
params = [P_in,T_in,e,P_a,T_a,Q];
[corel,kQ] = fun_GeneticPolyfit(n, params, r);
```

```
% Calculation of coefficients of the empirical model
```

```
kT_v(1) = kT_v(1) - T_v_max;
kT_out(1) = kT_out(1) - T_out_max;
```

```
% Limit for the rotor's frequency rotation at constraint T_out
nT_out = min(roots(kT_out));
% Limit for the rotor's rotation frequency at constraint T_v
nT_v = min(roots(kT_v));
% General limit for the rotor's rotation frequency
n_max = min([n_max nT_out nT_v]);
% Cost of a volumetric unit of fuel gas
c_g = 275;
% Preset volume of gas pumping
Q0 = 104000;

% Gamma-cut value
gamma = 0.8;
% Delta is the measurement accuracy of pressure differential on the confuser
delta = 0.15;
K = (delta / (4 * log(4) ^ 1 / 2)) * log(1 / gamma ^ 2) ^ (1/2);
```

```
options = optimset('Algorithm','interior-point','Display','off');
[res_n,G] = fmincon(@(n)fun_G(n, kG), 1000, [], [], [], [], n_min, n_max, @(n)fun_Qcond(n,kQ,K,Q0), options);
```

By solving the optimization problem, we obtained the following results: $n_1=3,200$ rpm; $n_2=3,550$ rpm.

6. Discussion of results of examining the optimization of the process of natural gas compression

Existing methods for solving optimization problems on the control over the process of natural gas compression employ deterministic models. However, the inaccuracy of measuring technological parameters allows us to consider performance of the compressor as a fuzzy magnitude with a triangular shape of the membership that makes it possible to build an adequate mathematical model of the process of natural gas compression.

A triangular shape of the membership function is inconvenient when used to solve the problem on the optimal

control over the process of natural gas compression. That is why, in the present work, we proposed approximating it with a Gaussian membership function. This allowed us to state the optimization problem in terms of fuzzy magnitudes and, based on it, to develop effective methods for its solution with consideration of constraints for controlling actions.

An introduction of fuzziness to the optimization problem changed a structure of the constraint, which determines a balance of gas flows through parallel-connected compressors. As a result, there appears an additive component, which is a kind of “payment” for the fuzziness in measuring productivity of compressors. The presence of such a component transforms a determined problem of discrete integral linear programming into a nonlinear discrete programming problem, which significantly complicates the process of solving such an optimization problem. It is natural that the introduction of fuzziness to the optimizing problem somewhat impairs efficiency of the process of control and such a deterioration can vary within 3–5 % compared with a problem in the determined statement, which does not take into account the specificity of measuring compressors’ productivity given the existing methods and technical means. Since technological parameters of the

gas pumping unit change over time, there is a need to recalculate the model’s parameters in order to solve the optimization problem on the process of natural gas compression. Determination of frequency of recalculation of the model’s parameters is an unsolved scientific problem at present and it is a promising task for further research.

7. Conclusions

Performance efficiency of centrifugal compressors are considered as fuzzy magnitudes with triangular membership functions, approximated by exponential functions, with their parameters calculated, which allowed us to formalize the problem on the optimal control over the process of natural gas compression taking into account the fuzziness in the productivity of compressors and constraints for controlling actions. We developed algorithmic provision and software in the MATLAB environment and solved the problem on the optimal control, which made it possible to determine rotation frequency of the rotors of centrifugal compressors of natural gas, at which total costs of the fuel gas for the natural gas compression are minimal.

References

1. Hovdiak, P. M. Stan i shliakhy pidvyshchennia efektyvnosti ta nadiynosti transportuvannia pryrodnoho hazu hazotransportnoiu systemoiu Ukrainy. Vol. 2 [Text] / P. M. Hovdiak, I. M. Kalapun, L. B. Chabanovych et. al. // Materialy mizhnarodnoi konferentsyi «Nafta i haz Ukrainy-2002». – Kyiv: UNHA, 2002. – P. 95–96.
2. Transportuvannia pryrodnoho hazu [Electronic resource]. – Available at: <http://www.naftogaz.com/www/3/nakweb.nsf?Open>
3. Priluckiy, M. H. Optimizacionnye zadachi operativnogo upravleniya raboty kompressoroy stanciey [Text] / M. H. Priluckiy, I. R. Buhvalova, L. G. Afraymovich, N. V. Starostin, A. V. Filimonov // Ehlektronny zhurnal «Issledovano v Rossii». – 2008.
4. Han, G. The optimization approach of casing gas assisted rod pumping system [Text] / G. Han, H. Zhang, K. Ling // Journal of Natural Gas Science and Engineering. – 2016. – Vol. 32. – P. 205–210. doi: 10.1016/j.jngse.2016.03.078
5. Mokhatab, S. Natural Gas Compression [Text] / S. Mokhatab, W. A. Poe, J. Y. Mak // Handbook of Natural Gas Transmission and Processing. – 2015. – P. 349–381. doi 10.1016/b978-0-12-801499-8.00011-0
6. Bekker, M. V. Vyznachennia optymalnogo rezhymu roboty kompresornoho tsekhu pry paralelnomu vkluchenni HPA [Text] / M. V. Bekker, V. V. Hulichev, V. I. Meleshko, A. O. Strilets, D. V. Artemenko // Naftova i hazova promyslovist. – 2005. – Issue 2. – P. 45–48.
7. Slobodchikov, K. Yu. Reshenie optimizacionnoy zadachi v sisteme upravleniya rezhimom kompressorного cekha gazoperekachivayushchih agregatov [Electronic resource] / K. Yu. Slobodchikov // Available at: http://model.exponenta.ru/slob_02.html
8. Vertepov, A. G. Algoritmicheskoe obespechenie optimal’noy zagruzki GPA [Text] / A. G. Vertepov, S. P. Zarickiy, I. Ch. Leshchenko, I. K. Lineckiy // Gazovaya promyshlennost’. – 2009. – Issue 4. – P. 57–60.
9. Samsonov, R. O. Upravlenie geoekologicheskimi riskami: optimizaciya raboty KS na magistral’nyh gazoprovodah [Electronic resource] / R. O. Samsonov // Ehlektronnyi nauchnyy zhurnal «Neftegazovoe delo». – 2007. – Available at: http://www.ogbus.ru/authors/SamsonovRO/SamsonovRO_2.pdf
10. Horbiychuk, M. I. Optymalne keruvannia robotoiu nahnitachiv pryrodnoho hazu iz vrakhuvanniam tekhnichnoho stanu HPA [Text] / M. I. Horbiychuk, Ya. I. Zaiachuk // Eastern-European Journal of Enterprise Technologies. – 2008. – Vol. 6, Issue 5 (30). – P. 7–11.
11. Horbiychuk, M. I. Metod syntezu empiyrychnykh modelei na zasadakh henetychnykh alhorytmiv [Text] / M. I. Horbiychuk, M. I. Kohutiak, O. B. Vasylenko, I. V. Shchupak // Rozvidka ta rozrobka naftovykh i hazovykh rodovyshch. – 2009. – Issue 4 (33). – P. 72–79.
12. Horbiychuk, M. I. Induktyvnyi metod pobudovy matematychnykh modelei hazoperekachuvalnykh ahreativ pryrodnoho hazu [Text] / M. I. Horbiychuk, M. I. Kohutiak, Ya. I. Zaiachuk // Naftova i hazova promyslovist. – 2008. – Issue 5. – P. 32–35.
13. Kompresorni stantsiyi. Kontrol teplotekhnichnykh ta ekolohichnykh kharakterystyk hazoperekachuvalnykh ahreativ: SOU 60.03-30019801-011:2004 [Text]. – Kyiv: DK «Ukrtranzgaz», 2004. – 117 p.
14. Ilchenko, B. S. Diahnostuvannia funktsionalno-tekhnichnoho stanu hazo-perekachuvalnykh arehativ [Text]: monohrafiya / B. S. Ilchenko. – Kharkiv: KhNAMH, 2011. – 228 p.
15. Ivahnenko, A. G. Spravochnik po tipovym programmam modelirovaniya [Text] / A. G. Ivahnenko, Yu. V. Koppa, V. S. Stepashko et. al.; A. G. Ivahnenko (Ed.). – Kyiv: Tekhnika, 1980. – 180 p.
16. Raskin, L. G. Nechetkaya matematika. Osnovy teorii. Prilozheniya [Text] / L. G. Raskin, O. V. Seraya. – Kharkiv: Parus, 2008. – 352 p.
17. Dyubua, D. Teoriya vozmozhnostey. Prilozhenie k predstavleniyu znaniy v informatike [Text] / D. Dyubua, A. Prad. – Moscow: Radio i svyaz’, 1990. – 286 p.