Розглядається динамічна система управління технологічними інноваціями. Її динаміка описується векторним лінійним дискретним рекурентним співвідношенням і схильна до впливу управлінських параметрів (управлінь) і неконтрольованого параметра (вектора ризиків або перешкоди). При цьому під ризиками в системі управління технологічними інноваціями будемо розуміти фактори, які впливають негативно або катастрофічно на результати розглянутих в ній процесів.

Для вирішення проблеми управління технологічними інноваціями запропоновано методи, засновані на побудові прогнозних множин – областей досяжності розглянутої динамічної моделі. Вони представляють собою множини всіх допустимих станів фазового вектора системи на заданий момент часу, що відповідають фіксованому програмному управлінню і всім допустимим векторам ризиків. Ця процедура супроводжена методом на основі мінімакса для знаходження гарантованого результату. Суть його полягає в тому, що значення найгіршого (максимального) вектора можливих ризиків є найменшим у порівнянні з аналогічними значеннями для інших при мінімально гарантованому оптимальному управлінні. Таким чином, мінімізуємо вплив ризиків у проблемі управління технологічними інноваціями, де ризики є неконтрольованими параметрами. Це здійснюється на основі вибору такого оптимального управління, яке б гарантувало отриманий результат під впливом будь-якого максимального ризику з набору допустимих.

Пропонований метод дає можливість розробляти ефективні чисельні процедури, що дозволяють реалізувати комп'ютерне моделювання динаміки розглянутої задачі, сформувати програмне мінімаксне управління технологічними інноваціями та отримати оптимальний гарантований результат.

Представлені результати можуть бути використані для економіко-математичного моделювання та вирішення інших завдань оптимізації процесів прогнозування даних і управління в умовах дефіциту інформації і наявності ризиків. Крім того, розроблений інструментарій моделювання може служити основою для розробки відповідних програмно-технічних комплексів для підтримки прийняття ефективних управлінських рішень в інноваційній діяльності

Ключові слова: програмне управління, технологічні інновації, динамічна система, мінімаксний результат, область досяжності

ASPECTS OF PROGRAM CONTROL OVER TECHNOLOGICAL INNOVATIONS WITH CONSIDERATION OF RISKS

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1. Introduction

Making decisions related to a technological innovation (TI) under conditions of uncertainty and risk that exists as a result of its implementation, necessitates solving the best choice problem under conditions of incomplete information about the examined system. In this case, basic existing approaches to solving similar tasks are based mainly on static models and the application of apparatus of stochastic modeling. To apply such mathematical toolset, it is necessary to know probabilistic characteristics of the main parameters of a model and special conditions for realization of an appro-

priate innovation process. In addition, the use of stochastic apparatus of modeling requires special conditions (for example, mass and the homogeneity of the values sample), which is usually difficult to execute under actual production conditions. Moreover, it is necessary to take into consideration the specificity of production innovation activity, where, specifically, risks can be independent on a decision maker and are uncontrollable parameters.

Modern production systems are known to be complex multifactor and multistage control objects. They may also be influenced by various kinds of risks. The requirement for effective control in such systems leads to the necessity of control selection within the influence of uncontrollable parameters (risks). Hence arise problem situations when solving optimization problems in complex control systems with consideration of risks.

The control system of the TI management leads to the need for further improvement of the concept of its optimization. The solution to this problem is associated with a decision-making problem within program control, in which possibilities of selection for a whole studied period of time should be taken into consideration. That is why there additionally arises the problem of processing large arrays of information when solving it. This fact creates considerable difficulties in automation of TI control in production systems. Thus, program control of technological innovations with respect to risks is a complex and relevant task of research.

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2. Literature review and problem statement

Let us consider the modern approaches to modeling socio-economic systems, in particularly those relating to the control of technological innovations (IT).

Certain practical problems were solved within the framework of static optimization, particularly, the apparatus of mathematical programming. However, one-stage selection of the optimal solution is characteristic both for this theory, and for the main range of its supplements, for example, distribution of the program of manufacturing a desired product between some production sections [1], the choice of an optimum variant of technology [2], calculation of the various networks [3], as well as the choice of prediction strategy in health care [4], planning of location of industrial enterprises, etc. [5]. We will note that these models and problems are static problems of conditional optimization. But, static models do not take into consideration the time factor that changes the basic characteristics of the studied object. However, as soon as the problem about the development of a system arises (not only in time, but also in space), and about control of a dynamic system, its one-stage description (modeling) becomes little suitable for solving the most actual practical problems of economy. In these problems, decision should be made for a certain number of steps ahead of time. Therefore, an optimization problem becomes multi-stage, i.e. dynamic. We will note that the problems of this kind include, for example, the problems of prospective and operative planning, as well as control of the technological TI, drafting the development programs for various systems, calculations of multi-stage technological complexes, etc. [6, 7].

It should be noted that any multi-stage problem can be also viewed as static, and to solve it, it is possible to use, for example, the method of mathematical programming. However, direct application of these methods for solution of multi-stage problems usually does not lead to the set goal. It is due to the fact that in this case, the problems of linear or nonlinear programming often have such large dimensionality, that they are very difficult to solve, even with the help of modern computing technology. That is why in this case it is necessary to develop special methods that take into consideration the dynamic nature of these problems and their specificity, for example, economic content, as well as possibilities of effective computer implementation of both modeling of processes and actual control.

It should be noted that whereas finding the optimal solution is most important for a one-stage problem, for a mutli-stage program, along with determining the program of the optimal development of a system, its practical implementation, i. e. the control process, is not less important. Thus, solution of multi-stage optimization problems must be based on the principles of both mathematical programming (on condition of the possibility of using), and on the theory of optimal control. Let us consider modeling of dynamics of the economic system based on differential equations [8]. It is possible to note that this approach is based on a rather approximate description of actual economic phenomena and does not offer any benefits compared to using discrete multi-stage equations.

Thus, equations of dynamics of an economic system from the very beginning are often formalized in a discrete multi-stage form [9]. In such models, the final series of discrete numeric values with assigned initial moment t_0, the interval between any adjacent moments, equal to τ, and final time T are often accepted as numerical parameters. To describe such a system (object), discrete multi-stage (recurrent) equations are used. The so-called terminal functional $F: \mathbf{R}^n \rightarrow \mathbf{R}^1$ is considered as the quality (optimization) criterion in problems of multi-stage optimization. Regarding the peculiarities of modeling control of the TI of enterprises, one of the most complex and important stages of development of a model is the selection of an optimization criterion (objective function) of the TI control. This determines not only economic efficiency of the modeled process of TI control, but also the methods of solution.

To study control optimization problems in multi-stage systems, it is possible to distinguish the basic ones among various approaches. The first of them is the method of dynamic programming, which is based on the principle of optimality of R. Bellman and leads to the need to solve functional equation of a special kind, the other is the variation approach.

The method of dynamic programming makes it possible to find extremum of a functional from many unknown functions by replacing the original problem with the sequence of simpler problems [10]. Thus, the method of dynamic programming makes it possible to reduce the whole n-dimensional problem of finding optimum control to the sequence of N one-dimensional problems, which substantially facilitates the solution of the researched problem. However, there is always an open problem whether it is possible to limit to the given number of iterations during the solution of the studied problem. In other words, a thorough content control of the results is needed. Otherwise, it is easy to obtain solutions that are far from optimal. A substantial drawback of the method is the need to memorize a large volume of information at every stage, which even got the name "dimensionality curse". This creates significant difficulties during realization of problems of great dimensionality on computational equipment under actual production conditions.

Another one, less used approach, is the variation approach [11]. It is based on spreading ideas and methods of mathematical programming for multi-stage problems and joins the apparatus of the maximum principle of L. S. Pontriagin, developed for solution of problems of optimum control in differential systems (with continuous time). This approach is commonly referred to as "discrete maximum principle". The maximum principle spreads variation methods for optimal problem for systems that are described by ordinary differential equations with arbitrary restrictions on control influence, as well as with some kinds of restrictions on variables of the process [12]. However, since the maximum principle determines only a necessary optimality condition, it does not follow that it is optimal from fact that a certain trajectory satisfies it. That is, the maximum principle gives a trajectory only a "suspicion" that it is optimal, an additional verification is necessary to determine the optimal trajectory from their number.

Paper [13] contains certain results regarding development of methods of variation problems in the field of optimal control, specifically, the Lavrentiev problems, as well as matrix properties of approximated solutions to variation problems. But when it comes to the issue of the practical use of classical variation methods in control problems, its solution is complicated by two factors. Firstly, by existence of restrictions of a type of inequalities (especially in cases, where the restricted value is at the admissible boundary). Secondly, by non-continuity of functions, which describe control influences (often, piecewise-continuous functions with a finite number of discontinuities of the first kind).

There are many known generalizations of the maximum principle by L. S. Pontriagin. Recently, the maximum principle has spread to discrete systems and the systems with distributed parameters [1]. However, this method is based on differential models for continuous processes, and in the problems of the TI control, the processes are essentially discrete. Thus, it is much better to use discrete economic-mathematical models at once. Moreover, during implementation by means of computer systems, continuous models must be still discretized anyway. In this case, discrete models, rather than continuous are used. Discrete economic-mathematical models and the methods of control are becoming increasingly important in the theory and practice of optimization of the control of different economic systems and processes. This is due to the fact that in the economy, many phenomena are discrete in nature, because in practice, the information about the state of a system and the processes, studied in it, as well as control, are implemented at discrete moments of time, i.e. by stages.

Let us consider other approaches to modeling innovative development of enterprises. Thus, article [13] proposed the models of formation and control of innovative systems for modernization of agricultural production. In this case, one uses the methods of the theory of one-criterion optimization on hypergraphs using a toolkit of deterministic and probabilistic automata of Moor, imitative probabilistic-automatic modeling and stochastic Petri networks. However, preliminary analysis of the exogenous factors proved the hypothesis about determinateness of the studied TI model [14]. That is why the solution to the problem of the program TI control at an enterprise requires an application of another mathematical toolkit. In addition, it is advisable to use the models of multicriterial optimization for a comprehensive study of effectiveness of TI control at an enterprise.

As it was noted, optimization methods are successfully used in the operation of various enterprises. Two different prediction strategies in health care were proposed in paper [14]. The proposed damage is estimated by using the algorithm of calculating net cash flow, which makes it possible to estimate fatigue of components. The general control purpose is changed by adding an additional criterion that takes into consideration the accumulated damage. The fact is that although the one-stage approach is improved with the help of the multilevel control scheme, where the solution of the problem of dynamic optimization is obtained from the model in two different temporal scales. However, the drawback of the proposed method of modeling is seen in the fact that it does not take into consideration the risks, associated with both strategies of control in order to achieve the desired compromise between minimum accumulated loss and operational costs.

Recent publications include scientific research, based on network methods of modeling. Specifically, modeling of neural networks and the concept of feedback were used to develop the models for prediction of operation efficiency of an enterprise and the search for innovative control strategies [15]. Even though the process is successfully modeled using artificial neural network, the problem is that it is not always possible to obtain sufficient data for modeling, based on the proposed methods under actual operation conditions of an enterprise.

Realization of the problems, associated with modeling TI control in practice, contributed to scientific development and wide application of hybrid models based on a combination of formal and informal texts, substantive and special graphic approaches. These tools offer simplified innovative models and, in many cases, include a graphic interpretation [16]. As a result, most models of TI control were implemented using descriptive tools or in a fragmentary form with insufficient analytical formalization [17]. Some of them are characterized by the lack of practical direction, integrity, and comprehensiveness in applying a specific innovation process [18]. In this case, there is no possibility to assess the impact and consequences of the various options for TI control in prospect with minimization of risks of innovative activity when making control decisions [19]. Thus, innovative modeling-based program control remains an unresolved challenge for researchers. In addition, existing models may be limited and difficult to adapt to conditions of activity of specific enterprises.

Thus, one might conclude that when resolving the problems of TI control, it is necessary to use special methods that take into consideration the dynamic nature of the researched problems and the specifics – taking into consideration the risk factor, variation of technologies and others, as well as possibilities of effective computer implementation of both processes of modeling and actual control. The problem of modeling TI control was most fundamentally studied in papers [20, 21], which deal with the common problems of multilevel and adaptive control of innovative processes at an enterprise. However, many problems, associated with formalization of the evidentiary part of the TI control process, remain unresolved.

3. The aim and objectives of the study

The aim of this research is to optimize the program control of TI with consideration of risks. It implies the devel-

opment of the method that makes it possible to proceed from a complex multi-stage problem of dynamic program control optimization with consideration of risks to implementation of a finite sequence of one-stage problems of discrete optimization.

To accomplish the set goal, the following tasks had to be solved:

– to formalize the problem of minimax program control for a dynamic system that describes an innovative technological process in the presence of risks;

– to develop a general scheme of solving a problem of minimax program control with consideration of risks;

– to develop an algorithm of solving the optimization problem of TI control in the presence of risks.

4. Materials and methods to study the problem of program control of TI in the presence of risks

Optimization of the program control over TI is implemented in the framework of solving a nonlinear multi-stage problem of minimax program terminal control for a dynamic system of risk [22].

For the problem on program control over TI, the methods were proposed, based on the construction of prediction sets (reachability regions) of the studied dynamic model. The sets of all permissible states of the phase vector of the system at a given moment of time, correspondent to fixed program control and all permissible vectors of risk, are presented.

The procedure of TI control is accompanied by the optimization process, which makes it possible to separate a result that is guaranteed, the best or preferred in some ways. Therefore, taking into consideration the specificity of the innovative production, where, in particular, risks are uncontrollable parameters, to solve the problem of TI control, we propose the application of the minimax approach or finding a guaranteed result: on the minimal guaranteed optimum control, the value of the worst (maximum) vector of dissimilar risks is the least in comparison with similar values for the others. Thus, the impact of risks in the problem of TI control, where risks are uncontrollable parameters, is minimized by selection of such optimum control, which would guarantee the found result when exposed to any maximum risks from the set of permissible risks.

4. 1. Statement of the problem on the minimax program terminal control to manage innovative technological process

Let us accept all definitions, introduced in paper [22], without changes and additional explanations.

The problem of minimax program control with consideration of risks. For fixed intervals of time $\overline{\tau, \vartheta} \subseteq \overline{0, T}$ $(\tau < \vartheta)$ and implementation of τ-position

$$g(\tau) = \left\{ \tau, \overline{x}(\tau) \right\} \in \hat{G}(\tau) \quad (g(0) = g_0)$$

of actor P in the dynamic system [22], which describes TI control, it is necessary to find set

$$U_F^{(e)}(\overline{\tau, T}, g(\tau)) \subseteq U(\overline{\tau, T})$$

of minimax program controls $\overline{u}^{(e)}(\cdot) \in U(\overline{\tau, T})$ of actor P, which is determined by ratio:

$$\mathbf{U}_F^{(e)}(\overline{\tau, T}, g(\tau)) = \{\overline{u}^{(e)}(\cdot) : \overline{u}^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau, T}),$$

$$\mathbf{F}_{\tau, T}^{(e)} = \max_{\overline{v}(\cdot) \in V(\overline{\tau, T}; \overline{u}^{(e)}(\cdot))} \mathbf{F}_{\overline{\tau, T}}(g(\tau), \overline{u}^{(e)}(\cdot), \overline{w}^{(e)}(\cdot), \overline{v}(\cdot)) =$$

$$= \min_{=\overline{w}(\cdot) \in W(\overline{\tau, T}; \overline{u}^{(e)}(\cdot))} \max_{\overline{v}(\cdot) \in V(\overline{\tau, T}; \overline{u}^{(e)}(\cdot))} \mathbf{F}_{\overline{\tau, T}}(g(\tau), \overline{u}^{(e)}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot)) =$$

$$= \min_{=\overline{u}(\cdot) \in U(\overline{\tau, T})} \min_{\overline{w}(\cdot) \in W(\overline{\tau, T}; \overline{u}(\cdot))} \max_{\overline{v}(\cdot) \in V(\overline{\tau, T}; u(\cdot))} \mathbf{F}_{\overline{\tau, T}}(g(\tau), \overline{u}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot)) =$$

$$= \min_{\overline{u}(\cdot) \in U(\overline{\tau, T})} \min_{\overline{w}(\cdot) \in W(\overline{\tau, T}; \overline{u}(\cdot))} \max_{\overline{v}(\cdot) \in V(\overline{\tau, T}; \overline{u}(\cdot))} \tilde{\mathbf{F}}(\overline{x}_{\overline{\tau, T}}(g(\tau), \overline{u}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot))) =$$

$$= \min_{\overline{w}(\cdot) \in W(\overline{\tau, T} \overline{u}^{(e)}(\cdot))} \max_{\overline{v}(\cdot) \in V(\overline{\tau, T}; \overline{u}^{(e)}(\cdot))} \tilde{\mathbf{F}}(\overline{x}_{\overline{\tau, T}}(g(\tau), \overline{u}^{(e)}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot))) =$$

$$= \max_{\overline{v}(\cdot) \in V(\overline{\tau, T}; \overline{u}^{(e)}(\cdot))} \tilde{\mathbf{F}}(\overline{x}_{\overline{\tau, T}}(g(\tau), \overline{u}^{(e)}(\cdot), \overline{w}^{(e)}(\cdot), \overline{v}(\cdot))) = c_F^{(e)}(\overline{\tau, T}, g(\tau))\}, \quad (1)$$

as implementation of a finite sequence of only one-stage operations.

Here, functional $\mathbf{F}_{\overline{\tau, T}}$ is determined by ratio:

$$\mathbf{F}_{\overline{\tau, T}}(g(\tau), \overline{u}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot)) =$$

$$= \sum_{k=1}^{r} \mu_k \cdot \Phi_{\overline{\tau, T}}^{(k)}(g(\tau), \overline{u}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot)) =$$

$$= \sum_{k=1}^{r} \mu_k \cdot F_{\overline{\tau, T}}^{(k)}(\overline{x}_{\overline{\tau, T}}(T; \overline{x}(\tau), \overline{u}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot)) =$$

$$= \sum_{k=1}^{r} \mu_k \cdot F_{\overline{\tau, T}}^{(k)}(\overline{x}(T)) = \tilde{\mathbf{F}}(\overline{x}(T)),$$

$$\forall k \in \overline{1, r} : \mu_k \geq 0, \sum_{k=1}^{r} \mu_k = 1, \quad (2)$$

where

$$\overline{x}(T) = \overline{x}_{\overline{\tau, T}}(T; \overline{x}(\tau), \overline{u}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot)),$$

and $\tilde{\mathbf{F}}$ is the convex functional, introduced in [22].

Let us remind that to assess the quality of TI control in time interval $\overline{\tau, T} \subseteq \overline{0, T}$, the vector terminal functional (quality indicator of the control process) was introduced

$$\mathbf{\Phi}_{\overline{\tau, T}} = (\Phi_{\overline{\tau, T}}^{(1)}, \Phi_{\overline{\tau, T}}^{(2)}, \dots, \Phi_{\overline{\tau, T}}^{(r)}),$$

a set of r convex functionals $\Phi_{\overline{\tau, T}}^{(k)}$:

$$\hat{\mathbf{G}}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times \mathbf{W}(\overline{\tau, T}; \overline{u}(\cdot)) \times \mathbf{V}(\overline{\tau, T}; \overline{u}(\cdot)) \rightarrow \mathbf{R}^1 (k \in \overline{1, r}),$$

so that to implement the set

$$(g(\tau), \overline{u}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot)) \in \hat{\mathbf{G}}(\tau) \times \mathbf{U}(\overline{\tau, T}) \times$$
$$\times \mathbf{W}(\overline{\tau, T}; \overline{u}(\cdot)) \times \mathbf{V}(\overline{\tau, T}; \overline{u}(\cdot)),$$

where $g(\tau) = \{\tau, \overline{x}(\tau)\} \in \hat{G}(\tau)$, the values are determined by the following ratios:

$$\Phi_{\overline{\tau, T}}^{(k)}(g(\tau), \overline{u}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot)) =$$

$$= F_{\overline{\tau, T}}^{(k)}(\overline{x}_{\overline{\tau, T}}(T; \overline{x}(\tau), \overline{u}(\cdot), \overline{w}(\cdot), \overline{v}(\cdot))) =$$

$$= F_{\overline{\tau, T}}^{(k)}(\overline{x}(T)),$$

$$k \in \overline{1, r}, \quad (3)$$

where

$$F_{\overline{\tau, T}}^{(k)} : \mathbf{R}^{\overline{n}} \rightarrow \mathbf{R}^1$$

is the convex functional for each $k \in \overline{1, r}$;

$$\overline{x}(T) = \overline{x}_{\tau,\overline{T}}(T;\overline{x}(\tau),\overline{u}(\cdot),\overline{w}(\cdot),\overline{v}(\cdot)).$$

Number $c_F^{(e)}(\overline{\tau,T},g(\tau)) = \mathbf{F}_{\tau,\overline{T}}^{(e)}$, will be called guaranteed (minimax) result of the process of minimax program terminal control for actor P in time interval $\overline{\tau,T}$ for a discrete dynamic system [22], regarding its τ-position $g(\tau)$ and functional $\boldsymbol{F}_{\overline{\tau,T}}$.

We will note that it is necessary to take into consideration the finiteness of the sets of permissible programming controls $\mathbf{U}(\overline{\tau,T})$ and permissible programming intensities of supplementing production and investment resources $\mathbf{W}(\overline{\tau,T};\overline{u}(\cdot))$, which correspond to the fixed program TI control $\overline{u}(\cdot) \in \mathbf{U}(\overline{\tau,T})$. Then taking into consideration ratios (1)–(3), it is possible to show that there is a solution of the problem of minimax program control with consideration of risks. It is reduced to a finite number of problems of linear and convex mathematical programming, as well as to a finite number of discrete optimization problems [20].

Let us consider schematic implementation of the explored nonlinear multi-stage problem of minimax program terminal control for a dynamic system [22], which describes the process of TI managing in the presence of risks (Fig. 1).
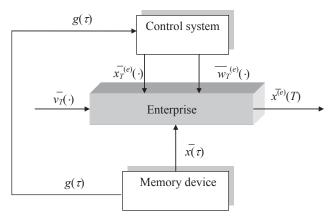

Fig. 1. Block diagram of minimax program terminal control of a technological innovation

In Fig. 1, τ-position of actor P $g(\tau) = \{\tau, \overline{x}(\tau)\} \in \hat{G}(\tau)$, $(\tau \in \overline{0,T-1})$; vector of TI control

$$\overline{u}_T^{(e)}(\cdot) = \{u(t)\}_{t \in \overline{\tau,T-1}} \in U_F^{(e)}(\overline{\tau,T},g(\tau)) = \tilde{U}_F^{(e)}(\overline{\tau,T},g(\tau));$$

vector of intensity of supplementing production and investment resources

$$\overline{w}_T^{(e)}(\cdot) \in \boldsymbol{W}(\overline{\tau,T};\overline{u}_T^{(e)}(\cdot));$$

vector of permissible risks

$$\overline{v}_T(\cdot) \in \boldsymbol{V}(\overline{\tau,T};\overline{u}_T^{(e)}(\cdot));$$

phase vector

$$\overline{x}^{(e)}(T) = \overline{x}_{\overline{\tau,T}}(T;\overline{x}(\tau),\overline{u}_T(\cdot),\overline{w}_T(\cdot),\overline{v}_T(\cdot))$$

and inequality is true:

$$F_{\overline{\tau,T}}(g(\tau),\overline{u}_T^{(e)}(\cdot),\overline{w}_T^{(e)}(\cdot),\overline{v}_T(\cdot)) \leq F_{\overline{\tau,T}}(g(\tau),\overline{u}_T^{(e)}(\cdot),\overline{w}_T^{(e)}(\cdot),\overline{v}_T^{(e)}(\cdot)) = F_{\overline{\tau,T}}^{(e)}.$$

Satisfaction of this inequality means that during realization of any risk vector $\overline{v}_T(\cdot) \in V(\overline{\tau,T};\overline{u}_T^{(e)}(\cdot))$, a decision maker obtains a guaranteed result not less than $F_{\overline{\tau,T}}^{(e)}$. It is the value of guaranteed (minimax) program TI control, which is achieved in the case if the risk vector is the worst for a decision maker.

$$\overline{v}_T^{(e)}(\cdot) \in V(\overline{\tau,T};\overline{u}_T^{(e)}(\cdot)).$$

4. 2. General scheme for solving the problem on the minimax program control with consideration of risks

For any fixed time intervals $\overline{\tau,\vartheta} \subseteq \overline{0,T}$ ($\tau < \vartheta$) and set

$$(\overline{X}(\tau),\overline{u}_{\overline{\tau,\vartheta}}(\cdot),\overline{w}_{\overline{\tau,\vartheta}}(\cdot)) \in 2^{\mathbf{R}^{\overline{n}}} \times \mathbf{U}(\overline{\tau,\vartheta}) \times \mathbf{W}(\overline{\tau,\vartheta};\overline{u}_{\overline{\tau,\vartheta}}(\cdot)),$$

where

$$\overline{X}(\tau) \subset \mathbf{R}^{\overline{n}} \cdot (\overline{X}(0) = \{\overline{x}_0, \mathrm{I}_0\})$$

is a convex closed and limited polyhedron (with a finite number of vertices) in space $\mathbf{R}^{\overline{n}}$; $2^{\mathbf{R}^{\overline{n}}}$ is the set of all subsets of space $\mathbf{R}^{\overline{n}}$; $\overline{u}_{\overline{\tau,\vartheta}}(\cdot) \in \mathbf{U}(\overline{\tau,\vartheta})$ is the permissible program control of TI in the time interval $\overline{\tau,\vartheta}$;

$$\overline{w}_{\overline{\tau,\vartheta}}(\cdot) \in \mathbf{W}(\overline{\tau,\vartheta};\overline{u}_{\overline{\tau,\vartheta}}(\cdot))$$

is the permissible program intensity of supplementing production and investment resources in the time interval $\overline{\tau,\vartheta}$. Based on a [22], we introduce the following set:

$$X_{(\overline{u}_{\overline{\tau,\vartheta}}(\cdot),\overline{w}_{\overline{\tau,\vartheta}}(\cdot))}^{(+)}(\tau,\overline{X}(\tau),\vartheta,V(\overline{\tau,\vartheta};\overline{u}_{\overline{\tau,\vartheta}}(\cdot))) = \{\overline{x}(\vartheta): \overline{x}(\vartheta) \in R^{\overline{n}},$$

$$\overline{x}(t+1) = \overline{A}(t)\overline{x}(t) + \overline{B}(t)\overline{u}(t) + \overline{C}(t)\overline{w}(t) + \overline{D}(t)\overline{v}(t) \in \mathbf{X}_1(t),$$

$$(\overline{x}(0) = \{\overline{x}_0, \mathrm{I}_0\}, \overline{u}_{\overline{\tau,\vartheta}}(\cdot) = \{\overline{u}(t)\}_{t \in \overline{\tau,\vartheta-1}}, \overline{w}_{\overline{\tau,\vartheta}}(\cdot) = \{\overline{w}(t)\}_{t \in \overline{\tau,\vartheta-1}}\}, \quad (4)$$

which will be called direct reachability region of forecasting set [9, 21] of phase states of system [22] at moment ϑ, which corresponds to the set of

$$(\overline{X}(\tau),\overline{u}_{\overline{\tau,\vartheta}}(\cdot),\overline{w}_{\overline{\tau,\vartheta}}(\cdot)) \in 2^{\mathbf{R}^{\overline{n}}} \times U(\overline{\tau,\vartheta}) \times W(\overline{\tau,\vartheta};\overline{u}_{\overline{\tau,\vartheta}}(\cdot)).$$

Taking into consideration the linearity of the recurrent dynamic system [22] and the introduced condition of $V_1(\overline{u}(t))$, which for each $\overline{u}(t) \in U_1(t)$ is convex, closed and limited polyhedron in space $\mathbf{R}^{\overline{q}}$, similarly to [20], it is possible to show that for the fixed set of

$$(\overline{X}(\tau),\overline{u}_{\overline{\tau,\vartheta}}(\cdot),\overline{w}_{\overline{\tau,\vartheta}}(\cdot)) \in 2^{\mathbf{R}^{\overline{n}}} \times U(\overline{\tau,\vartheta}) \times W(\overline{\tau,\vartheta};\overline{u}_{\overline{\tau,\vartheta}}(\cdot)).$$

The following properties of the introduced set [22] are true:

$- X_{(\overline{u}_{\overline{\tau,\vartheta}}(\cdot),\overline{w}_{\overline{\tau,\vartheta}}(\cdot))}^{(+)}(\tau,\overline{X}(\tau),\vartheta,V(\overline{\tau,\vartheta};\overline{u}_{\overline{\tau,\vartheta}}(\cdot))) = X_{(\overline{u}_{\overline{\tau,\vartheta}}(\cdot),\overline{w}_{\overline{\tau,\vartheta}}(\cdot))}^{(+)}(t)$ for all $t \in \overline{\tau+1,\vartheta}$ is a non-empty, convex, closed and limited polyhedron (with a limited number of vertices) in space $\mathbf{R}^{\overline{n}}$;

$-$ for all $t \in \overline{\tau,\vartheta-1}$ and $X_{(\overline{u}_{0,\tau}(\cdot),\overline{w}_{0,\tau}(\cdot))}^{(+)}(\tau) = \overline{X}(\tau)$, the recurrent ratio is true:

$$\mathbf{X}_{(\overline{u}_{\overline{\tau,t+1}}(\cdot),\overline{w}_{\overline{\tau,t+1}}(\cdot))}^{(+)}(\tau,\overline{X}(\tau),t+1,\mathbf{V}(\overline{\tau,t+1};\overline{u}_{\overline{\tau,t+1}}(\cdot))) =$$

$$= \mathbf{X}_{(\overline{u}_{\overline{t,t+1}}(\cdot),\overline{w}_{\overline{t,t+1}}(\cdot))}^{(+)}(t,\overline{X}(t),t+1,\mathbf{V}(\overline{t,t+1};\overline{u}_{\overline{t,t+1}}(\cdot))) =$$

$$= \mathbf{X}_{(\overline{u}_{\overline{t,t+1}}(t),\overline{w}_{\overline{t,t+1}}(t))}^{(+)}(t,\overline{X}(t),t+1,\mathbf{V}_1(t;\overline{u}_{\overline{t,t+1}}(t))), \quad (5)$$

where

$$\mathbf{X}^{(+)}_{(\overline{u}_{\overline{\tau}_t}(\cdot),\overline{w}_{\overline{\tau}_t}(\cdot))}(\tau,\overline{X}(\tau),t,\mathbf{V}(\overline{\tau,t};\overline{u}_{\overline{\tau,t}}(\cdot)))=\overline{X}(t).$$

Then from ratio (5), it follows that a multi-stage problem of construction of the reachability region

$$\boldsymbol{X}^{(+)}_{(\overline{u}_{\overline{\tau,\vartheta}}(\cdot),\overline{w}_{\overline{\tau,\vartheta}}(\cdot))}(\tau,\overline{X}(\tau),\vartheta,V(\overline{\tau,\vartheta};\overline{u}_{\overline{\tau,\vartheta}}(\cdot)))$$

is reduced to solving a finite recurrent sequence of only one-stage problems of construction according to the following reachability regions of:

$$\overline{X}(t+1)=X^{(+)}_{(\overline{u}_{\overline{\tau,t+1}}(\cdot),\overline{w}_{\overline{\tau,t+1}}(\cdot))}(t+1)=$$
$$=\mathbf{X}^{(+)}_{(\overline{u}_{\overline{\tau,t+1}}(\cdot),\overline{w}_{\overline{\tau,t+1}}(\cdot))}(t,\overline{X}(t),t+1,\mathbf{V}(\overline{t,t+1};\overline{u}_{\overline{t,t+1}}(\cdot)))=$$
$$=\mathbf{X}^{(+)}_{(\overline{u}_{\overline{\tau,t+1}}(\cdot),\overline{w}_{\overline{\tau,t+1}}(\cdot))}(t,X^{(+)}_{(\overline{u}_{\overline{\tau,t}}(\cdot),\overline{w}_{\overline{\tau,t}}(\cdot))}(t),t+1,\mathbf{V}(\overline{t,t+1};\overline{u}_{\overline{t,t+1}}(\cdot)))=$$
$$=\mathbf{X}^{(+)}_{(\overline{u}_{\overline{\tau,t+1}}(t),\overline{w}_{\overline{\tau,t+1}}(t))}(t,\overline{X}(t),t+1,\mathbf{V}_1(t;\overline{u}_{\overline{t,t+1}}(t)),$$
$$t\in\overline{\tau,\vartheta-1},\ \overline{X}(\tau)=X^{(+)}_{(\overline{u}_{\overline{0,\tau}}(\cdot),\overline{w}_{\overline{0,\tau}}(\cdot))}(\tau). \tag{6}$$

Based on these properties, the general scheme of solving problem 3.1 for the dynamic system [22] that describes TI control in the presence of risks can be described in the form of realization of the following sequence of actions [20].

4. 3. The algorithm for solving a problem on the optimization of management of technological innovations in the presence of risks
1. Write in order of ascending of natural index *i*, the finite set $U(\overline{\tau,T})$, which consists of N_τ – permissible program controls

$$\overline{u}_T^{(i)}(\cdot)=\{\overline{u}_T^{(i)}(t)\}_{t\in\overline{\tau,T-1}}\in\mathbf{U}(\overline{\tau,T})$$

in the time interval $\overline{\tau,T}\subseteq\overline{0,T}$ ($\tau<T$), that is, we have

$$\mathbf{U}(\overline{\tau,T})=\{\overline{u}_T^{(i)}(\cdot)\}_{i\in\overline{1,N_\tau}}.$$

2. For each permissible control $\overline{u}_T^{(i)}(\cdot)\in\mathbf{U}(\overline{\tau,T})$ ($i\in\overline{1,N_\tau}$), write in order of ascending of natural index *j*, the finite set $\mathbf{W}(\overline{\tau,T};\overline{u}_T^{(i)}(\cdot))$, which consist of $M_\tau(i)$ ($M_\tau(i)\in\mathbf{N},\ i\in\overline{1,N_\tau}$) – permissible program intensities of supplementing production and investment resources

$$\overline{w}_T^{(j)}(\cdot)=\{\overline{w}_T^{(j)}(t)\}_{t\in\overline{\tau,T-1}}\in W(\overline{\tau,T};\overline{u}_T^{(i)}(\cdot))$$

In time interval $\overline{\tau,T}$, that is, we have

$$\mathbf{W}(\overline{\tau,T};\overline{u}_\tau^{(i)}(\cdot))=\{\overline{w}_T^{(j)}(\cdot)\}_{j\in\overline{1,M_\tau(i)}}.$$

3. For the fixed and permissible set

$$(\overline{X}(\tau),\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot)\in 2^{R^{\overline{n}}}\times U(\overline{\tau,\vartheta})\times W(\overline{\tau,T};\overline{u}_T^{(i)}(\cdot))$$

($i\in\overline{1,N_\tau},\ j\in\overline{1,M_\tau(i)}$), due to the above property, the reachability region

$$\mathbf{X}^{(+)}_{(\overline{u}_{\overline{\tau,T}}^{(i)}(\cdot),\overline{w}_{\overline{\tau,T}}^{(j)}(\cdot))}(\tau,\overline{X}(\tau),T,\mathbf{V}(\overline{\tau,T};\overline{u}_{\overline{\tau,T}}^{(i)}(\cdot)))$$

of the studied dynamic system [22] at the final moment of time *T* is a convex, closed and limited polyhedron (with the finite number of vertices) in space $\mathbf{R}^{\overline{n}}$, which is constructed based on recurrent formulas by implementation of construction (T-τ) of one-stage reachability regions:

$$\mathbf{X}^{(+)}_{(\overline{u}_{t+1}^{(i)}(\cdot),\overline{w}_{t+1}^{(j)}(\cdot))}(\tau,\overline{X}(\tau),t+1,\mathbf{V}(\overline{\tau,t+1};\overline{u}_{t+1}^{(i)}(\cdot)))=$$
$$=\mathbf{X}^{(+)}_{(\overline{u}_{t+1}^{(i)}(t),\overline{w}_{t+1}^{(j)}(t))}(t,\overline{X}(t),t+1,\mathbf{V}(\overline{t,t+1};\overline{u}_{t+1}^{(i)}(t)))=$$
$$=\mathbf{X}^{(+)}_{(\overline{u}_{t+1}^{(i)}(t),\overline{w}_{t+1}^{(j)}(t))}(t,\overline{X}(t),t+1,\mathbf{V}_1(t;\overline{u}_{t+1}^{(i)}(t)))$$
$$t\in\overline{\tau,T-1},\overline{X}(\tau)=X^{(+)}_{(\overline{u}_{\overline{0,\tau}}(\cdot),\overline{w}_{\overline{0,\tau}}(\cdot))}(\tau), \tag{7}$$

where

$$\overline{X}(0)=\{\overline{x}_0\};\ \ \overline{X}(t)=\mathbf{X}^{(+)}_{(\overline{u}_t^{(i)}(\cdot),\overline{w}_t^{(j)}(\cdot))}(\tau,\overline{X}(\tau),t,\mathbf{V}(\overline{\tau,t};\overline{u}_t^{(i)}(\cdot))).$$

4. For τ-position

$$g(\tau)=\{\tau,\overline{x}(\tau)\}\in\hat{\mathbf{G}}(\tau)$$

of actor *P*, of the selected fixed and permissible set

$$(\overline{X}(\tau),\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))\in 2^{R^{\overline{n}}}\times\mathbf{U}(\overline{\tau,\vartheta})\times W(\overline{\tau,T};\overline{u}_\tau^{(i)}(\cdot))$$

$$(\overline{X}(0)=\{\overline{x}_0,\ I_0\},\overline{X}(\tau)=\overline{x}(\tau),\ i\in\overline{1,N_\tau},j\in\overline{1,M_\tau}),$$

based on [22] thorough implementation of a finite sequence of problems of linear mathematical programming, we construct the reachability region

$$\mathbf{X}^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))}(\tau,\overline{X}(\tau),T,\mathbf{V}(\overline{\tau,T};\overline{u}_T^{(i)}(\cdot))).$$

Which is a convex, closed and limited polyhedron (with a finite number of vertices) in space $\mathbf{R}^{\overline{n}}$.
5. To construct the reachability region

$$\boldsymbol{X}^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))}(\tau,\overline{X}(\tau),T,V(\overline{\tau,T};\overline{u}_T^{(i)}(\cdot)))$$

we form the correspondent a finite system of linear inequalities-restrictions $\mathbf{L}^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))}(T)$, determined on the elements-vectors of space $\mathbf{R}^{\overline{n}}$, the set of solutions to which will be designated $\mathbf{S}^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))}(T)$. It is possible to prove that the equality is true:

$$\mathbf{S}^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))}(T)=\mathbf{X}^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))}(\tau,\overline{X}(\tau),T,\mathbf{V}(\overline{\tau,T};\overline{u}_T^{(i)}(\cdot))). \tag{8}$$

6. According to the introduced functional (2), for the permissible set

$$(\overline{X}(\tau),\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))\in 2^{\mathbf{R}^{\overline{n}}}\times\mathbf{U}(\overline{\tau,\vartheta})\times\mathbf{W}(\overline{\tau,T};\overline{u}_T^{(i)}(\cdot))$$

$$(\overline{X}(0)=\{\overline{x}_0,\ I_0\},$$

$$\overline{X}(\tau)=\overline{x}(\tau),\ \ i\in\overline{1,N_\tau},\ \ j\in\overline{1,M_\tau}$$

and permissible implementation of vector

$$\overline{v}_T(\cdot)=\{\overline{v}_T(t)\}_{t\in\overline{\tau,T-1}}\in\mathbf{V}(\overline{\tau,T};\overline{u}_T^{(i)}(\cdot))),$$

we consider the functional:

$$\boldsymbol{F}_{\overline{\tau,T}}(g(\tau),\ \overline{u}_T^{(i)}(\cdot),\ \overline{w}_T^{(j)}(\cdot),\ \overline{v}(\cdot))=$$
$$=\sum_{i=1}^{k}\mu_i\cdot\Phi^{(i)}_{\overline{\tau,T}}(g(\tau),\ \overline{u}_T^{(i)}(\cdot),\ \overline{w}_T^{(j)}(\cdot),\ \overline{v}_T(\cdot))=$$
$$=\sum_{k=1}^{r}\mu_k\cdot F^{(k)}_{\overline{\tau,T}}(\overline{x}_{\overline{\tau,T}}(T;\overline{x}(\tau),\overline{u}_T^{(i)}(\cdot),\ \overline{w}_T^{(j)}(\cdot),\overline{v}_T(\cdot))=$$
$$=\sum_{i=1}^{r}\mu_i\cdot F^{(i)}_{\overline{\tau,T}}(\overline{x}(T))=\tilde{\mathbf{F}}(\overline{x}(T)),$$

$$\forall k \in \overline{1,r}: \mu_k \geq 0, \sum_{k=1}^{r} \mu_k = 1,$$

$$\overline{x}(T) \in \mathbf{S}^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))}(T) =$$

$$= \mathbf{X}^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))}(\tau, \overline{X}(\tau), T, \mathbf{V}(\overline{\tau,T}; \overline{u}_T^{(i)}(\cdot))), \qquad (9)$$

where $\overline{x}(T) = \overline{x}_{\overline{\tau,T}}(T; \overline{x}(\tau), \overline{u}_T^{(i)}(\cdot), \overline{w}_T^{(j)}(\cdot), \overline{v}_T(\cdot));$ $\tilde{\mathbf{F}}$ is the convex functional, formed from functional $\Phi = (\Phi_1, \Phi_2, ..., \Phi_r)$ based on (2), (3).

Then form the solution to the problem of convex mathematical programming with the convex terminal functional, which has the form (9), and a finite system of linear restrictions $\mathbf{L}^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(j)}(\cdot))}(T),$ according to (1), (8), (9), we also find the values of this functional:

$$\mathbf{F}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(i)}(\cdot))} = \mathbf{F}_{\overline{\tau,T}}(g(\tau), \overline{u}_T^{(i)}(\cdot), \overline{w}_T^{(j)}(\cdot), \overline{v}_T^{(e)}(\cdot)) =$$

$$= \max_{\overline{v}_T(\cdot) \in V(\overline{\tau,T}; \overline{u}_T^{(i)}(\cdot))} \mathbf{F}_{\overline{\tau,T}}(g(\tau), \overline{u}_T^{(i)}(\cdot), \overline{w}_T^{(j)}(\cdot), \overline{v}_T(\cdot)) =$$

$$= \max_{\overline{v}_T(\cdot) \in V(\overline{\tau,T}; \overline{u}_T^{(i)}(\cdot))} \sum_{k=1}^{r} \mu_k \cdot \Phi_{\overline{\tau,T}}^{(k)}(g(\tau), \overline{u}_T^{(i)}(\cdot), \overline{w}_T^{(j)}(\cdot), \overline{v}_T(\cdot)) =$$

$$= \max_{\overline{x}(T) \in X^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(i)}(\cdot))}(\tau, \overline{X}(\tau), T, V(\overline{\tau,T}\overline{u}_T^{(i)}(\cdot)))} \sum_{k=1}^{r} \mu_k \cdot F_{\overline{\tau,T}}^{(k)}(\overline{x}(T)) =$$

$$= \max_{\overline{x}(T) \in S^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(i)}(\cdot))}(T)} \sum_{k=1}^{r} \mu_k \cdot F_{\overline{\tau,T}}^{(k)}(\overline{x}(T)) =$$

$$= \max_{\overline{x}(T) \in S^{(+)}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(i)}(\cdot))}(T)} \tilde{\mathbf{F}}(\overline{x}(T)), \qquad (10)$$

$$\forall k \in \overline{1,r}: \mu_k \geq 0, \sum_{k=1}^{r} \mu_k = 1,$$

where

$$\overline{x}(T) = \overline{x}_{\overline{\tau,T}}(T; \overline{x}(\tau), \overline{u}_T^{(i)}(\cdot), \overline{w}_T^{(j)}(\cdot), \overline{v}_T(\cdot)),$$

and $\tilde{\mathbf{F}}$ is the convex functional, formed from functional $\Phi_{\overline{\tau,T}} = (\Phi_{\overline{\tau,T}}^{(1)}, \Phi_{\overline{\tau,T}}^{(2)}, ..., \Phi_{\overline{\tau,T}}^{(r)})$ based on (2), (3).

We will note that to solve the problem (10), it is possible to use, for example, the Zoitendenk method [8].

7. Based on (1), (10), from the solution to the finite discrete optimization problem, we find the set

$$\tilde{\mathbf{U}}_{\mathbf{F}}^{(e)}(\overline{\tau,T}, g(\tau)) \subseteq \mathbf{U}(\overline{\tau,T})$$

of program controls $\overline{u}_T^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau,T})$ and numeric value $\tilde{\mathbf{F}}_{\overline{\tau,\mathbf{T}}}^{(e)}$:

$$\tilde{\mathbf{U}}_{\mathbf{F}}^{(e)}(\overline{\tau,T}, g(\tau)) = \{\overline{u}_T^{(e)}(\cdot): \overline{u}_T^{(e)}(\cdot) \in \mathbf{U}(\overline{\tau,T}),$$

$$\mathbf{F}_{\overline{\tau,T}}(g(\tau), \overline{u}_T^{(e)}(\cdot), \overline{w}_T^{(e)}(\cdot), \overline{v}_T^{(e)}(\cdot)) =$$

$$= \min_{\overline{u}_T^{(i)}(\cdot) \in U(\overline{\tau,T})} \min_{\overline{w}_T^{(j)}(\cdot) \in W(\overline{\tau,T};\overline{u}_T^{(i)}(\cdot))} \mathbf{F}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(i)}(\cdot))} =$$

$$= \min_{i \in 1,N_\tau} \min_{j \in 1,M_\tau(i)} \mathbf{F}_{(\overline{u}_T^{(i)}(\cdot),\overline{w}_T^{(i)}(\cdot))} = \mathbf{F}_{(\overline{u}_T^{(e)}(\cdot),\overline{w}_T^{(e)}(\cdot))} = \tilde{\mathbf{F}}_{\overline{\tau,\mathbf{T}}}^{(e)}. \qquad (11)$$

Based on relations (1) to (3), (4) to (11), it is possible to show that the following equality is true [21]:

$$\mathbf{U}_{\mathbf{F}}^{(e)}(\overline{\tau,T}, g(\tau)) = \tilde{\mathbf{U}}_{\mathbf{F}}^{(e)}(\overline{\tau,T}, g(\tau)), F_{\overline{\tau,\mathbf{T}}}^{(e)} = \tilde{\mathbf{F}}_{\overline{\tau,\mathbf{T}}}^{(e)}. \qquad (12)$$

Satisfaction of equalities (12) means that as a result of implementation of the proposed general scheme, the complete solution to the explored problem of minimax program control with consideration of risks was found. That is, to the nonlinear multi-stage problem of minimax program terminal control for a dynamic system [22], which describes the process of TI control in the presence of risks.

We will note that construction of one-stage reachability region

$$\mathbf{X}^{(+)}_{(\overline{u}_{t+1}^{(i)}(t),\overline{w}_{t+1}^{(j)}(t))}(t, \overline{X}(t), t+1, \mathbf{V}(\overline{t,t+1}; \overline{u}_{t+1}^{(i)}(t))) =$$

$$= \mathbf{X}^{(+)}_{(\overline{u}_{t+1}^{(i)}(t),\overline{w}_{t+1}^{(j)}(t))}(t, \overline{X}(t), t+1, \mathbf{V}_1(t; \overline{u}_{t+1}^{(i)}(t))),$$

$$t \in \overline{\tau,T-1}, \quad \overline{X}(\tau) = X^{(+)}_{(\overline{u}_{0,\tau}(\cdot),\overline{w}_{0,\tau}(\cdot))}(\tau) \quad (i \in \overline{1,N_\tau}, j \in \overline{1,M_\tau(i)}),$$

which exists in formula (7), it is possible to implement similarly to the calculation algorithm [20], which reduces the solution of this problem to implementation of solutions of a finite number of problems of linear mathematic programming.

Then it is possible to make a general conclusion that the solution to the problem of minimax program control of TI in the presence of risks is found by realization of the proposed general scheme. We will reduce it to implementation of solutions of a finite number of problems of linear and convex mathematical programming, as well as a finite number of discrete optimization problems.

Let us note, that the proposed general scheme of solution of the stated problem of minimax program control with consideration of risks makes it possible to develop efficient numerical methods. This enables realization of computer modeling of the solution of the problem of minimax program control of TI and development of computer-based information systems of support of making effective control decisions.

5. Discussion of results of studying the program control over technological innovations with consideration of risks

It should be noted that the application of the method based on construction of reachability region makes it possible to get a convex polyhedron. In his vertices, the optimizing functions take extreme values. Subsequently, from this set of sets we choose such a pair that the maximum value of the objective function is a guaranteed value of the objective function. The program control, which is included in this pair, satisfies the minimax condition, i. e., is a minimax control or a guaranteed control in the explored problems. Such approach will make it possible to reduce the original multistage problem to realization of a finite sequence of one-stage problems of discrete optimization.

When applying modern computation equipment, it is possible to solve such problems by the methods of linear or nonlinear (depending on the type of objective functions) mathematical programming. Thus, the main result of the application of the method of construction of reachability regions is that the problem of TI control is solved within a certain number of iterations.

Thus, the synthesis of these approaches and performed analysis of the state of the problem of economic-mathematical modeling in TI control made it possible to apply the proposed optimization model for TI control. The basis is formed by the optimization of portfolio of release of new products, using formalization of a sequence of stages of the process and moments of decision making as for TI implementation with respect to innovation and investment aspects.

The minimax condition (principle) is used in problems of the canonical form, when we study an objective function for a minimum. In the actual practical statement of the problem

of optimization of technological innovations control, there can occur a situation when it is necessary to maximize the objective function, that is, to solve the maximin problem (in this case, the worst risk vectors minimize the objective function). In order to reduce such problem to a minimax problem, it is necessary to transform the original objective function into the opposite by multiplying its values by (−1), forming in this way a new objective function. Then it is known that minimax control will coincide with maximin, therefore, it is sufficient to explore only minimax problems.

It is necessary to pay attention to the restrictions of the proposed methods. They apply to taking into consideration only deterministic risks in the proposed model. But in practice, there can occur a situation, associated with considerable difficulty in finding the values of input parameters of the vector of deterministic risks. Such a situation requires additional research and further improvement of the model of TI control. So, the prospects of the performed research are related to the possibility of introducing the parameters of the vector of stochastic risks into the model of TI control.

6. Conclusions

1. The proposed economic-mathematical model makes it possible to describe dynamics of multi-stage TI control depending on assigned initial conditions of investment resources and other initial values of the parameters of the state of a system and selection of specific implementations of managerial influences. Different kinds of functionals (linear or convex) can be considered as the criterion of control efficiency optimization.

2. Given the multicriterial optimization of the studied process, the dynamic model of multicriterial optimization of program control of technological innovation was proposed in the work. It was proposed to form objective function F in the form of the convex scalar convolution of vector functional $\Phi=(\Phi_1, \Phi_2, ..., \Phi_r)$. It is formed according to the method of scalarization of vector objective functions with non-negative weight coefficients μ_I, $i \in \overline{1,r}$, which can be determined, for example, in an expert way. The proposed general scheme of solution of the problem of multicriterial optimization of the program minimax control of technological innovation in the presence of risks makes it possible to apply various types of convolutions of objective function, which are limited by capabilities of information resources for the studied process.

3. It is expedient to specify the most important indicators, characterizing effectiveness of TI control, as optimization criteria. They include: the level of labor productivity during TI implementation; intensity, profitability of an innovation process, as well as specific consumption of raw materials, semi-finished products and power per unit of innovative produce or products, manufactured based on the innovative technology; output (amount) and quality of the finished innovative product. In addition, production costs during the use of innovative technologies; cost value of innovative products, etc. But the investment component is one of the key components for implementation of the TI control process. That is why discounted indicators that reflect the money change in time are often used as criteria for the TI control optimization. They include: net current value of TI, cost of cash flows, generated by an innovative process, TI profitability, the return index, investment costs for TI implementation, as well as other technical and economic indicators of production: characteristics of financial sustainability of an enterprise, the amount of innovative products, cost value of an innovative product, meeting consumer standards by the product, manufactured with the use of TI, etc. Thus, in the process of TI control, there is a need to consider the problem of achieving the extremum simultaneously by several objective functions.

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EXPONENTIAL AND HYPERBOLIC TYPES OF DISTRIBUTION IN MACRO SYSTEMS: THEIR COMBINED SYMMETRY AND FINITE PROPERTIES

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Запропоновано розширений ентропійний метод, що виявляє деякі нові зв'язки в організації макросистем, тим самим проливаючи світло на ряд існуючих питань теорії. Зокрема, показано, що тип розподілу всередині макросистеми визначається співвідношенням кінетичних властивостей її агентів – «носіїв» і «ресурсів». Якщо час релаксації менше у «носіїв» – формується експонентний тип розподілу, якщо менше у «ресурсів» – формується тип розподілу з важким хвостом.

Виявлено існування комбінованої симетрії цих двох типів розподілів, які можна розглядати як два різні статистичні трактування єдиного стану макросистеми. Розподіли реальних макросистем мають фінітні властивості - у них природним чином формуються праві межі. Запропонований метод враховує праві межі фінітних розподілів як продукт самоорганізації макросистем, координати яких визначаються на основі екстремального принципу.

Отримано аналітичні вирази для цих двох типів розподілів і їх спектрів, для яких знайдено вдалий спосіб параметричного запису через модальні характеристики. Отримано аналітичні вирази, що враховують фінітні особливості розподілів, де фігурують лише два параметри – середня кількість «ресурсів» та формпараметр як відношення модальної і граничної координат.

Цінність отриманих результатів полягає в тому, що вони проливають світло на ряд проблемних питань статистичної теорії макросистем, та містять набір зручних інструментів для аналізу двох типів розподілів з фінітними властивостями

Ключові слова: макросистема, ентропія, ентропійне моделювання, фінітні розподіли, гіперболічні розподіли, розподіли з важким хвостом

1. Introduction

The growing demand for quantitative predictions in natural, economic, humanitarian, and other fields, has prompted interest in the theory of macro systems as the ideological basis of these studies. Predicting the state of large systems with stochastic behavior of separate elements became possible owing to the tools developed in statistical physics. One such powerful tool is the extreme entropy principle, underlying classic distributions by Maxwell-Boltzmann,