

The object of this study is to robotize the technological operation of removing the oxide film from the surface of a magnesium melt poured into continuously moving molds of a casting conveyor for the production of commercial magnesium. To robotize this technological operation, it is proposed to use a two-armed manipulation robot with a spherical coordinate system, which has six degrees of mobility. Software trajectories have been developed according to the degrees of mobility of the manipulation robot in terms of position, speed, and acceleration to perform the technological operation of removing the oxide film from the surface of the magnesium melt poured into the moving molds of the foundry conveyor. Programmed trajectories are described by quadratic polynomials that satisfy restrictions on the values of the generalized coordinate, velocity, and acceleration. These limitations are determined by the design features and energy capabilities of the degrees of mobility drives of the manipulation robot. Programmed trajectories along the first and second degrees of freedom compensate for the continuous movement of the molds of the foundry conveyor. Programmed trajectories along the third and fourth degrees of mobility enable the collection of the oxide film from the surface of the magnesium melt. Programmed trajectories along the fifth and sixth degrees of freedom enable the discharge of the collected oxide film into a special container. The reliability of the developed programmed trajectories is confirmed by the simulation results using MATLAB version R2015b. Based on the results, a cyclogram for controlling a manipulation robot has been constructed to perform the technological operation of removing the oxide film in the production of commercial magnesium. The results could be used in the robotization of technological processes for removing the oxide film in the production of commercial magnesium or similar foundries

Keywords: *foundry conveyor, oxide film, manipulation robot, trajectory planning, quadratic interpolation*

DEVELOPMENT OF PROGRAMMED TRAJECTORIES BASED ON THE MOBILITY DEGREES OF MANIPULATION ROBOT WITH A SPHERICAL COORDINATE SYSTEM FOR REMOVING OXIDE FILM IN THE PRODUCTION OF COMMERCIAL MAGNESIUM

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Received date 08.12.2023

Accepted date 19.02.2024

Published date 28.02.2024

How to Cite: Beisembayev, A., Yerbossynova, A., Pavlenko, P., Baibatshayev, M. (2024). Development of programmed trajectories based on the mobility degrees of manipulation robot with a spherical coordinate system for removing oxide film in the production of commercial magnesium. *Eastern-European Journal of Enterprise Technologies*, 1 (1 (127)), 67–88. doi: <https://doi.org/10.15587/1729-4061.2024.298912>

1. Introduction

In the foundry production of non-ferrous metals such as lead, zinc, magnesium, there are a number of technological operations, such as the collection of oxide films from the surface of melts poured into the molds of a casting machine, performed manually. These technological operations relate to low-skilled labor, are of a monotonous, monotonous nature, and are performed under conditions harmful to the health of the worker.

One of the methods for solving this problem is the robotization of these technological operations using manipulation robots. In the case of a manipulation robot reproducing movements to remove the oxide film, when the worker visu-

ally sees the formed oxide film and collects it with a scraper and dumps the collected oxide film into a special container, it is difficult and practically impossible to implement.

For this reason, the kinematic structure of the manipulation robot is designed in connection with the casting machine, taking into account the features of the technological operation of removing the oxide film from the surface of the melt poured into the molds. Next, a sequence of movements is developed to collect the oxide film from the surface of the melt poured into the molds and dump the collected oxide film into a special container. On this basis, it is possible:

– to develop programmed trajectories according to the degrees of mobility of the proposed manipulation robot;

- using mathematical modeling methods to evaluate the reliability of the developed programmed trajectories;
- to construct a control cyclogram for the manipulation robot;
- to evaluate the results from the point of view of coordinating the functioning of the manipulation robot together with the casting machine.

Therefore, it is a relevant task to carry out studies on the design of kinematic structures of manipulation robots that take into account the characteristics of the robotic object. As well as on the development of programmed trajectories according to the degrees of mobility of a manipulation robot that implements the technological process of removing oxide films from the surface of melts poured into the molds of a casting machine.

2. Literature review and problem statement

A description of the technology for the production of commercial magnesium is given in [1]. In particular, the design of the casting conveyor, technological modes, and basic requirements for the process of casting magnesium melt are described. A technological operation is described for removing the oxide film from the surface of a magnesium melt, which is performed manually by a worker, which represents an unsolved problem.

In work [2], an analysis was performed of the dependence of quality of magnesium workpieces on the operating regulations of foundry conveyor. In particular, the issues of compliance of geometric dimensions, surface deformations from the temperature regime of pouring magnesium melt into the molds of a foundry conveyor are considered. The dependence of the quality of magnesium workpieces on the technical characteristics of the material of the molds of the foundry conveyor is also shown. When developing the kinematic structure of MR, as well as programmed trajectories according to the degree of mobility of MR, it is necessary to take into account the features of the foundry production of commercial magnesium given in [1, 2].

In work [3], the issues of robotization of technical removal of the oxide film from the surface of a magnesium melt using a two-armed MR having 4 degrees of mobility are considered. The oxide film is removed from the surface of the magnesium melt poured into the continuously moving molds of a foundry conveyor. In this case, the task of accompanying the moving molds of a foundry conveyor is simplified. However, from the point of view of the structural implementation of translational hinges of the degrees of freedom of MR, it becomes significantly more complicated. This is due to ensuring movement with the application of significant forces and moments to implement the required values of speeds and accelerations according to the degrees of MR mobility.

The problem of removing the oxide film in the production of commercial lead and zinc using a two-armed MR having 6 degrees of freedom is considered in [4]. The kinematic structure of this MR has only one degree of mobility, implemented in the form of a translational hinge. However, commercial lead and zinc are produced using rotary casting machines, in which the molds are placed around a turntable. The oxide film is removed from the surface of the melt poured into the stationary molds of a carousel casting machine. To carry out maintenance of removing the oxide film from the surface of a magnesium melt, one can use the proposed kinematic structure of MR. However, it is required to link this MR to a foundry conveyor for the production of commercial magnesium. It is also ne-

cessary to develop programmed trajectories according to the degrees of MR mobility to perform maintenance of removing the oxide film from the surface of the magnesium melt.

One of the tasks in robotic maintenance is the task of planning programmed trajectories according to the degrees of MR mobility. The task of planning programmed trajectories according to degrees of MR mobility is to determine control laws in the form of algebraic polynomials of a given degree of change in the generalized coordinate in position, speed, acceleration. In this case, it must be ensured that the moving parts do not collide with existing obstacles and the required orientations of MR working body [5]. Using the proposed methods, arbitrary programmed trajectories are described in the form of algebraic polynomials in degrees of MR mobility. However, it should be noted that the programmed trajectories depend on the kinematic structure of MR and the characteristics of the robotic object. Therefore, the task of developing programmed trajectories strongly depends on the statement of the research problem.

Automatic generation of programmed trajectories of MR movement, under conditions of uncertainty, which are clarified using an explicit description of the set of interrelations of objects in the workspace, are considered in [6]. Based on an explicit description of a set of objects and connections between them, the problem of manipulation is represented in the form of an algorithm for moving along the degrees of MR mobility. In [7], a heuristic approach to planning MR motion trajectories is proposed, taking into account the obstacles described approximately in the workspace, taking into account which the inverse kinematics problem is solved. Based on the results of the inverse problem of MR kinematics, a heuristic algorithm for planning trajectories of a two-armed MR has been developed, ensuring the absence of collisions with existing obstacles in the workspace. The description of the program trajectory in the form of algebraic polynomials was not considered in [7, 8].

A method for tracking the program trajectory of MR, using the method of training and implementing movement, using the ideas of the LFD method, is considered in [8]. MR motion trajectory is tracked using a three-layer neural network, minimizing position and speed errors in real time. MR control algorithm is formed using neural network training methods. Next, simulation modeling of the operation of MR is carried out in order to assess the feasibility of the resulting control algorithms. In this case, the program trajectory is also formed in the form of a control algorithm. Its analytical description in the form of algebraic polynomials is not considered.

The use of finite impulse response filters to develop a planner for MR programmed trajectories was proposed in [9]. In this case, the programmed trajectory is divided into several separate sections and for each section its own polynomial is compiled, which takes into account restrictions on speed and acceleration, while minimizing movement time. This made it possible to plan trajectories in the form of an algebraic polynomial, providing maximum performance for each degree of MR mobility. However, problems in which it is necessary to develop programmed trajectories that enable the movement of MR along a trajectory that is constantly moving at a given linear speed have not been considered.

The development of MR programmed trajectories for performing assembly operations is considered in [10]. This method can be applied to existing software path scaling algorithms to give them predictive properties when software paths change during assembly maintenance. Features of assembly mainte-

nance require solving the problem of predicting changes in programmed trajectories. Therefore, using scaling algorithms, the procedure for predicting programmed trajectories of MR is performed. This work does not consider cases when it is necessary to carry out the process of tracking a moving trajectory, using an analytical description of the programmed trajectories of MR.

Training methods, in which the operator's hand movements are copied and then reproduced by the exoskeleton, are discussed in [11]. This is of interest from the point of view of reproducing the operator's movements to remove the oxide film from the surface of the metal melt. Based on the kinematic structure of the exoskeleton, it is possible to develop the kinematic structure of an anthropomorphic robot for removing the oxide film from the surface of a magnesium melt. However, in this case, the costs of developing the kinematic structure of an anthropomorphic robot, which is much more complex than the proposed MR, will be significant.

Optimal tracking of programmed trajectories using differential evolution (DE) optimization techniques and the MATLAB toolbox with a robot operating system (ROS) application is discussed in [12]. This makes it possible to develop programmed motion trajectories without colliding with existing obstacles in the workspace of the manipulation robot. This approach is interesting in the case of existing obstacles that change their position. In the case of commercial magnesium foundry, all obstacles are known in advance and their positions do not change.

Work [13] considers approaches to planning programmed trajectories of MR passing through nodal points while minimizing time and energy consumption. Quintic NURBS curves adapt to match the software path to the node points while maintaining continuity in position, velocity, and acceleration. This approach can be used to develop a software trajectory for MR with a complex configuration of the movement trajectory. In the case of movement to remove the oxide film from the surface of the magnesium melt, the movement trajectories consist of separate sections in which only acceleration, movement at a given speed and braking are required. Complication of the programmed trajectory is possible when using a MR having a more complex kinematic structure, which represents a separate scientific problem.

When planning programmed trajectories, it is necessary to ensure that MR does not collide with existing obstacles, and also to enable the required orientation of the working body [14]. This work proposes a method for jointly solving the problem of enabling the required movements with the problem of ensuring the required orientation of MR working body. However, further development of this approach is required for the case of using two-armed MRs, which have a predetermined kinematic structure tied to a given TO for removing the oxide film.

The development of programmed trajectories for several MRs jointly performing one operation is considered in [15]. Using modeling methods, coordinated programmed trajectories were developed using the method of coordinating the trajectory of a multimanipulator, for the case of two MPs performing one circular drawing operation. This approach is interesting from the point of view of performing technical removal of the oxide film from the surface of a magnesium melt using two MRs. However, it is necessary to take into account the difficulties of linking MR by geometric dimensions to the foundry conveyor for the production of commercial magnesium.

An algorithm for planning the trajectory of a multi-link manipulator based on the ant colony algorithm, a metaheuristic

algorithm for solving problems on graphs, was proposed in [16]. The results of the algorithm's operation for various sets of parameters are analyzed. Based on the analysis, recommendations are given for selecting parameters to effectively solve the planning problem. This method also assumes the presence of uncertainties in specifying the obstacles present in MR workspace. In the case of commercial magnesium foundry, all obstacles are predetermined.

In [17], a method for developing programmed trajectories with optimization of energy consumption of MR drives was devised. A new method for optimizing energy consumption is presented, based on dynamic time scaling of trajectories. The development of a new "whip" method based on the movement of a robot's arm, similar to the movement of a whip, is given in [18]. This leads to the achievement of an optimal programmed trajectory in time, by increasing the speed of movement along the degrees of MR mobility. A method for developing programmed trajectories, consisting of two stages, was proposed in [19]. At the first stage, the programmed trajectory is optimized using static methods. At the second stage, optimization is carried out taking into account the orientation of MR working body, minimizing energy consumption and task completion time. In the case of production of commercial magnesium, all parameters of movement along the degrees of MR mobility are specified and limited by the operating parameters of the casting conveyor. This leads to unambiguous solutions, then it is difficult to pose the problem of optimizing programmed trajectories using the methods proposed in [17–19].

The development of programmed trajectories of several robots performing a common task can be solved by Super Intendo semantic programming with multiple demonstration (SRP-MD) methods [20]. Based on the developed demonstration models, with the analysis of the given initial locations of MR, using training methods, programmed trajectories of each MR are generated. In this case, the technological operation of removing the oxide film from the surface of the magnesium melt is proposed to be performed with one two-armed MR. Therefore, this method is not applicable.

From the review of works [3, 4], we can conclude that it is necessary to develop kinematic structures of MR that meet the requirements of the foundry production of commercial magnesium. The application of the kinematic structure of MR proposed in [4] and its modification to the requirements of the foundry production of commercial magnesium is of significant interest.

Taking into account the review of papers [6–8], we can conclude that it is advisable to analytically describe MR programmed trajectories. This will allow us to evaluate the kinematic structure of MR for robotization of the production process under consideration. The possibility of describing programmed trajectories in the form of algebraic polynomials is shown in [9, 10]. However, cases where it is necessary to perform operations associated with a constantly moving trajectory have not been considered. The use of an anthropomorphic robot [11], a collaborative robot [12], which are universal and have a sufficiently large number of degrees for robotization of production processes, may be redundant since only part of their capabilities will be used.

The development of programmed trajectories using Quintic NURBS curves of the fifth power [13], a method for jointly solving the problem of ensuring the required movements with the task of ensuring the required orientation of MR working body [14] also leads to obtaining results in

geometric form. Using these approaches, the problems of developing programmed trajectories for two-armed MR have not been considered. Modeling methods and the development of coordinated programmed trajectories of a multimanipulator are given in [15], the coordinated use of several robots using semantic programming methods Super Intendo is proposed in [20]. However, the use of two manipulators to perform many operations seems redundant.

Application of the approaches proposed in [16] using the ant colony algorithm to select parameters for effectively solving the scheduling problem. In this case, there must be a set of initial options for programmed trajectories. In some cases, this condition is not met. A method for optimizing MR energy consumption was proposed in [17, 19], and the whip method was also proposed for optimizing the movements of a robot's arm in [18]. Their use is relevant when not only the kinematic structure but also the design of MR has been developed. In this case, taking into account the drives used in MR, it is possible to solve the problem of optimizing the energy consumption of the developed MR design. As can be seen from the literature, it is of interest to develop software technologies for degrees of MR mobility in the form of analytical expressions. In this case, the robotic process is associated with performing movements along trajectories that move linearly at a given speed. It should also be noted that the robotic process does not repeat the movements of the worker to perform this operation.

Therefore, it is advisable to use MR kinematic structures that meet the requirements of the foundry production of commercial magnesium for robotic technological operation of removing the oxide film from the surface of a magnesium melt. To assess the compliance of the developed kinematic structure of MR with the requirements of the foundry production of commercial magnesium, it is necessary to develop programmed trajectories according to the degrees of MR mobility, simulate them, with the further construction of a cyclogram for controlling MR. This will make it possible to evaluate the capabilities of the proposed kinematic structure of MR to robotize technical operation of removal of the oxide film from the surface of magnesium melt.

3. The aim and objectives of the study

The purpose of our study is to develop programmed trajectories according to the degrees of MR mobility during robotization of the process of removing the oxide film from the surface of a magnesium melt poured into continuously moving molds of a foundry conveyor. This will make it possible to robotize low-skilled manual labor to remove the oxide film from the surface of the magnesium melt.

To achieve these goals, the following tasks were set:

- to design the kinematic structure of MR with the determination of the sequence of movements according to the degrees of MR mobility to perform technological operation of removing the oxide film from the surface of the magnesium melt;
- to determine the laws of change of the generalized coordinate, its speed and acceleration according to the degrees of MR mobility;
- to assess reliability of the resulting programmed trajectories by modeling them in MATLAB;
- to construct a cyclogram for MR control to perform technological operation of removing the oxide film from the surface of the magnesium melt.

4. The study materials and methods

The object of our study is the process of removing the oxide film from the surface of a magnesium melt poured into continuously moving molds of a casting conveyor for the production of commercial magnesium. Based on an analysis of production processes, the process of removing the oxide film from the surface of a magnesium melt was determined as an object of robotization. Taking into account the specific features of the thermal removal of the oxide film from the surface of the magnesium melt, it is necessary to design the kinematic structure of MR. It is necessary to take into account that the oxide film is collected from the surface of the magnesium melt poured into the continuously moving molds of the casting conveyor.

Of course, the preferred option is to use commercially produced industrial robots. However, this approach is not always applicable. In this case, to robotize the technological process under consideration, it is possible to design a kinematic structure of a manipulation robot that satisfies all the requirements of the robotic technological process, and to devise a sequence of movements of the manipulation robot to perform this technological operation.

For further research, a method was chosen in which the kinematic structure of a manipulation robot is designed that fully complies with the requirements of the technological process. Next, research is carried out on the development of programmed trajectories at the level of kinematics of the manipulation robot. The drive is believed to be ideal.

When designing a kinematic structure, there is an element of bias since the choice of kinematic structure is made based on an analysis of the implementation of the technological process. This process is personal, subjective, and based on the developer's experience.

The required trajectory of movement of the working body of the manipulation robot is set with a given accuracy by nodal points. The accuracy of reproducing the trajectory of movement depends on the number of nodal points. The programmed trajectory according to degrees of mobility is developed from the condition of passing through given nodal points of a given trajectory of movement. The trajectory between nodal points is not controlled. For this reason, a mismatch arises between the given and actual movement of the robot, which determines the accuracy of reproducing the given trajectory of movement. To increase the accuracy of reproducing a given motion trajectory, one can increase the number of nodal points. As the number of nodal points increases, the procedure for developing programmed trajectories becomes more complicated.

To assess the performance of the proposed kinematic structure, develop programmed trajectories according to the degrees of mobility of the manipulation robot, which can be described by algebraic polynomials. These polynomials depend on the values of generalized coordinates, velocities, and accelerations for a given degree of mobility of the manipulation robot. The reliability of the developed programmed trajectories can be assessed using mathematical modeling methods. Based on the resulting programmed trajectories, a control cyclogram can be constructed. Using the control cyclogram, it is possible to assess the compliance of the manipulation robot with the requirements of the technological process in terms of time parameters.

After the kinematic structure of MR is selected, a sequence of movements is developed according to the degrees of

mobility so that to MR performs the technological operation of removing the oxide film from the surface of the magnesium melt poured into the continuously moving molds of the casting conveyor. The limits of change in generalized coordinates according to the degrees of MR mobility are determined. Restrictions are imposed on the values of speeds and accelerations according to the degree of MR mobility, with the fulfillment of the condition of complete collection of the oxide film from the surface of the magnesium melt and ensuring the discharge of the collected oxide film into a special container.

Based on these data, programmed trajectories are formed according to the degrees of MR mobility, which can be approximated by quadratic algebraic polynomials.

The reliability of the developed programmed trajectories according to the degrees of MR mobility is carried out by modeling in the MATLAB software environment. Based on the simulation results, adjustments can be made to the mathematical description of programmed trajectories according to the degrees of MR mobility.

Next, based on the analysis of the developed programmed trajectories by degrees of mobility, it is possible to construct a cyclogram for MR control to perform technological operation of removing the oxide film from the surface of the magnesium melt.

5. Results of investigating the task of robotization of the technological operation of removing the oxide film from the surface of a magnesium melt

5.1. Kinematic structure of a manipulation robot for removing an oxide film from the surface of a magnesium melt

Foundry production of commercial magnesium is a process of casting molten magnesium from an inclined crucible on a foundry conveyor, shown in Fig. 1. The foundry conveyor is belt 1 stretched over drive 2 and tension 3 drums, on which molds 5 are attached. During the rotation of drive drum 2, endless belt 1 moves molds 5 along the conveyor belt. Commercial magnesium is produced in the form of magnesium castings called ingots.

The reason for the formation of an oxide film is the pouring of magnesium in an open jet, the surface of which is oxidized, resulting in the creation of a stocking consisting of frozen magnesium oxide. As the mass increases, the oxide film falls into the mold and floats to the surface. The amount of oxide on the surface of the molds and between the molds is distributed unevenly.

The oxide film is manually collected with a scraper and dumped into a special container. The magnesium melt poured into the mold passes into the solid phase due to heat exchange with the mold. Next, mold 4, having reached the drive drum, is overturned by rotation around axis 5, colliding with stop 6, and the magnesium ingot falls out of the mold (Fig. 1).

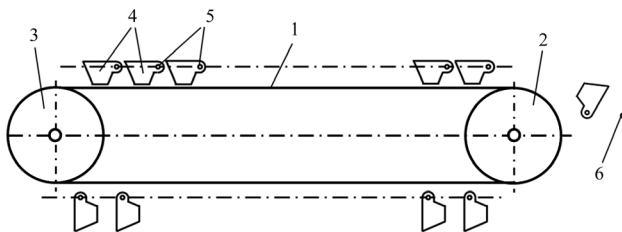


Fig. 1. Diagram of a foundry conveyor for the production of commercial magnesium

To perform technological operation of removing the oxide film from the surface of a magnesium melt, it is proposed to use a two-armed manipulation robot (MR) with a spherical coordinate system. The kinematic structure of MR is shown in Fig. 2. The proposed MR consists of fixed base 1, with fixed rotary hinge 2. Rotary hinge 3 is fixed to rotary hinge 2. This hinge is connected to the next rotary hinge 4, on which a translational hinge is attached for linear movement of rod 6, which represents 1 arm of MR, to which movable blades 7 are attached. Rotational hinge 8 is connected to translational hinge 5, to which rotational hinge 9 is attached, which represents the 2nd arm of MR, with fixed rotational blades 10.

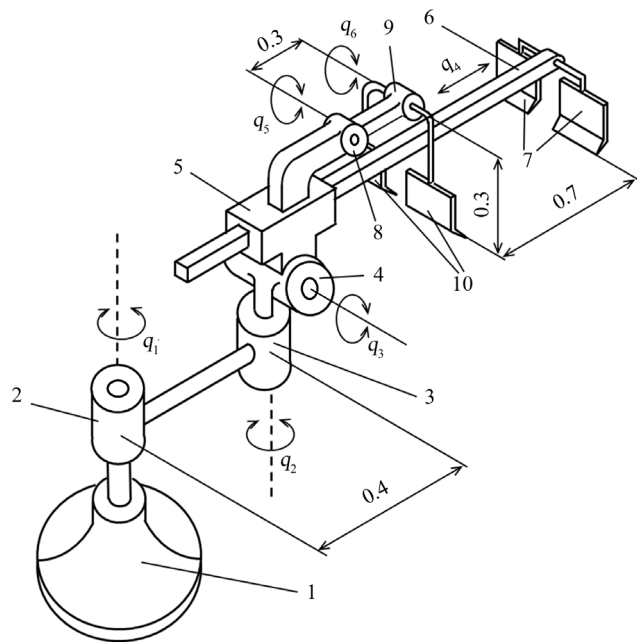


Fig. 2. Manipulation robot for removing the oxide film from the surface of the magnesium melt

In this case, the oxide film is removed from the surface of the metal melt poured into the continuously moving molds of casting conveyor [3]. The time period for moving one mold is 5 s. This is a fairly short period of time, so it is proposed to remove the oxide film simultaneously from the surfaces of two molds, so the time period increases to 10 s.

The sequence of movements to implement the process of removing the oxide film from the surface of the magnesium melt is specified by different relative positions in terms of degrees of MR mobility. The initial position of MR for performing technological operation of removing the oxide film from the surface of the magnesium melt is shown in Fig. 3. In this case, due to the rotation of the 1st degree of freedom clockwise, and the 2nd degree of freedom counterclockwise, the movable and rotary blades are positioned along the molds. By clockwise rotation of the 3rd degree of freedom, both blades are raised above the molds; the positions of the 4th and 5th degrees of freedom orient the rotary blade in its original position [4].

Fig. 4 shows the position of the degrees of MR mobility, at which the movable and rotating blades are lowered onto the surface of the magnesium melt poured into the mold. This is realized due to the counterclockwise rotational movement of the 3rd degree of MR mobility [4].

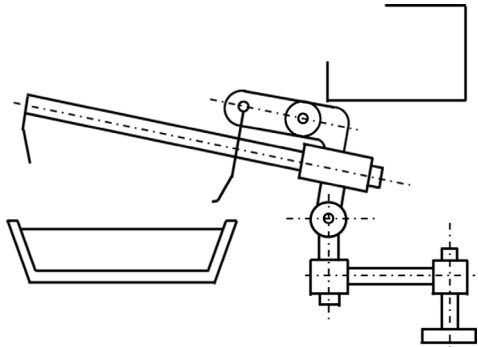


Fig. 3. Initial position of the manipulation robot

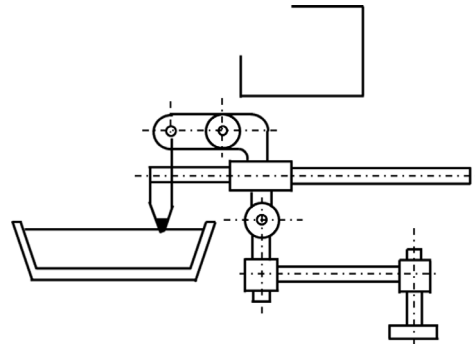


Fig. 6. The process of collecting the oxide film from the surface of the magnesium melt

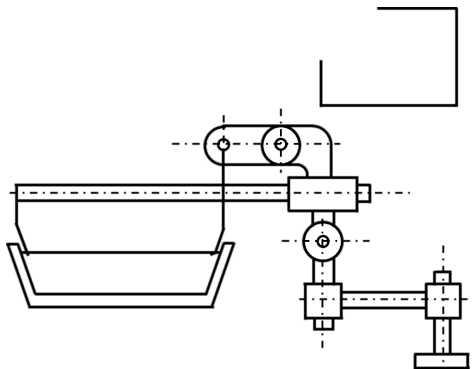


Fig. 4. The process of lowering the blades to the surface of the magnesium melt

Fig. 5 shows a top view of MR position in which the blades are lowered onto the surface of the magnesium melt. As can be seen from Fig. 5, the blades are lowered on the surface of the magnesium melt poured into two molds. This shows the speed of continuous movement of the molds of the foundry conveyor.

Fig. 6 shows the position of MR, at which, by linear movement along the 4th degree of freedom with the counter movement of the movable blade (Fig. 2) along the 4th degree of MR mobility, the oxide film from the surface of the magnesium melt is collected on the rotary blade [4].

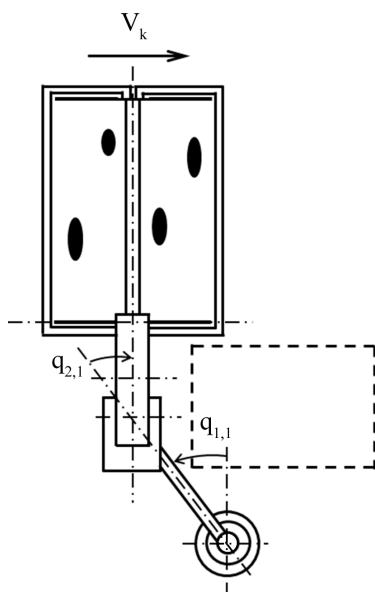


Fig. 5. Top view of the process of lowering the blades onto the surface of a magnesium melt poured into two molds

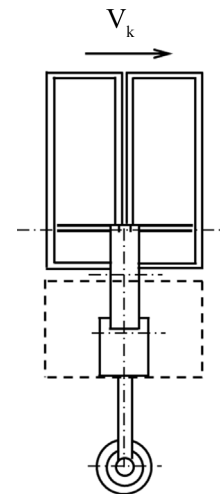


Fig. 7. Top view of the process of collecting the oxide film from the surface of the magnesium melt

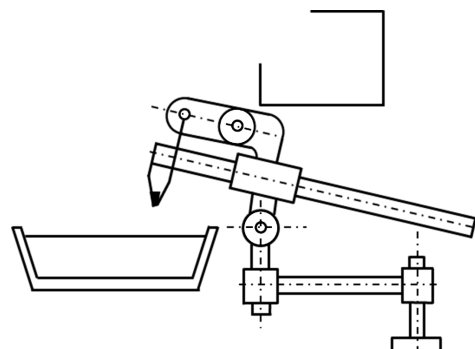


Fig. 8. The process of lifting the blades with the collected oxide film

At the same time, rotary joint 8 is rotated clockwise (Fig. 2), which will lead to the rotation of the rotary blade with the collected oxide film to the position shown in Fig. 9 [4].

Next, the oxide film collected on the rotary blade is dropped into a special container by clockwise rotation of the 6th degree of MR mobility, as shown in Fig. 10 [4].

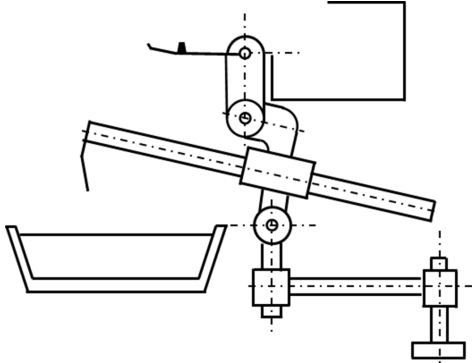


Fig. 9. The process of turning a hinge clockwise with a fixed rotary blade

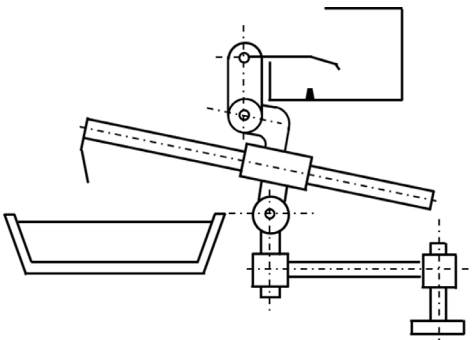


Fig. 10. The process of dumping the oxide film into a special container

Carrying out this sequence of movements according to the degrees of MR mobility will allow collecting the oxide film from the surface of the magnesium melt poured into the moving molds of the casting conveyor. As well as accompany the moving molds of MR casting conveyor and carry out further discharge of the collected oxide film into a special container.

5. 2. Determining the laws of change in the generalized coordinate according to the degrees of mobility of the manipulation robot, in the form of quadratic polynomials

For each degree of MR mobility, the limits of change in the values of generalized coordinates, velocities, and accelerations are determined, which are specified by the design features of the kinematic structure of MR and the energy characteristics of the drives, on the basis of which the following was obtained:

$$\left\{ \begin{array}{l} -\frac{\pi}{2} \leq q_1 \leq \frac{\pi}{2} \text{ rad}, -\frac{\pi}{2} \leq q_2 \leq \frac{\pi}{2} \text{ rad}, \\ -\frac{\pi}{6} \leq q_3 \leq \frac{\pi}{6} \text{ rad}, \\ 0 \leq q_4 \leq 0.7 \text{ m}, -\frac{\pi}{2} \leq q_5 \leq 0 \text{ rad}, \\ -\frac{\pi}{2} \leq q_6 \leq \frac{\pi}{2} \text{ rad}. \end{array} \right. \quad (1)$$

By velocity:

$$\left\{ \begin{array}{l} -\frac{\pi}{2} \leq \dot{q}_1 \leq \frac{\pi}{2} \text{ rad/s}, -\frac{\pi}{2} \leq \dot{q}_2 \leq \frac{\pi}{2} \text{ rad/s}, \\ -\frac{\pi}{4} \leq \dot{q}_3 \leq \frac{\pi}{4} \text{ rad/s}, \\ -0.3 \leq \dot{q}_4 \leq 0.3 \text{ m/s}, -\pi \leq \dot{q}_5 \leq \pi \text{ rad/s}, \\ -\pi \leq \dot{q}_6 \leq \pi \text{ rad/s}. \end{array} \right. \quad (2)$$

By acceleration:

$$\left\{ \begin{array}{l} -\frac{\pi}{4} \leq \ddot{q}_1 \leq \frac{\pi}{4} \text{ rad/s}^2, -\frac{\pi}{4} \leq \ddot{q}_2 \leq \frac{\pi}{4} \text{ rad/s}^2, \\ -\frac{\pi}{6} \leq \ddot{q}_3 \leq \frac{\pi}{6} \text{ rad/s}^2, \\ -0.3 \leq \ddot{q}_4 \leq 0.3 \text{ m/s}^2. \end{array} \right. \quad (3)$$

To develop program motion trajectories, the laws of change of generalized coordinates according to the degrees of MR mobility are determined. These laws are described by quadratic polynomials, which provided the condition for the coincidence of the values of generalized coordinates, values of velocities and accelerations at the nodal points of conjugation of sections of MR motion trajectory [4, 5].

The initial position of MR is specified by the values of the generalized coordinates according to the degrees of mobility $q_{1,1}=0.6047 \text{ rad}$, $q_{2,1}=-0.6047 \text{ rad}$, $q_{3,1}=0.1745 \text{ rad}$, $q_{4,1}=0.605 \text{ m}$, $q_{5,1}=0 \text{ rad}$, $q_{6,1}=-1.57 \text{ rad}$ (Fig. 3).

Since the removal of the oxide film was carried out from the surface of the poured metal melt into moving molds, at the first step it was necessary to compensate for the speed of movement of the casting conveyor molds equal to $v_k=0.04 \text{ m/s}$. This is accomplished by rotating the 1st degree of freedom clockwise. Counterclockwise rotation at the same angular speed along the 2nd degree of freedom enables the rectilinear arrangement of the movable and rotary blades along the line of symmetry of the molds. In this way, support was ensured for the linearly moving molds of the foundry conveyor.

To accompany moving molds, it is necessary to initially accelerate to a given speed $V_k=0.04 \text{ m/s}$. After acceleration to the preset speed, the movement of the molds is accompanied until the oxide film is completely removed from the surface of the magnesium melt and the blades with the collected oxide film are lifted (Fig. 5). Next, the collected oxide film was dumped into a special container.

The required angle for 1 degree of mobility was equal to:

$$q_{1,2} = -0.5628 \text{ rad}.$$

Then:

$$q_{1,1} = -0.6047 \text{ rad}.$$

In this case, the linear displacement was computed as follows:

$$l_{1,1} = 0.4 \cdot (\sin q_{1,1} - \sin q_{1,2}) = 0.012 \text{ m},$$

which corresponded to linear movement with speed $V_k=0.04 \text{ m/s}$, over a period of time:

$$\Delta t_{1,1} = 0.35 \text{ s}.$$

Then, the rotation angle for the 1st degree of motion is calculated as follows:

$$\Delta q_{1,1} = q_{1,1} - q_{1,2} = -0.0419 \text{ rad,}$$

hence:

$$-0.0419 = \frac{\ddot{q}_{1,1} \cdot 0.35^2}{2},$$

from this expression, the following is calculated:

$$\ddot{q}_{1,1} = -0.684 \text{ rad/s}^2.$$

Then the trajectory of movement in the 1st degree of mobility took the following form:

$$\begin{cases} t = 0: q_{1,1} = 0.6047 \text{ rad,} \\ \forall t \in [0, 0.35]: q_{1,1}^o = 0.6047 - \frac{0.684t^2}{2}, \\ t = 0.35: q_{1,2} = 0.5628 \text{ rad,} \end{cases} \quad (4)$$

where $q_{1,1}^o$ is a polynomial that describes the change in the generalized coordinate during acceleration in 1 section of the programmed trajectory along 1 degree of MR mobility.

Similar to expression (4), the trajectory of movement along the 2nd degree of mobility is obtained in the following form:

$$\begin{cases} t = 0: q_{2,1} = -0.6047 \text{ rad,} \\ \forall t \in [0, 0.35]: q_{2,1}^o = -0.6047 + \frac{0.684t^2}{2}, \\ t = 0.35: q_{2,2} = -0.5628 \text{ rad,} \end{cases} \quad (5)$$

where $q_{2,1}^o$ is a polynomial that describes the change in the generalized coordinate during acceleration in the 1st section of the programmed trajectory along the 2nd degree of MR mobility.

Next, the process of accompanying moving molds was carried out at a speed of $V_k=0.04$ m/s, due to clockwise rotation along the 1st degree of mobility and counterclockwise, along the 2nd degree of mobility (Fig. 5). In this case, the final location of the 1st and 2nd degrees of mobility corresponded to Fig. 7.

By dividing the trajectory of movement along the 1st and 2nd degrees of freedom into 10 sections, specifying 11 nodal points at which the coincidence of the values of the generalized coordinate and the positions of the moving molds is ensured. Then the increment in the value of the generalized coordinate was calculated as follows:

$$\Delta q_1 = \frac{q_{2,2}}{10} = 0.05628 \text{ rad.}$$

An expression has been obtained for determining the values of generalized coordinates corresponding to the nodal

points along the first degree of MR mobility, which takes the following form:

$$q_{1,i+1} = q_{1,i} - \Delta q_1, i = \overline{2,11}.$$

By substituting into this equation, the values of generalized coordinates for the first degree of MR mobility were calculated, which are summarized in Table 1.

An expression was obtained for determining the time intervals corresponding to the values of the generalized coordinates according to the first degree of MR mobility (Table 1), which takes the following form:

$$\Delta t_{1,i+1} = \frac{0.4 \cdot (\sin q_{1,i} - \sin q_{1,i+1})}{0.04}, i = \overline{2,11}.$$

By substituting into this equation, the values of time intervals for the first degree of MR mobility were calculated, which are summarized in Table 2.

Based on the resulting necessary time intervals between adjacent nodal points, expressions are determined that define programmed trajectories for performing movement along the 1st degree of mobility. To do this, starting from 1 node point, the initial speed of which was determined from the following expression:

$$\dot{q}_{1,2} = \ddot{q}_{1,1} \cdot \Delta t_{1,1} = -0.2394 \text{ rad/s.}$$

Next, the value of the required acceleration is determined, satisfying the values of the increment of the generalized coordinate, the required time interval, and the initial speed from the expression:

$$-0.05628 = -0.2394 \cdot 0.485 + \frac{\ddot{q}_{1,2} \cdot 0.485^2}{2}.$$

Based on this expression, the value of the required acceleration was calculated $\ddot{q}_{1,2} = 0.5087 \text{ rad/s}^2$. Taking into account the obtained values of the required time interval, initial speed and required acceleration, the trajectory of movement along the 1st degree of mobility was obtained in the following form:

$$\begin{cases} t = 0.35: q_{1,2} = 0.5628 \text{ rad,} \\ \forall t \in [0.35, 0.835]: q_{1,2}^b = 0.5628 - 0.2394 \cdot t + \frac{0.5087t^2}{2}, \\ t = 0.835: q_{1,3} = 0.5065 \text{ rad,} \end{cases} \quad (6)$$

where $q_{1,2}^b$ is a polynomial that describes the change in the generalized coordinate during braking on the 2nd section of the programmed trajectory along the 1st degree of MR mobility.

Table 1

Values of generalized coordinates for the first degree of MR mobility, corresponding to the nodal points of trajectory

$q_{1,i}$	$q_{1,3}$	$q_{1,4}$	$q_{1,5}$	$q_{1,6}$	$q_{1,7}$	$q_{1,8}$	$q_{1,9}$	$q_{1,10}$	$q_{1,11}$	$q_{1,12}$
rad	0.5065	0.4502	0.394	0.3377	0.2814	0.2251	0.1688	0.1125	0.056285	0

Table 2

Values of time intervals for the first degree of MR mobility, corresponding to the values of generalized coordinates

$t_{1,i}$	$t_{1,3}$	$t_{1,4}$	$t_{1,5}$	$t_{1,6}$	$t_{1,7}$	$t_{1,8}$	$t_{1,9}$	$t_{1,10}$	$t_{1,11}$	$t_{1,12}$
s	0.4842	0.499	0.513	0.526	0.536	0.545	0.552	0.557	0.56	0.562

Between the following nodal points, we defined:

$$\begin{aligned}\dot{q}_{1,3} &= \dot{q}_{1,2} + \ddot{q}_{1,2} \cdot \Delta t_{1,2} = 0.0073 \text{ rad/s}, \\ -0.05628 &= 0.0073 \cdot 0.499 + \frac{\ddot{q}_{1,3} \cdot 0.499^2}{2}, \\ \ddot{q}_{1,3} &= -0.4813 \text{ rad/s}^2.\end{aligned}$$

Then the trajectory of movement in the 1st degree of mobility in this section is obtained in the following form:

$$\left\{ \begin{array}{l} t = 0.835 : q_{1,3} = 0.5065 \text{ rad}, \\ \forall t \in [0.835, 1.334] : \dot{q}_{1,3} = \\ = 0.5065 + 0.0073 \cdot t - \frac{0.4813t^2}{2}, \\ t = 1.334 : q_{1,4} = 0.4502 \text{ rad}, \end{array} \right. \quad (7)$$

where $q_{1,3}^o$ is a polynomial that describes the change in the generalized coordinate during acceleration in the 3rd section of the programmed trajectory along the 1st degree of MR mobility.

In the following section between the nodal points, we defined:

$$\begin{aligned}\dot{q}_{1,4} &= \dot{q}_{1,3} + \ddot{q}_{1,3} \cdot \Delta t_{1,3} = -0.2328 \text{ rad/s}, \\ -0.05628 &= -0.2328 \cdot 0.513 + \frac{\ddot{q}_{1,4} \cdot 0.513^2}{2}, \\ \ddot{q}_{1,4} &= 0.4799 \text{ rad/s}^2.\end{aligned}$$

Then the trajectory of movement in the 1st degree of mobility in this section is obtained in the following form:

$$\left\{ \begin{array}{l} t = 1.334 : q_{1,4} = 0.4502 \text{ rad}, \\ \forall t \in [1.334, 1.847] : \dot{q}_{1,4} = \\ = 0.4502 - 0.2328 \cdot t + \frac{0.4799t^2}{2}, \\ t = 1.847 : q_{1,5} = 0.394 \text{ rad}, \end{array} \right. \quad (8)$$

where $q_{1,4}^b$ is a polynomial that describes the change in the generalized coordinate during braking on the 4th section of the programmed trajectory along the 1st degree of MR mobility.

The next section between the nodal points is then considered, for which it is defined:

$$\begin{aligned}\dot{q}_{1,5} &= \dot{q}_{1,4} + \ddot{q}_{1,4} \cdot \Delta t_{1,4} = 0.0133 \text{ rad/s}, \\ -0.05628 &= 0.0133 \cdot 0.526 + \frac{\ddot{q}_{1,5} \cdot 0.526^2}{2}, \\ \ddot{q}_{1,5} &= -0.4574 \text{ rad/s}^2.\end{aligned}$$

Then the trajectory of movement in the 1st degree of mobility in this section is obtained in the following form:

$$\left\{ \begin{array}{l} t = 1.847 : q_{1,5} = 0.394 \text{ rad}, \\ \forall t \in [1.847, 2.373] : \dot{q}_{1,5} = \\ = 0.394 + 0.0133 \cdot t - \frac{0.4574t^2}{2}, \\ t = 2.373 : q_{1,6} = 0.3377 \text{ rad}, \end{array} \right. \quad (9)$$

where $q_{1,5}^o$ is a polynomial that describes the change in the generalized coordinate during acceleration in the 5th section of the programmed trajectory along the 1st degree of MR mobility.

Next, the section between the following nodal points is considered, for which we defined:

$$\begin{aligned}\dot{q}_{1,6} &= \dot{q}_{1,5} + \ddot{q}_{1,5} \cdot \Delta t_{1,5} = -0.2273 \text{ rad/s}, \\ -0.05628 &= -0.2273 \cdot 0.536 + \frac{\ddot{q}_{1,6} \cdot 0.536^2}{2}, \\ \ddot{q}_{1,6} &= 0.4563 \text{ rad/s}^2.\end{aligned}$$

Then the trajectory of movement in the 1st degree of mobility in this section is obtained in the following form:

$$\left\{ \begin{array}{l} t = 2.373 : q_{1,6} = 0.3377 \text{ rad}, \\ \forall t \in [2.373, 2.909] : \dot{q}_{1,6} = \\ = 0.3377 - 0.2273 \cdot t + \frac{0.4563t^2}{2}, \\ t = 2.909 : q_{1,7} = 0.2814 \text{ rad}, \end{array} \right. \quad (10)$$

where $q_{1,6}^b$ is a polynomial that describes the change in the generalized coordinate during braking on the 6th section of the programmed trajectory along the 1st degree of MR mobility.

In the following section between the nodal points, we defined:

$$\begin{aligned}\dot{q}_{1,7} &= \dot{q}_{1,6} + \ddot{q}_{1,6} \cdot \Delta t_{1,6} = 0.0172 \text{ rad/s}, \\ -0.05628 &= 0.0172 \cdot 0.545 + \frac{\ddot{q}_{1,7} \cdot 0.545^2}{2}, \\ \ddot{q}_{1,7} &= -0.442 \text{ rad/s}^2.\end{aligned}$$

Then the trajectory of movement in the 1st degree of mobility in this section is obtained in the following form:

$$\left\{ \begin{array}{l} t = 2.909 : q_{1,7} = 0.2814 \text{ rad}, \\ \forall t \in [2.909, 3.454] : \dot{q}_{1,7} = 0.2814 + 0.0172 \cdot t - \frac{0.442t^2}{2}, \\ t = 3.454 : q_{1,8} = 0.2251 \text{ rad}, \end{array} \right. \quad (11)$$

where $q_{1,7}^o$ is a polynomial that describes the change in the generalized coordinate during acceleration in the 7th section of the programmed trajectory along the 1st degree of MR mobility.

We defined between the following nodal points:

$$\begin{aligned}\dot{q}_{1,8} &= \dot{q}_{1,7} + \ddot{q}_{1,7} \cdot \Delta t_{1,7} = -0.2236 \text{ rad/s}, \\ -0.05628 &= -0.2236 \cdot 0.552 + \frac{\ddot{q}_{1,8} \cdot 0.552^2}{2}, \\ \ddot{q}_{1,8} &= 0.44 \text{ rad/s}^2.\end{aligned}$$

Then the trajectory of movement in the 1st degree of mobility in this section is obtained in the following form:

$$\left\{ \begin{array}{l} t = 3.454 : q_{1,8} = 0.2251 \text{ rad}, \\ \forall t \in [3.454, 4.006] : \dot{q}_{1,8} = 0.2251 - 0.2236 \cdot t + \frac{0.44t^2}{2}, \\ t = 4.006 : q_{1,9} = 0.1688 \text{ rad}, \end{array} \right. \quad (12)$$

where $q_{1,8}^b$ is a polynomial that describes the change in the generalized coordinate during braking on the 8th section of the programmed trajectory along the 1st degree of MR mobility.

In the following section between the nodal points, we defined:

$$\begin{aligned} \dot{q}_{1,9} &= \dot{q}_{1,8} + \ddot{q}_{1,8} \cdot \Delta t_{1,8} = 0.0192 \text{ rad/s}, \\ -0.05628 &= 0.0192 \cdot 0.557 + \frac{\ddot{q}_{1,9} \cdot 0.557^2}{2}, \\ \ddot{q}_{1,9} &= -0.4317 \text{ rad/s}^2. \end{aligned}$$

Then the trajectory of movement along the 1st degree of mobility in this section is obtained in the following form:

$$\begin{cases} t = 4.006 : q_{1,9} = 0.1688 \text{ rad}, \\ \forall t \in [4.006, 4.563] : q_{1,9}^o = \\ = 0.1688 + 0.0192 \cdot t - \frac{0.4317t^2}{2}, \\ t = 4.563 : q_{1,10} = 0.1125 \text{ rad}, \end{cases} \quad (13)$$

where $q_{1,9}^o$ is a polynomial that describes the change in the generalized coordinate during acceleration in the 9th section of the programmed trajectory along the 1st degree of MR mobility.

We defined between the following nodal points:

$$\begin{aligned} \dot{q}_{1,10} &= \dot{q}_{1,9} + \ddot{q}_{1,9} \cdot \Delta t_{1,9} = -0.2212 \text{ rad/s}, \\ -0.05628 &= -0.2212 \cdot 0.561 + \frac{\ddot{q}_{1,10} \cdot 0.561^2}{2}, \\ \ddot{q}_{1,10} &= 0.4309 \text{ rad/s}^2. \end{aligned}$$

Then the trajectory of movement along the 1st degree of mobility in this section is obtained in the following form:

$$\begin{cases} t = 4.563 : q_{1,10} = 0.1125 \text{ rad}, \\ \forall t \in [4.563, 5.124] : q_{1,10}^b = \\ = 0.1125 - 0.2212 \cdot t + \frac{0.4309t^2}{2}, \\ t = 5.124 : q_{1,11} = 0.05628 \text{ rad}, \end{cases} \quad (14)$$

where $q_{1,10}^b$ is a polynomial that describes the change in the generalized coordinate during braking on the 10th section of the programmed trajectory along the 1st degree of MR mobility.

Between the last nodal points, the following is defined:

$$\begin{aligned} \dot{q}_{1,11} &= \dot{q}_{1,10} + \ddot{q}_{1,10} \cdot \Delta t_{1,10} = 0.0205 \text{ rad/s}, \\ -0.05628 &= 0.0205 \cdot 0.562 + \frac{\ddot{q}_{1,11} \cdot 0.562^2}{2}, \\ \ddot{q}_{1,11} &= -0.4293 \text{ rad/s}^2. \end{aligned}$$

Then the trajectory of movement along the 1st degree of mobility in this section is obtained in the following form:

$$\begin{cases} t = 5.124 : q_{1,11} = 0.05628 \text{ rad}, \\ \forall t \in [5.124, 5.686] : q_{1,11}^o = \\ = 0.05628 + 0.0205 \cdot t - \frac{0.4293t^2}{2}, \\ t = 5.686 : q_{1,12} = 0 \text{ rad}, \end{cases} \quad (15)$$

where $q_{1,11}^o$ is a polynomial describing the change in the generalized coordinate during acceleration in the 11th section of the programmed trajectory along the 1st degree of MR mobility.

Next, braking is carried out in the 1st degree of mobility, to zero speed, then:

$$\begin{aligned} \dot{q}_{1,12} &= \dot{q}_{1,11} + \ddot{q}_{1,11} \cdot \Delta t_{1,10} = -0.2207 \text{ rad/s}, \\ -0.2207 &+ 0.7853 \cdot \Delta t_{1,12} = 0, \\ \Delta t_{1,12} &= 0.281 \text{ s}, \\ q_{1,13} &= -0.2207 \cdot 0.281 + \frac{0.7853 \cdot 0.281^2}{2} = -0.031 \text{ rad}. \end{aligned}$$

In this case, the trajectory of movement along the 1st degree of mobility in this section is obtained in the following form:

$$\begin{cases} t = 5.686 : q_{1,12} = 0 \text{ rad}, \\ \forall t \in [5.124, 5.686] : q_{1,12}^b = -0.2207 \cdot t + \frac{0.7853t^2}{2}, \\ t = 5.967 : q_{1,13} = -0.031 \text{ rad}, \end{cases} \quad (16)$$

where $q_{1,12}^b$ is a polynomial describing the change in the generalized coordinate during braking on the 12th section of the programmed trajectory along the 1st degree of MR mobility.

For the 2nd degree of freedom, counterclockwise rotation is performed, the programmed trajectories will be similar to the expressions of the programmed trajectories for the 1st degree of freedom. At the same time, the equality of the nodal points of the trajectory, the values of increments of generalized coordinates, the values of velocities and accelerations in absolute value is ensured.

The programmed trajectory of movement along the 2nd degree of mobility corresponding to the section specified by expression (6) is obtained in the following form:

$$\begin{cases} t = 0.35 : q_{2,2} = -0.5628 \text{ rad}, \\ \forall t \in [0.35, 0.835] : q_{2,2}^b = \\ = -0.5628 + 0.2394 \cdot t - \frac{0.5087t^2}{2}, \\ t = 0.835 : q_{2,3} = -0.5065 \text{ rad}, \end{cases} \quad (17)$$

where $q_{2,2}^b$ is a polynomial that describes the change in the generalized coordinate during braking on the 2nd section of the programmed trajectory along the 2nd degree of MR mobility.

The programmed trajectory of movement along the 2nd degree of mobility, corresponding to the section specified by expression (7), is obtained in the following form:

$$\begin{cases} t = 0.835 : q_{2,3} = -0.5065 \text{ rad}, \\ \forall t \in [0.835, 1.334] : q_{2,3}^o = \\ = -0.5065 - 0.0073 \cdot t + \frac{0.4813t^2}{2}, \\ t = 1.334 : q_{2,4} = -0.4502 \text{ rad}, \end{cases} \quad (18)$$

where $q_{2,3}^o$ is a polynomial that describes the change in the generalized coordinate during acceleration in the 3rd section of the programmed trajectory along the 2nd degree of MR mobility.

The programmed trajectory of movement along the 2nd degree of mobility, corresponding to the section specified by expression (8), is obtained in the following form:

$$\left\{ \begin{array}{l} t = 1.334 : q_{2,4} = -0.4502 \text{ rad}, \\ \forall t \in [1.334, 1.847] : q_{2,4}^b = \\ = -0.4502 + 0.2328 \cdot t - \frac{0.4799t^2}{2}, \\ t = 1.847 : q_{2,5} = 0.394 \text{ rad}, \end{array} \right. \quad (19)$$

where $q_{2,4}^b$ is a polynomial describing the change in the generalized coordinate during braking on the 4th section of the programmed trajectory along the 2nd degree of MR mobility. The programmed trajectory of movement along the 2nd degree of mobility, corresponding to the section specified by expression (9), is obtained in the following form:

$$\left\{ \begin{array}{l} t = 1.847 : q_{2,5} = -0.394 \text{ rad}, \\ \forall t \in [1.847, 2.373] : q_{2,5}^o = \\ = -0.394 - 0.0133 \cdot t + \frac{0.4574t^2}{2}, \\ t = 2.373 : q_{2,6} = -0.3377 \text{ rad}, \end{array} \right. \quad (20)$$

where $q_{2,5}^o$ is a polynomial that describes the change in the generalized coordinate during acceleration in the 5th section of the programmed trajectory along the 2nd degree of MR mobility.

The programmed trajectory of movement along the 2nd degree of mobility, corresponding to the section specified by expression (10), is obtained in the following form:

$$\left\{ \begin{array}{l} t = 2.373 : q_{2,6} = -0.3377 \text{ rad}, \\ \forall t \in [2.373, 2.909] : q_{2,6}^b = \\ = -0.3377 + 0.2273 \cdot t - \frac{0.4563t^2}{2}, \\ t = 2.909 : q_{2,7} = -0.2814 \text{ rad}, \end{array} \right. \quad (21)$$

where $q_{2,6}^b$ is a polynomial that describes the change in the generalized coordinate during braking on the 6th section of the programmed trajectory along the 2nd degree of MR mobility.

The programmed trajectory of movement along the 2nd degree of mobility, corresponding to the section specified by expression (11), is obtained in the following form:

$$\left\{ \begin{array}{l} t = 2.909 : q_{2,7} = -0.2814 \text{ rad}, \\ \forall t \in [2.909, 3.454] : q_{2,7}^o = \\ = -0.2814 - 0.0172 \cdot t + \frac{0.442t^2}{2}, \\ t = 3.454 : q_{2,8} = -0.2251 \text{ rad}, \end{array} \right. \quad (22)$$

where $q_{2,7}^o$ is a polynomial that describes the change in the generalized coordinate during acceleration in the 7th section of the programmed trajectory along the 2nd degree of MR mobility.

The programmed trajectory of movement along the 2nd degree of mobility, corresponding to the section specified by expression (12), is obtained in the following form:

$$\left\{ \begin{array}{l} t = 3.454 : q_{2,8} = -0.2251 \text{ rad}, \\ \forall t \in [3.454, 4.006] : q_{2,8}^b = \\ = -0.2251 + 0.2236 \cdot t - \frac{0.44t^2}{2}, \\ t = 4.006 : q_{2,9} = -0.1688 \text{ rad}, \end{array} \right. \quad (23)$$

where $q_{2,8}^b$ is a polynomial that describes the change in the generalized coordinate during braking on the 8th section of the programmed trajectory along the 2nd degree of MR mobility.

The programmed trajectory of movement along the 2nd degree of mobility, corresponding to the section specified by expression (13), is obtained in the following form:

$$\left\{ \begin{array}{l} t = 4.006 : q_{2,9} = -0.1688 \text{ rad}, \\ \forall t \in [4.006, 4.563] : q_{2,9}^o = \\ = -0.1688 - 0.0192 \cdot t + \frac{0.4317t^2}{2}, \\ t = 4.563 : q_{2,10} = -0.1125 \text{ rad}, \end{array} \right. \quad (24)$$

where $q_{2,9}^o$ is a polynomial describing the change in the generalized coordinate during acceleration in the 9th section of the programmed trajectory along the 2nd degree of MR mobility.

The programmed trajectory of movement along the 2nd degree of mobility, corresponding to the section specified by expression (14), is obtained in the following form:

$$\left\{ \begin{array}{l} t = 4.563 : q_{2,10} = -0.1125 \text{ rad}, \\ \forall t \in [4.563, 5.124] : q_{2,10}^b = \\ = -0.1125 + 0.2212 \cdot t - \frac{0.4309t^2}{2}, \\ t = 5.124 : q_{2,11} = -0.05628 \text{ rad}, \end{array} \right. \quad (25)$$

where $q_{2,10}^b$ is a polynomial that describes the change in the generalized coordinate during braking on the 10th section of the programmed trajectory along the 2nd degree of MR mobility.

The programmed trajectory of movement along the 2nd degree of mobility, corresponding to the section specified by expression (15), is obtained in the following form:

$$\left\{ \begin{array}{l} t = 5.124 : q_{2,11} = -0.05628 \text{ rad}, \\ \forall t \in [5.124, 5.686] : q_{2,11}^o = \\ = -0.05628 - 0.0205 \cdot t + \frac{0.4293t^2}{2}, \\ t = 5.686 : q_{2,12} = 0 \text{ rad}, \end{array} \right. \quad (26)$$

where $q_{2,11}^o$ is a polynomial describing the change in the generalized coordinate during acceleration in the 11th section of the programmed trajectory along the 2nd degree of MR mobility.

The programmed trajectory of movement along the 2nd degree of mobility, corresponding to the section specified by expression (16), is obtained in the following form:

$$\left\{ \begin{array}{l} t = 5.686 : q_{2,12} = 0 \text{ rad}, \\ \forall t \in [5.124, 5.686] : q_{2,12}^b = 0.2207 \cdot t - \frac{0.7853t^2}{2}, \\ t = 5.967 : q_{2,13} = 0.031 \text{ rad}, \end{array} \right. \quad (27)$$

where $q_{2,12}^b$ is a polynomial that describes the change in the generalized coordinate during braking on the 12th section of the programmed trajectory along the 2nd degree of MR mobility.

In parallel with the beginning of the process of accompanying the movement of the molds, the process of lowering the blades onto the surface of the melt was carried out to collect the oxide film (Fig. 4). This process is realized by changing the value of the generalized coordinate for the 3rd degree of MR mobility $q_{3,1}=0.1745$ rad to the value $q_{3,2}=0$ rad.

The increment of the generalized coordinate was obtained in the following form:

$$\Delta q_{3,1} = q_{3,2} - q_{3,1} = -0.1745 \text{ rad.}$$

In this case, acceleration is carried out first, then braking is performed with maximum acceleration. Then the time intervals and increments of the generalized coordinate for acceleration $\Delta q_{3,1}^o$ and deceleration $\Delta q_{3,1}^b$ are given by the following expression:

$$\Delta q_{3,1}^o = \frac{\Delta q_{3,1}}{2} = -0.08725 \text{ rad, } \Delta q_{3,1}^b = \frac{\Delta q_{3,1}}{2} = -0.08725 \text{ rad.}$$

Whence:

$$\Delta q_{3,1}^o = -\frac{\ddot{q}_3 (\Delta t_{3,1}^o)^2}{2}, \Delta t_{3,1}^o = 0.58 \text{ s,}$$

where $\Delta t_{3,1}^o$ is the time period for acceleration through the 3rd degree of MR mobility.

Then:

$$\dot{q}_3 = \ddot{q}_3 \Delta t_{3,1}^o = 0.303 \text{ rad/s,}$$

$$\dot{q}_3 - \ddot{q}_3 \Delta t_{3,1}^b = 0, \Delta t_{3,1}^b = 0.58 \text{ s,}$$

where $\Delta t_{3,1}^b$ is the time interval for braking according to the 3rd degree of MR mobility.

Consequently, the trajectory of movement along the 3rd degree of mobility is found in the following form:

$$\left\{ \begin{array}{l} t = 0.35: q_{3,1} = 0.1745 \text{ rad,} \\ \forall t \in [0.35, 0.93]: q_{3,1}^o = 0.1745 - \frac{0.5236t^2}{2}, \\ \forall t \in [0.93, 1.51]: q_{3,1}^b = 0.08725 - 0.303t + \frac{0.5236t^2}{2}, \\ t = 1.51: q_{3,2} = 0 \text{ rad,} \end{array} \right. \quad (28)$$

where $q_{3,1}^o, q_{3,1}^b$ are polynomials describing changes in generalized coordinates during acceleration and deceleration in 1 section of the programmed trajectory, respectively, according to the 3rd degree of MR mobility.

Next, the oxide film was collected from the surface of the melt due to the translational movement of the movable blade onto the rotary blade (Fig. 6, 7). This movement is realized by changing the value of the generalized coordinate for the 4th degree of MR mobility from $q_{4,1} = 0.605 \text{ m}$ to $q_{4,2} = 0 \text{ m}$, in this case it is obtained:

$$\Delta q_{4,1} = q_{4,2} - q_{4,1} = -0.605 \text{ m.}$$

In this case, first acceleration is carried out to a given speed, then movement is carried out at a given speed, and finally braking is implemented to zero speed. Acceleration is carried out from zero speed to a value $\dot{q}_4 = 0.3 \text{ m/s}$, since the movement of the movable blade at a higher speed could lead to splashing of the melt; in this case the following is obtained:

$$\dot{q}_4 = \ddot{q}_4 \Delta t_{4,1}^o, \Delta t_{4,1}^o = 1 \text{ s,}$$

where $\Delta t_{4,1}^o$ is the time interval for acceleration through the 4th degree of MR mobility.

Hence

$$\Delta q_{4,1}^o = -\frac{\ddot{q}_4 (\Delta t_{4,1}^o)^2}{2} = 0.15 \text{ m,}$$

where $\Delta q_{4,1}^o$ is the increment value of the generalized coordinate for acceleration through the 4th degree of MR mobility.

Having considered the braking process, in which the initial speed is equal to $\dot{q}_4 = 0.3 \text{ m/s}$, and the final speed is equal to zero, we obtained:

$$\dot{q}_4 = \ddot{q}_4 \Delta t_{4,1}^b, 0.3 = 0.3 \cdot \Delta t_{4,1}^b, \Delta t_{4,1}^b = 1 \text{ s,}$$

where $\Delta t_{4,1}^b$ is the time interval for braking according to the 4th degree of MR mobility.

Then:

$$\Delta q_{4,1}^b = \dot{q}_4 \Delta t_{4,1}^o - \frac{\ddot{q}_4 (\Delta t_{4,1}^b)^2}{2}, \Delta q_{4,1}^m = 0.15 \text{ m,}$$

where $\Delta q_{4,1}^b$ is the increment value of the generalized coordinate for braking according to the 4th degree of MR mobility.

In this case, the distance of movement at a given speed $\dot{q}_4 = 0.3 \text{ m/s}$ was equal to:

$$\Delta q_{4,1}^d = \Delta q_{4,1}^o - \Delta q_{4,1}^b - \Delta q_{4,1}^m = -0.305 \text{ m.}$$

Then:

$$-0.305 = -0.3 \cdot \Delta t_{4,1}^d, \Delta t_{4,1}^d = 1.01 \text{ s,}$$

where $\Delta t_{4,1}^d$ is the time interval for movement at a given speed $\dot{q}_4 = 0.3 \text{ m/s}$ along the 4th degree of MR mobility.

From here, the trajectory of movement along the fourth degree of mobility was obtained in the following form:

$$\left\{ \begin{array}{l} t = 1.51: q_{4,1} = 0.605 \text{ m,} \\ \forall t \in [1.51, 2.51]: q_{4,1}^o = 0.605 - \frac{0.3t^2}{2}, \\ \forall t \in [2.51, 3.52]: q_{4,1}^d = 0.455 - 0.3t, \\ \forall t \in [3.52, 4.52]: q_{4,1}^b = 0.15 - 0.3t + \frac{0.3t^2}{2}, \\ t = 4.52: q_{4,2} = 0 \text{ m,} \end{array} \right. \quad (29)$$

where $q_{4,1}^o, q_{4,1}^d, q_{4,1}^b$ are polynomials that describe changes in generalized coordinates during acceleration, movement at a given speed and braking on 1 section of the programmed trajectory, respectively, along the 4th degree of MR mobility.

At the next step, the process of lifting the blades was carried out (Fig. 8), then the programmed trajectory of movement along the 3rd degree of mobility, similar to the derivation of expression (28), was obtained in the following form:

$$\left\{ \begin{array}{l} t = 4.52: q_{3,2} = 0 \text{ rad,} \\ \forall t \in [4.52, 5.1]: q_{3,2}^o = \frac{0.5236t^2}{2}, \\ \forall t \in [5.1, 5.68]: q_{3,2}^b = \\ = 0.08725 + 0.303t - \frac{0.5236t^2}{2}, \\ t = 5.68: q_{3,3} = 0.1745 \text{ rad,} \end{array} \right. \quad (30)$$

where $q_{3,2}^o$, $q_{3,2}^b$ are polynomials describing changes in generalized coordinates during acceleration and deceleration in section 2 of the programmed trajectory, respectively.

Next, the reverse translational movement of the movable blade is performed, in this case the programmed trajectory of movement along the 4th degree of freedom is obtained similar to expression (29), in the following form:

$$\left\{ \begin{array}{l} t = 5.68: q_{4,2} = 0 \text{ m}, \\ \forall t \in [5.68, 6.68]: q_{4,2}^o = \frac{0.3t^2}{2}, \\ \forall t \in [6.68, 7.69]: q_{4,2}^d = 0.15 + 0.3t, \\ \forall t \in [7.69, 8.69]: q_{4,2}^b = 0.455 + 0.3t - \frac{0.3t^2}{2}, \\ t = 8.69: q_{4,3} = 0.605 \text{ m}, \end{array} \right. \quad (31)$$

where $q_{4,2}^o$, $q_{4,2}^d$, $q_{4,2}^b$ are polynomials that describe changes in generalized coordinates during acceleration, movement at a given speed and braking on the 2nd section of the programmed trajectory, respectively, along the 4th degree of MR mobility.

In parallel with the movement (31) along the 4th degree of mobility, a movement was performed along the 5th degree of mobility, turning counterclockwise from the angle $q_{5,1} = 0$ rad to the angle $q_{5,2} = 1.36$ rad (Fig. 9). In this case, the time of the beginning of movement along a given degree of mobility must correspond to the condition of mutual non-collision of the rotating and moving blades. The trajectory of movement along the 5th degree of MR mobility, which has a pneumatic drive, was obtained in the following form:

$$\left\{ \begin{array}{l} t = 6.0: q_{5,1} = 0 \text{ rad}, \\ \forall t \in [6.0, 6.43]: q_{5,1}^{tc} = 3.14t, \\ t = 6.43: q_{5,2} = 1.36 \text{ rad}, \end{array} \right. \quad (32)$$

where $q_{5,1}^{tc}$ is a polynomial that describes the change in the generalized coordinate when turning counterclockwise on the 1st section of the programmed trajectory along the 5th degree of MR mobility.

Next, movement was performed along the 6th degree of mobility, turning counterclockwise from the angle $q_{6,1} = -1.57$ rad to the angle $q_{6,2} = 1.57$ rad (Fig. 10). This ensures that the oxide film is discharged into a special container. Next, a clockwise rotation was performed to a value of $q_{6,3} = -1.57$ rad. The trajectory of movement along the 6th degree of MR mobility, which has a pneumatic drive, was obtained in the following form:

$$\left\{ \begin{array}{l} t = 6.43: q_{6,1} = -1.57 \text{ rad}, \\ \forall t \in [6.43, 7.43]: q_{6,1}^{rc} = -1.57 + 3.14t, \\ t = 7.43: q_{6,2} = 1.57, \\ \forall t \in [7.43, 8.43]: q_{6,2}^{rc} = 1.57 - 3.14t, \\ t = 8.43: q_{6,3} = -1.57 \text{ rad}, \end{array} \right. \quad (33)$$

where $q_{6,1}^{rc}$, $q_{6,2}^{rc}$ are polynomials that describe changes in generalized coordinates when turning counterclockwise on the 1st section of the programmed trajectory and when turning clockwise on the 2nd section of the programmed trajectory, respectively, according to the 6th degree of MR mobility.

At the next step, movement is performed along the 5th degree of mobility, turning clockwise, similar to expression (32), the programmed trajectory of which was obtained in the following form:

$$\left\{ \begin{array}{l} t = 8.43: q_{5,2} = 1.36 \text{ rad}, \\ \forall t \in [8.43, 8.86]: q_{5,2}^{rc} = 1.36 - 3.14t, \\ t = 8.86: q_{5,3} = 0 \text{ rad}, \end{array} \right. \quad (34)$$

where $q_{5,2}^{rc}$ is a polynomial that describes the change in the generalized coordinate when turning counterclockwise on the 2nd section of the programmed trajectory along the 5th degree of MR mobility.

Then, in parallel with the movement along the 5th and 6th degrees of freedom, movement along the 1st degree of freedom was carried out, from the value $q_{1,13} = -0.31$ rad to the value $q_{1,14} = 0.6047$ rad. This ensured the transition at this degree of mobility to the initial state. In this case, acceleration is performed first, then deceleration is performed with maximum acceleration. Then the increment of generalized coordinates was obtained in the following form:

$$\Delta q_{1,13}^o = \Delta q_{1,13}^b = \frac{0.6357}{2} = 0.31785 \text{ rad},$$

hence:

$$\Delta q_{1,13}^o = 0.31785 = \frac{0.7853(\Delta t_{1,13}^o)^2}{2},$$

then:

$$\Delta t_{1,13}^o = 0.9 \text{ s},$$

where $\Delta t_{1,13}^o$ is the time period for acceleration through 1 degree of MR mobility.

To implement braking at this degree of mobility, it is necessary:

$$\dot{q}_{1,13}^b = 0.7853\Delta t_{1,13}^o = 0.7067 \text{ rad/s},$$

hence:

$$0.7067 - 0.7853\Delta t_{1,13}^b = 0,$$

then:

$$\Delta t_{1,13}^b = 0.9 \text{ s},$$

where $\Delta t_{1,13}^b$ is the time period for braking according to the 1st degree of MR mobility.

Consequently, the trajectory of movement along the 1st degree of mobility was obtained in the following form:

$$\left\{ \begin{array}{l} t = 8.0: q_{1,13} = -0.031 \text{ rad}, \\ \forall t \in [8.0, 8.9]: q_{1,13}^o = \frac{0.7853t^2}{2}, \\ \forall t \in [8.9, 9.8]: q_{1,13}^b = 0.31785 + 0.7067t - \frac{0.7853t^2}{2}, \\ t = 9.8: q_{1,14} = 0.6047 \text{ rad}, \end{array} \right. \quad (35)$$

where $q_{1,13}^o$, $q_{1,13}^b$ are polynomials that describe changes in generalized coordinates during acceleration and deceleration in the 13th section of the programmed trajectory, respectively, according to the 1st degree of MR mobility.

Next, the transition to the initial position was carried out according to the 2nd degree of mobility, that is, the generalized

coordinate was changed from the value $q_{2,13}=0.031$ rad to the value $q_{2,14}=-0.6047$ rad, similar to expression (35), the programmed trajectory of which was obtained in the following form:

$$\begin{cases} t = 8.0: q_{2,13} = 0.031 \text{ rad,} \\ \forall t \in [8.0, 8.9]: q_{2,13}^o = 0.031 - \frac{0.7853t^2}{2}, \\ \forall t \in [8.9, 9.8]: q_{2,13}^b = -0.31785 - 0.7067t + \frac{0.7853t^2}{2}, \\ t = 9.8: q_{2,14} = -0.6047 \text{ rad,} \end{cases} \quad (36)$$

where $q_{2,13}^o, q_{2,13}^b$ are polynomials that describe changes in generalized coordinates during acceleration and deceleration in the 13th section of the programmed trajectory, respectively, according to the 2nd degree of MR mobility.

Thus, the removal of the oxide film from the surface of the poured magnesium melt poured into the molds of the casting conveyor is ensured.

5. 3. Modeling of programmed trajectories according to the degrees of mobility of a manipulation robot using MATLAB

To check the reliability of the resulting expressions describing the programmed trajectories according to the degrees of MR mobility, they were simulated in the MATLAB software environment. The first two degrees of MR mobility are used to accompany constantly moving molds and orient the moving and rotating blades along the mold.

Accompaniment of linearly moving molds is carried out by rotational movement along the 1st degree of mobility. Acceleration to a given speed, with further accompaniment of the moving molds of the casting conveyor, corresponds to the MP configuration shown in Fig. 5, with the value of the generalized coordinate $q_{1,1}=0.61$ rad. The moving molds are accompanied until the oxide film is completely collected from the surface of the magnesium melt; in this case, MR configuration is shown in Fig. 7.

Modeling of the software trajectory of the 1st degree of mobility by position was performed using the Tr_q1.m program. Speed simulation was performed using the Tr_speed_q1.m program. Acceleration modeling was performed using the Tr_acceleration_q1.m program. These programs are developed based on expressions (4), (6) to (16), (35).

The results of modeling the programmed trajectory for the 1st degree position are shown in Fig. 11. As can be seen from Fig. 11, the programmed trajectory is a continuous curve in which there are sections of acceleration, deceleration, and a stationary state. These sections are consistent with each other in terms of the magnitude of displacement, speed, and acceleration at the nodal points of the movement trajectory.

The results of modeling the programmed trajectory for speed of the 1st degree are shown in Fig. 12. As can be seen from Fig. 12, the programmed trajectory represents piecewise linear segments that are conjugated at the nodal points of the trajectory. These segments correspond to areas with an increase or decrease in the speed of movement for a given degree of mobility. At nodal points, the direction of movement speed is reversed for a given degree of mobility. There is also a section of the trajectory where the speed is zero.

The results of modeling the programmed trajectory for acceleration of the 1st degree are shown in Fig. 13. As can be seen from Fig. 13, the programmed trajectory consists of

piecewise linear segments that change abruptly at the nodal points of the trajectory. These segments correspond to the acceleration and deceleration sections for a given degree of mobility. At the nodal points there is an abrupt change in the sign and magnitude of the acceleration for a given degree of mobility. There is also a section of the trajectory where the acceleration is zero.

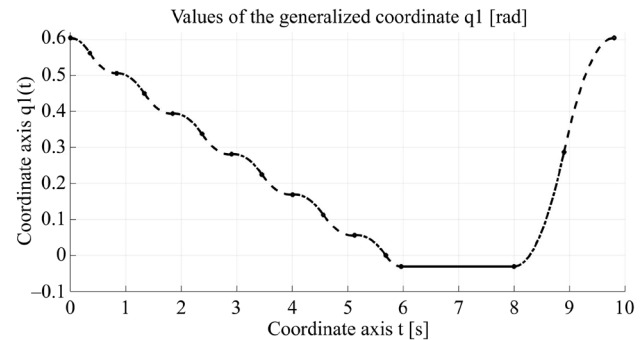


Fig. 11. Programmed trajectory according to the position of the first degree of motion of the manipulation robot

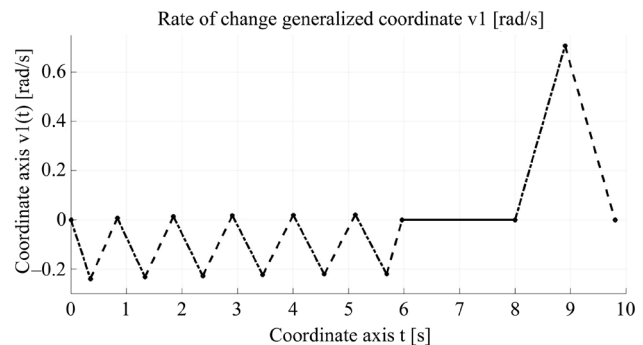


Fig. 12. Programmed trajectory based on the speed of the first degree of mobility of a manipulation robot

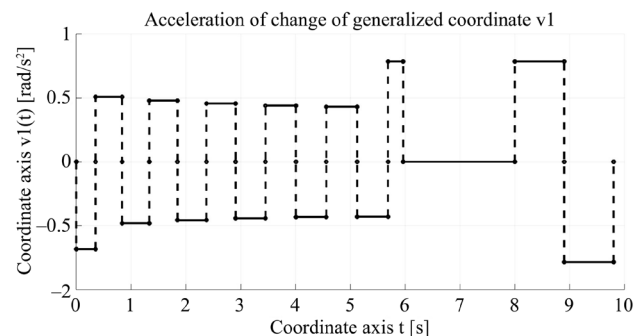


Fig. 13. Programmed trajectory to accelerate the first degree of mobility of a manipulation robot

Since the 1st degree of mobility compensates for the continuous linear movement of the molds of the foundry conveyor, it is necessary to evaluate the accuracy of this process. To solve this problem, the program Error_Tr_q1 was developed. The simulation results are shown in Fig. 14. As can be seen from Fig. 14, the error in tracking the movement of the molds coincides at the nodal points and varies between the values of the nodal points. The maximum error for a given choice of the number of nodal points of the trajectory is 0.007 m. This is a fairly large value.

The rotational movement along the 2nd degree of freedom enables the orientation of MR blades along the moving molds of the casting conveyor. The initial value of the generalized coordinate for the 2nd degree of mobility is equal to $q_{2,1} = -0.61$ rad (Fig. 5). The final position of the 2nd degree of mobility with complete collection of the oxide film from the surface of the magnesium melt corresponds to Fig. 7. Due to the equality of the value of the generalized coordinate of the 2nd degree of freedom with the opposite sign to the value of the generalized coordinate for the 1st degree of freedom, the orientation of MR blades along the moving molds of the foundry conveyor is ensured.

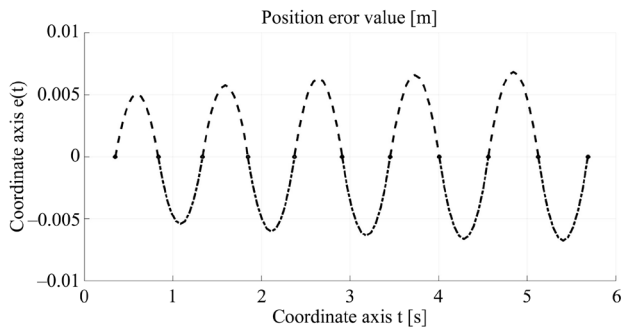


Fig. 14. Plot of change in the tracking error by the first degree of mobility of the manipulation robot of the linear movement of the molds of the casting conveyor

Modeling of the programmed trajectory of the 2nd degree of mobility in position was performed using the Tr_q2.m program, in speed using the Tr_speed_q2.m program, and in acceleration using the Tr_acceleration_q2.m program. These programs are developed based on expressions (5), (17) to (27), (36).

The results of modeling the programmed trajectory for the 2nd degree position are shown in Fig. 15. As can be seen from Fig. 15, this programmed trajectory is similar to the programmed trajectory for 1 degree of mobility presented in Fig. 11 and differs only in the opposite direction of the continuous curve in identical sections of the motion trajectory.

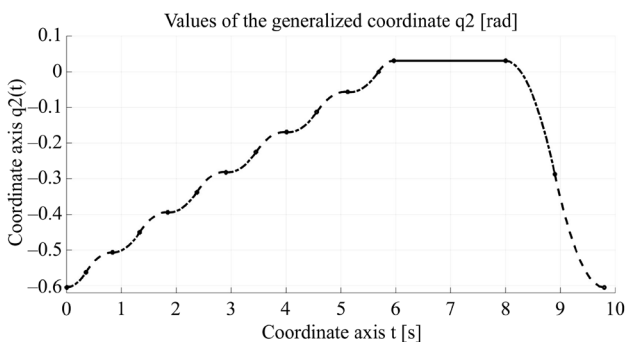


Fig. 15. Programmed trajectory according to the position of the second degree of mobility of the manipulation robot

The results of modeling the programmed trajectory for speed of the 2nd degree are shown in Fig. 16. As can be seen from Fig. 16, the programmed trajectory is similar to the programmed trajectory along the 1st degree of freedom presented in Fig. 12. It differs only in the opposite direction of the speed change in the same sections of the movement trajectory. This ensures the required orientation of MR blades.

The results of modeling the programmed trajectory for acceleration of the 2nd degree are shown in Fig. 17. As can be seen from Fig. 17, the programmed trajectory is similar to the programmed trajectory along the 1st degree of freedom presented in Fig. 13, it differs only in the opposite sign of the acceleration values in the considered sections of the motion trajectory.

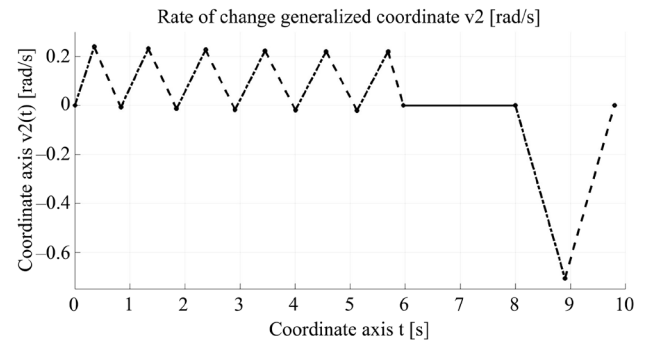


Fig. 16. Programmed trajectory based on the speed of the second degree of mobility of a manipulation robot

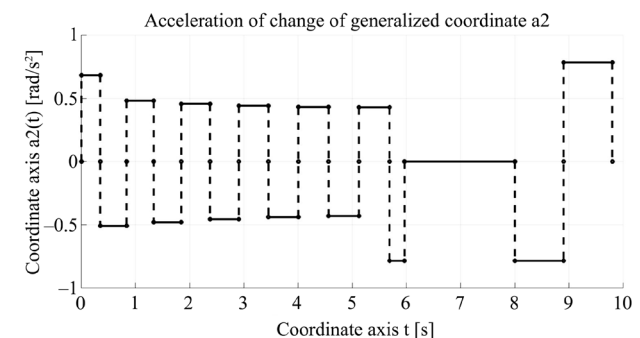


Fig. 17. Programmed trajectory to accelerate the second degree of mobility of a manipulation robot

This ensures the required orientation of MR blades relative to the moving molds of the casting conveyor.

The lowering and lifting of the blades to collect the oxide film from the surface of the magnesium melt is carried out by a rotary movement in the 3rd degree of mobility. The lowering of the blades onto the surface of the magnesium melt poured into the molds of the casting conveyor is carried out by turning counterclockwise to the 3rd degree of MR mobility (Fig. 4). After the oxide film is collected from the surface of the magnesium melt poured into the molds, turning clockwise in the 3rd degree of mobility ensures the lifting of the blades with the collected oxide film (Fig. 8).

Modeling of the programmed trajectory of the 3rd degree of mobility in position was performed using the Tr_q3.m program, in speed using the Tr_speed_q3.m program, and in acceleration using the Tr_acceleration_q3.m program. These programs are developed based on expressions (28), (30).

The results of modeling the programmed trajectory for the 3rd degree position are shown in Fig. 18. As can be seen from Fig. 18, the programmed trajectory is a continuous curve in which there are sections of acceleration, deceleration, and a stationary state. This ensures that the blades are lowered onto the surface of the magnesium melt, and after collecting the oxide film, the blades are lifted above the surface of the magnesium melt.

The results of modeling the programmed trajectory for speed of the 3rd degree are shown in Fig. 19. As can be seen

from Fig. 19, the programmed trajectory consists of piecewise linear segments that are conjugated at the nodal points of the trajectory. At nodal points, the direction of movement speed is reversed for a given degree of mobility. There are also three sections of the trajectory where the speed is zero.

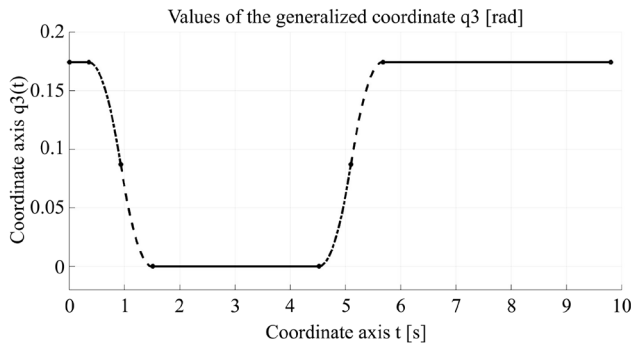


Fig. 18. Programmed trajectory according to the position of the third degree of mobility of the manipulation robot

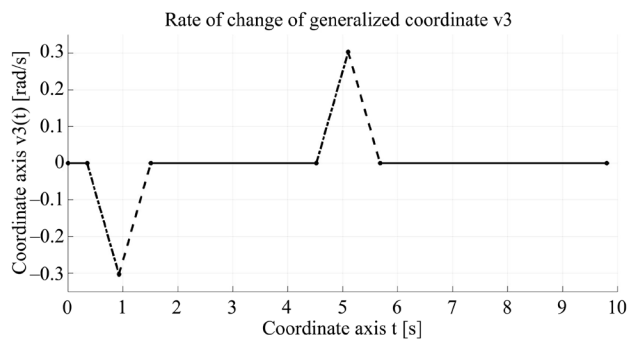


Fig. 19. Programmed trajectory based on the speed of the third degree of mobility of the manipulation robot

The results of modeling the programmed trajectory for acceleration of the 3rd degree are shown in Fig. 20. As can be seen from Fig. 20, the programmed trajectory consists of piecewise linear segments that change abruptly at the nodal points of the trajectory. At the nodal points there is an abrupt change in the sign and magnitude of the acceleration for a given degree of mobility. There are also 3 sections of the trajectory where the acceleration is zero.

By linear movement along the 4th degree of MR mobility, the oxide film from the surface of the magnesium melt is collected by a movable blade onto a rotating blade (Fig. 6, 7). After the oxide film is collected from the surface of the magnesium melt poured into the molds, the blades with the collected oxide film are lifted (Fig. 8). Then, with a linear movement along the 4th degree of mobility, the movable blade returns to its original position (Fig. 3).

Modeling of the programmed trajectory of the 4th degree of mobility in position was performed using the Tr_q4.m program, in speed using the Tr_speed_q4.m program, and in acceleration using the Tr_acceleration_q4.m program. These programs are developed based on expressions (29), (31).

The results of modeling the programmed trajectory for the 4th degree position are shown in Fig. 21. As can be seen from Fig. 21, the programmed trajectory is a continuous curve in which there are sections of acceleration, movement at a given speed, braking, and a stationary state. This ensures the collection of the oxide film from the surface of the magnesium melt; after collecting the oxide film, lifting the blades

above the surface of the magnesium melt, the moving blade is moved to its original state.

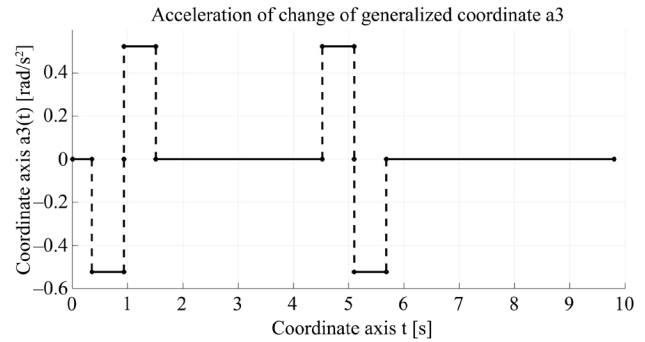


Fig. 20. Programmed trajectory for accelerating the third degree of mobility of a manipulation robot

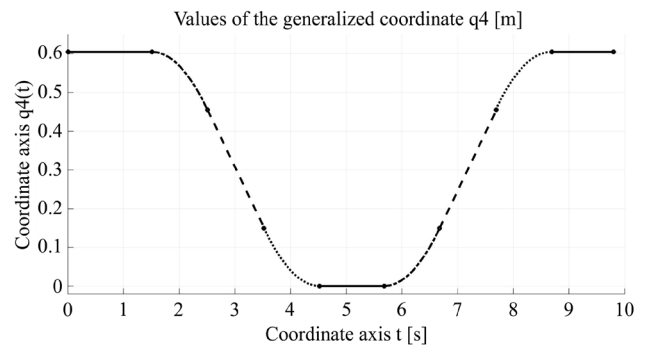


Fig. 21. Programmed trajectory at the position of the fourth degree of mobility of the manipulation robot

The results of modeling the programmed trajectory for speed of the 4th degree are shown in Fig. 22. As can be seen from Fig. 22, the programmed trajectory is piecewise linear segments that correspond to sections of increasing speed, moving at a given speed, and decreasing speed. These sections are mated to each other at the nodal points of the trajectory. There are also three sections of the trajectory where the speed is zero.

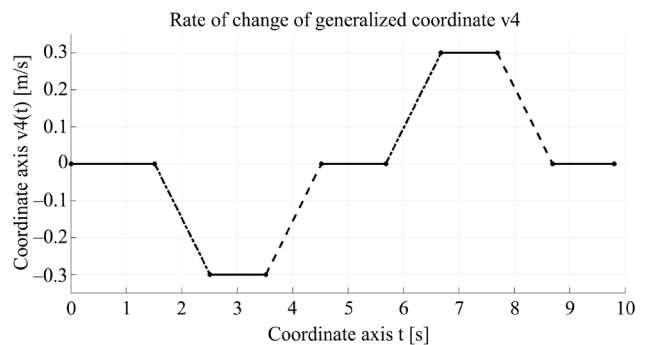


Fig. 22. Programmed trajectory based on the speed of the fourth degree of mobility of a manipulation robot

The results of modeling the programmed trajectory for acceleration of the 4th degree are shown in Fig. 23. As can be seen from Fig. 23, the programmed trajectory represents piecewise linear segments corresponding to the process of removing the oxide film from the surface of the magnesium melt and returning the movable blade to its original position. There are also 5 sections of the trajectory where the acceleration is zero.

By rotating clockwise at the 5th degree of MR mobility, the collected oxide film on the rotary blade rises above the level of the magnesium melt (Fig. 9). Then, after dumping the oxide film into a special container, turning it counter-clockwise, it returns to its original position (Fig. 3).

Modeling of the programmed trajectory of the 5th degree of mobility by position was performed using the Tr_q5.m program, and by speed using the Tr_speed_q5.m program. These programs are developed based on expressions (32), (34). Since the drive of the 5th degree of mobility MR is pneumatic, the movement in this case is carried out from lock to lock, which is not controlled by the magnitude of acceleration. Therefore, modeling the acceleration of this degree of mobility is not feasible.

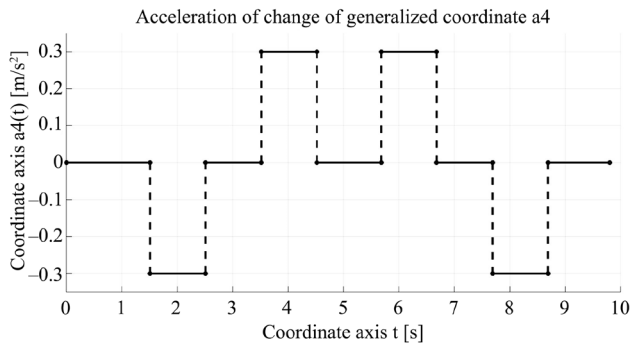


Fig. 23. Programmed trajectory to accelerate the fourth degree of mobility of a manipulation robot

The results of modeling the programmed trajectory for the 5th degree position are shown in Fig. 24. From Fig. 24, the programmed trajectory consists of piecewise linear segments, in which there are sections of clockwise rotation, counter-clockwise rotation and a stationary state. This ensures that the collected oxide film rises above the surface of the magnesium melt and returns to its original state.

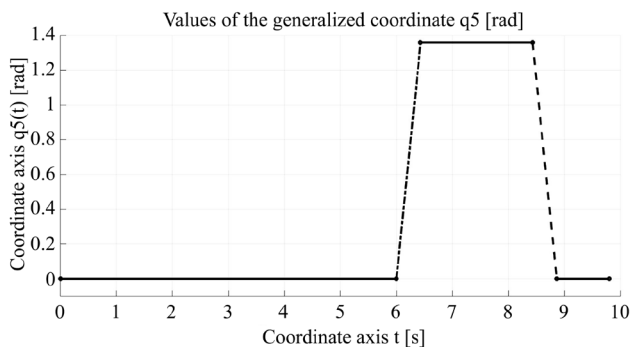


Fig. 24. Programmed trajectory according to the position of the fifth degree of mobility of the manipulation robot

The results of modeling the programmed trajectory for speed of the 5th degree are shown in Fig. 25. As can be seen from Fig. 25, the programmed trajectory is piecewise linear segments that correspond to the speeds of clockwise rotation, counter-clockwise rotation, and stationary state. These sections are mated to each other at the nodal points of the trajectory. There are also three sections of the trajectory where the speed is zero.

By rotating clockwise at the 6th degree of MR mobility, the collected oxide film is dumped onto the rotary blade into a special container (Fig. 10). Then, by turning it counter-clockwise, it returns to its original position (Fig. 3).

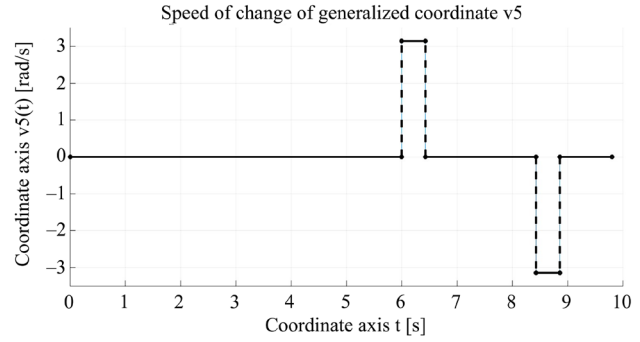


Fig. 25. Programmed trajectory at the speed of the fifth degree of mobility of a manipulation robot

Modeling of the programmed trajectory of the 6th degree of mobility by position was performed using the Tr_q6.m program, and by speed using the Tr_speed_q6.m program. These programs are developed based on expression (33). Since the drive of the 6th degree of mobility is pneumatic, modeling for the acceleration of this degree of mobility is not feasible.

The results of modeling the programmed trajectory for the 6th degree position are shown in Fig. 26. From Fig. 26, the programmed trajectory consists of piecewise linear segments, in which there are sections of clockwise rotation, counter-clockwise rotation and a stationary state. This ensures that the collected oxide film is discharged into a special container and returned to its original state.

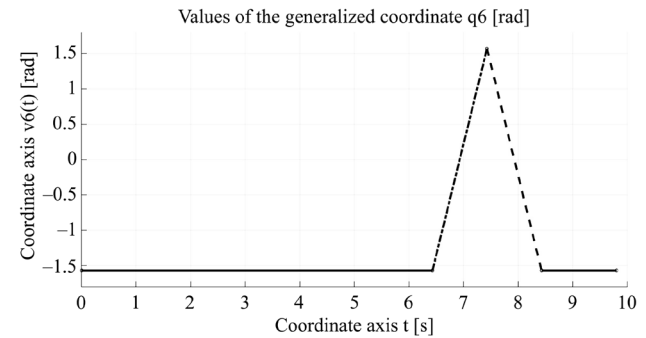


Fig. 26. Programmed trajectory at the position of the sixth degree of mobility of the manipulation robot

The results of modeling the programmed trajectory for speed of the 6th degree are shown in Fig. 27. The programmed trajectory in terms of speed of this degree of mobility is similar to the programmed trajectory in terms of speed of the 5th degree of MR mobility.

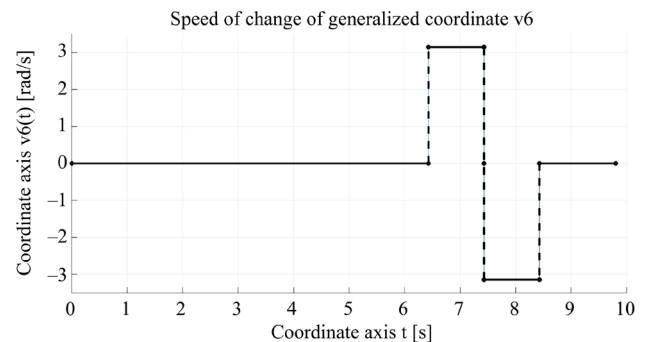


Fig. 27. Programmed trajectory at the speed of the sixth degree of mobility of the manipulation robot

The resulting plots of programmed trajectories according to the degrees of MR mobility make it possible to perform technological operation of removing the oxide film from the surface of the magnesium melt poured into the moving molds of the casting conveyor. Based on our results, it is possible to construct an MR control cyclogram for performing technological operation of the removal of the oxide film during the production of commercial magnesium.

5. 4. Construction of a cyclogram for controlling a manipulation robot for removing an oxide film from the surface of a magnesium melt

Summarizing expressions (4) to (36), as well as the results of modeling (Fig. 11, 15, 18, 21, 24, 26), a cyclogram for controlling the MR mobility for removing the oxide film has been constructed, which is shown in Fig. 28. The ordinate axis shows transitions in degrees of MR mobility. The abscissa axis represents time t , s.

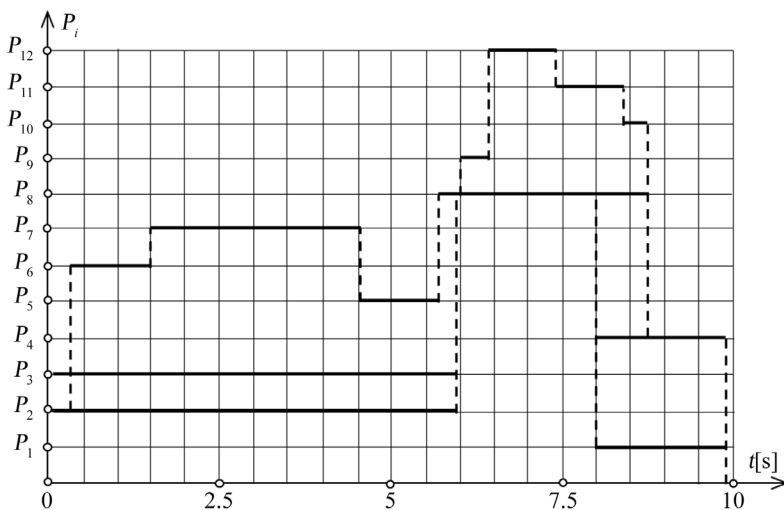


Fig. 28. Cyclogram of control by degrees of mobility of the manipulation robot for removing the oxide film: P_1 – counterclockwise rotation of the 1st degree of mobility, P_2 – clockwise rotation of the first 1st degree of mobility, P_3 – counterclockwise rotation of the 2nd degree of mobility, P_4 – clockwise rotation of the 2nd degrees of mobility, P_5 – counterclockwise rotation of 3 degree of freedom, P_6 – clockwise rotation of 3 degree of freedom, P_7 – retraction of movable blades of 4 degree of freedom, P_8 – extension of movable blades, 4 degree of freedom, P_9 – counterclockwise rotation, 5 degree of freedom, P_{10} – clockwise rotation of the 5th degree of freedom, P_{11} – counterclockwise rotation of the 6th degree of freedom, P_{12} – clockwise rotation of the 6th degree of freedom of the manipulator

As can be seen from Fig. 28, the full cycle of the process of removing the oxide film from the surface of the magnesium melt requires 9.8, which is enough to remove the oxide film from the surface of two moving molds. The time limit is 10 s.

6. Discussion of results of robotization of the process of removing the oxide film from the surface of a magnesium melt

Based on taking into account the features of the technical process of removing the oxide film from the surface of a magnesium melt poured into moving molds of a foundry conveyor, a kinematic structure of a two-armed MR having 6 degrees of freedom is proposed. This kinematic structure

of MR was proposed in [4] for removing the oxide film from the surface of lead and zinc melts poured into the molds of a carousel casting machine. In this case, the oxide film is removed from the surface of the melt poured into the stationary mold of a rotary casting machine. There is no time limit on performing this operation. Programmed trajectories in terms of degrees of mobility are determined by restrictions on the positions specified by the relative position of MR relative to the rotary casting machine, on the speed values that enable the removal of the oxide film without splashing the melt poured into the mold [4].

In the case of removing the oxide film from the surface of the magnesium melt, the molds of the casting conveyor move continuously at a given speed and an additional time limitation is also imposed on the performance of this operation. To increase the time limit, it is proposed to remove the oxide film simultaneously from the surfaces of melts poured into two molds. In this case, the time limit is 10 seconds. In the case of a single mold, the time limit is 5 seconds, which is not enough to complete the entire operation under consideration.

To remove the oxide film from the surface of a magnesium melt poured into linearly moving molds of a foundry conveyor, the kinematic structure of MR proposed in [4] was linked to the foundry conveyor for the production of commercial magnesium. It is proposed to compensate for the linear movement of the molds due to the clockwise rotational movement of the first degree of MR mobility. At the same time, counterclockwise movement with the same values of speeds and accelerations along the second degree of MR mobility ensures the orientation of the blades perpendicular to the linear movement of the molds of the casting conveyor.

The programmed trajectory along the first degree of mobility represents a rotational movement along a circular arc, which provides support for the linearly moving molds of the casting conveyor. To determine sections of the programmed trajectory by position, the values of the nodal points were determined, as well as the time intervals for accelerating to a given speed, accompanying moving molds, and braking linearly moving molds of a casting conveyor.

Movement along a circular arc along the first degree of mobility coincides with the linear movement of the molds at the nodal points and does not coincide between the values of the nodal points. Next, the accuracy of reproducing a linear trajectory due to rotational motion along the line of a circular arc was assessed. Based on the obtained programmed trajectories according to the degree of MR mobility, a cyclogram for controlling MR was constructed for removing the oxide film from the surface of the magnesium melt poured into the moving molds of the foundry conveyor.

The same functions are also performed by the kinematic structure of MR, which has 4 degrees of mobility, proposed in [3]. MR under consideration contains 3 translational and one rotational joints. A distinctive feature of the proposed kinematic structure is the replacement of a translational

hinge designed to compensate for the linear movement of the casting conveyor molds with two rotational hinges. Also, replacing the translational hinge for lifting the blades with a rotary hinge that performs the same function. It should be noted that rotational hinges are structurally simpler than translational hinges and can realize fairly high values of developed moments and forces [5].

A distinctive feature of this study is the consideration of technical support for robotization, which is not applicable to common approaches using commercially produced industrial robots. This is due to the complexity of the considered technological operation, which requires collecting the oxide film from the surface of the magnesium melt poured into the moving molds of the casting conveyor. To perform this TO, a kinematic structure of a two-armed MR is proposed. A distinctive feature of this MR is the presence of degrees of mobility that compensate for the movement of the molds, as well as degrees of mobility that enable the collection and discharge of the oxide film into a special container.

For this reason, these restrictions make it possible, on the one hand, to simplify the development of programmed trajectories by position, speed, acceleration, and obtain their analytical description. In this case there is no need to search for an optimal solution, in contrast to works [17–19]. There is no need to search for a solution to the problem in the presence of uncertainties in MR workspace [6, 7]. Or the development of programmed trajectories in real time since trajectories are subject to change [6–10].

To implement the technological operation of removing the oxide film from the surface of the magnesium melt [1, 2], the program control system (PCS) for 1 and 2 degrees of MR mobility must be contour since when performing this technological operation, it is necessary to continuously monitor the position of the moving molds. According to the 3rd degree of mobility, MR SPUs must be positional since it is necessary to smoothly lower the blades into the magnesium melt and smoothly lift the blades with the collected oxide film. According to the 4th degree of mobility, MR SPU must be contour since there is a linear movement of movable blade 8 along the mold to collect oxide film 10 onto the rotary blade. For degrees of freedom 5 and 6, MR SPU must be cyclic since the rotational movement of the rotary blades is carried out to discharge the collected oxide film into the container. Taking these circumstances into account, the first four degrees of mobility should have an electric drive, and the 5th and 6th degrees of mobility MR should have a pneumatic drive [5].

Since linear movement is compensated by rotational movement, that is, the linear speed of the conveyor belt is compensated by the angular speed of rotation in 1 degree of mobility. This leads to a mismatch between the position of the mold and the position of the movable and rotating MR blades. The magnitude of the mismatch in the considered case when specifying a rectilinear trajectory with 10 nodal points was 0.007 m. This limits the use of the developed programmed trajectories for practical implementation when MR performs the process under consideration.

A further direction of research to improve the accuracy of reproducing the rectilinear trajectory of movement of blades at a given speed can be carried out by increasing the number of nodal points of the trajectory of the movement of the blades [5]. Of interest is the option of increasing the number of nodal points to 20 or 30, with determining the accuracy of the rectilinear movement of the movable blade.

The process of collecting the oxide film from the surface of the magnesium melt is carried out by the translational movement of the movable blade onto the rotary blade. Next, the collected oxide film located between the movable and rotating blades is lifted from the surface of the magnesium melt (Fig. 8, 9). Further reversal of the moving blade may not guarantee that some fragments of the oxide film will fall from the rotating blade back into the mold. This will require choosing a rotating blade shape that will avoid this occurrence. This drawback will require additional experimental research in the field of hydrodynamics of the process of collecting the oxide film from the surface of the magnesium melt.

The process of dumping the oxide film into a special container also has its own difficulties (Fig. 9, 10). It is necessary to enable complete removal of the oxide film; this may not happen due to the adhesion of the oxide film to the surface of the rotary blade. Therefore, it is necessary to select a material for the manufacture of a rotating blade on which the magnesium oxide film does not stick.

In this work, programmed trajectories are approximated by quadratic polynomials. In further research, programmed trajectories can be approximated by cubic polynomials. In this case, it is possible to enable the continuity of program acceleration trajectories. Based on a comparative analysis of errors in reproducing a given trajectory, it will be possible to draw conclusions about the advantages of this approach.

Programmed trajectories are developed on the basis of laws of kinematics, which assume that the mechanism of the manipulator and the drive of the degrees of mobility are ideal. Therefore, in the future it will be of interest to study the reproduction of trajectories taking into account the use of a real drive, as well as the presence of errors in the manipulator mechanism. This will make it possible to more accurately evaluate the proposed kinematic structure from the point of view of its implementation in the form of an MR.

One of the areas for the development of this research is the task of reducing the speed of movement of the moving blade when collecting the oxide film. This guarantees the possible appearance of waves on the surface of the magnesium melt and its splashing. To do this, one can increase the reverse speed to perform a return to the initial position according to the degrees of MR mobility. In this case, it will be necessary to develop programmed trajectories according to the degrees of MR mobility, taking into account the new statement of the problem.

It is also possible to further complicate the kinematic structure of MR, for example, replacing the translational joint with two rotational joints, as in Delta robots [9, 13]. This will lead to the need to develop programmed trajectories for the added degrees of MR mobility. At the same time, it should be noted that it is difficult to implement due to rotational movements and enable high accuracy of reproducing the rectilinear trajectory of movement of the rotary blade for collecting the oxide film from the surface of the magnesium melt.

The process of casting magnesium melt occurs in an aggressive environment, in the presence of hydrochloric acid vapor. Therefore, it is necessary to conduct experimental studies of the proposed version of the kinematic structure of MR, the design of the elements of which must satisfy the condition of resistance to an aggressive external environment.

Experimental studies of the use of this kinematic structure for removing the oxide film from the surface of a magnesium melt have not been carried out. In the future, the results of this study could be used for robotization of foundry production of commercial magnesium.

7. Conclusions

1. To robotize the removal of the oxide film from the surface of the magnesium melt poured into the moving molds of a foundry conveyor, a kinematic structure of a two-armed MR with 6 degrees of freedom is proposed. Its features are that the first two degrees of mobility are designed to compensate for the speed of movement of the molds of the foundry conveyor. The third degree of mobility is intended for raising MR arms. The fourth degree of mobility is designed to collect the oxide film from the surface of the magnesium melt. The fifth and sixth degrees of mobility are for dumping the collected oxide film into a special container.

2. Determination of the laws of change in the generalized coordinate along 6 degrees of MR mobility to perform technological operation of the removal of the oxide film from the surface of the magnesium melt poured into the moving molds of the casting conveyor, in the form of quadratic polynomials. According to the first degree, the law of change of the generalized coordinate is determined from the condition of ensuring the support of linearly moving molds with a given speed $v_k=0.04$ m/s. The linear movement of the molds is compensated by the rotational movement of the first degree of freedom clockwise, that is, movement along a circular arc. The resulting arc is divided into 10 sections using 11 nodal points. Next, the law of change of the generalized coordinate in all 10 sections is determined from the condition of equality at the nodal points of the values of the generalized coordinate, velocities, and accelerations of movement along the circular arc and linear movement of the molds. The second degree of mobility ensures the orientation of the blades perpendicular to the axis of linear movement of the casting conveyor molds. The law of change of the generalized coordinate along the second degree of mobility coincides with the law of change of the generalized coordinate along the first degree of mobility. In this case, the rotational movement in the second degree of freedom is performed counterclockwise. The law of changing the generalized coordinate along the third degree of mobility ensures that the blades are lowered onto the surface of the magnesium melt by a counterclockwise rotational movement, as well as the blades with the collected oxide film are raised by a clockwise rotational movement. The law of change according to the fourth degree of mobility ensures the translational movement of the movable blade to collect the oxide film from the surface of the magnesium melt, and also ensures the return of the movable blade to its original position. The fifth and sixth degrees of mobility of the manipulation robot are pneumatically driven. This drive is controlled only by changing the rotation speed according to these degrees of freedom. The laws of change for these degrees of mobility are determined from the condition of performing counterclockwise rotational movements when discarding the collected oxide film, as well as clockwise rotational movements when returning to the original position.

3. The reliability of the obtained laws of change in the generalized coordinate, the values of velocities and accelerations according to the degrees of mobility of the manipulation robot was assessed by modeling in MATLAB. By modeling in MATLAB, plots of the laws of change of the generalized coordinate for 6 degrees of mobility of the manipulation robot were built. Based on them, it was established that the process of accompanying moving molds is carried out over a time interval $t_1 \in [0, 5.967]$, s. Further on the time interval $t_1 \in [5.967, 8.0]$, s, no changes occur in this degree of mobility.

Then, at time interval $t_1 \in [8.0, 9.8]$, s, a return to the initial state is carried out. The process of accompanying moving molds of a foundry conveyor consists of acceleration and deceleration sections. This ensures equality of positions, velocities, and accelerations at the nodal points of the trajectory of movement along a given degree of mobility. Return to the starting position is carried out by acceleration and deceleration at a given degree of mobility. Since the accompaniment of the linear movement of the molds is compensated by the rotational movement along the first degree of freedom along a circular arc, an error occurs. Thus, when dividing the tracking section into 10 parts, an error of about 0.008 m occurs.

Plots of the laws of change of the generalized coordinate of the second degree of mobility in position, speed, and acceleration were constructed. This degree of mobility ensures that the blades are oriented perpendicular to the axis of linear movement of the molds. These plots are symmetrical to the plots for the first degree of mobility, with the same time intervals. The difference is the direction of rotation, in the first case it is clockwise, in the second it is counterclockwise.

The law of change of the generalized coordinate of the third degree in position was obtained, which showed that according to this degree of mobility in the time interval $t_3 \in [0.35, 1.51]$, s, the lowering of the blades onto the surface of the magnesium melt, which consists of an acceleration and deceleration section, is ensured. Next, at time interval $t_3 \in [4.52, 5.68]$, s, the blades with the collected oxide film are lifted, which also consists of an acceleration and deceleration section.

The established law of change in the generalized coordinate of the fourth degree of mobility in position, speed, and acceleration showed that in the time interval $t_4 \in [1.52, 4.52]$, s, the oxide film is collected from the surface of the magnesium melt, which consists of sections of acceleration, movement at a given speed and braking. Return to the starting position is performed over a time interval $t_4 \in [5.68, 8.69]$, s, which also consists of acceleration, movement at a given speed and braking sections.

The laws of change of the generalized coordinate according to the fifth degree of mobility in position and speed are obtained. This degree of mobility has a pneumatic drive, so control is carried out by changing the speed along this degree of mobility. At time interval $t_5 \in [6.0, 6.43]$, s, the rotary blade is raised above the level of the poured magnesium melt, which consists of acceleration and deceleration sections. At time interval $t_5 \in [8.43, 8.86]$, s, a return to the starting position is performed, which also consists of acceleration and deceleration sections.

The laws of variation in the generalized coordinate in the sixth degree of mobility in position and speed are obtained. This degree of mobility is also pneumatically driven, so control is carried out by changing the speed along this degree of mobility. At time interval $t_6 \in [6.43, 8.43]$, s, the oxide film collected on the rotating blade is dumped into a special container and returned to its original position. Here, the law of change in the generalized coordinate along the sixth degree of mobility also consists of acceleration and deceleration sections.

4. A control cyclogram for a 6-step two-arm MR has been constructed to perform technological operation of removing the oxide film from the surface of a magnesium melt poured into moving molds of a foundry conveyor. The cyclogram shows the sequence of movements and their direction according to the degrees of mobility of the manipulation robot. From the cyclogram it follows that movement along the first and second degrees of mobility have different directions and are

performed in parallel. This ensures tracking of moving molds and orientation of the blades perpendicular to the axis of movement of the molds. In parallel with the movement along the first and second degrees of freedom, movement is carried out through the third degree of freedom, lowering the blades onto the surface of the magnesium melt. Next, movement is performed along the fourth degree of mobility; by linear movement of the movable blade, the oxide film is collected on the rotating blade. The collected oxide film rises above the level of the magnesium melt by a rotary movement along the third degree of mobility. Next, movement begins along the fourth degree of mobility, returning to the starting position. When the blades are moved apart, a rotary movement is performed in the fifth degree of freedom, the oxide film is completely collected on the rotary blade. At this time, the process of returning the movable blade to its original position continues. Next, the rotary blade rotates around the sixth degree of freedom and is dropped into a special container. The rotary blade returns to its original position in the sixth degree of mobility. In parallel, the return to the starting position begins in the first and second degrees of mobility. Next, the rotary blade returns to its original position according to the fifth degree of mobility. At this time, the return to the starting position in the fourth degree of mobility is completed. Next, the return to the starting position in the first and second degrees of mobility is completed. The duration of the control cyclogram was 9.8 s. The accepted time limit is 10 s.

Therefore, the manipulation robot performed the oxide film collection operation within the specified limited time.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

The data will be provided upon reasonable request. The manuscript has associated data in the data warehouse.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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