ISSN 2078–5097. Вісн. Львів. ун-ту. Сер. прикл. матем. та інф. 2014. Вип. 21. С. 118–124
Visnyk of the Lviv University. Series Appl. Math. and Informatics. Issue 21. P. 118–124

UDC 519.832.3

AN EXPECTED VALUE INVARIANT BY THE PROBABILITY DISTRIBUTION AS A SECOND PLAYER OPTIMAL STRATEGY IN THE GAME WITH MINIMIZING ASSUREDLY THE ABSOLUTE DEVIATIONS

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There is investigated a task whether the expected value by the probability distribution, determined with minimizing assuredly the absolute deviations among the fixed N values of an object factor, stays unvarying. For that there has been proved the expected value as a scalar product of the N-dimensional point and the second player optimal strategy by the corresponding matrix $N \times N$ game is an invariant of this strategy on the convex compact, being the set of all the second player optimal strategies. The proof helps to evaluate the object factor, although the invariant brings the investigator to the choice problem over that convex compact, when the specific probability distribution is needed.

Key words: expected value, probability distribution, assuredly minimized absolute deviations, second player optimal strategies, an optimal strategy support, convex compact, invariant.

1. PREFACING

Sometimes it is important to obtain or determine a probability distribution over values of an object factor, when the object identification access is restricted or just being launched [1, 2]. This problem is essentially simplified for factors with the finite number of their values. However, restrictions on watching the object bring to situations when there spring up continual sets of relatively admissible probability distributions [3, 4]. The present paper is going to give a manifestation of that in a specific way of obtaining the class of such sets, those corresponding probability distributions have the common numerical feature.

2. AN UNSOLVED TASK

There is an unsolved task in investigating the features of sets of probability distributions over the object factor values, when these distributions are obtained under uncertainties [3, 5, 6] or random circumstances combination. The spoken above common numerical feature of those probability distributions is the expected value of the factor, produced by them. And there should be vindicated that expected value whether to stay unvarying for a minimax method [3, 4, 7, 8] of determining the probability distribution.

3. PUTTING A TARGET

May there be N values of the factor, describing the object. Let the set

$$\left\{v_i\right\}_{i=1}^{N} \text{ at } v_i \in \mathbb{R}, \ \forall\, i = \overline{1, N} \text{ and } v_j < v_{j+1}, \ \forall\, j = \overline{1, N-1} \tag{1}$$

be the sorted in ascending order set of those values of the factor v, where $N \in \mathbb{N} \setminus \left\{1\right\}$. The probability distribution over the values in the set (1) is determined through playing [1, 4] the matrix $N \times N$ game

$$\left\langle \left\{m_k\right\}_{k=1}^{N}, \left\{c_j\right\}_{j=1}^{N}, \left[u_{kj}\right]_{N \times N} \right\rangle = \left\langle \left\{m_k\right\}_{k=1}^{N}, \left\{c_j\right\}_{j=1}^{N}, \mathbf{U} \right\rangle \tag{2}$$

in which the second player, using its pure strategy c_j to assign the factor value v_j, possesses an optimal strategy [4] over the set of its pure strategies $\{c_j\}_{j=1}^{N}$ to minimize assuredly the absolute deviations

$$u_{kj} = \left|v_k - v_j\right|, \ \forall \ k = \overline{1, N} \ \text{ and } \ \forall \ j = \overline{1, N}, \tag{3}$$

while the first player, personifying the random circumstances combination [2, 3, 7] generator, uses its pure strategy m_k from its pure strategies set $\{m_k\}_{k=1}^{N}$ to assign the factor value v_k. The being sought probability distribution over the values in the set (1) is assigned to a second player optimal strategy in the game (2) with (3). The target is to clear up whether the expected value of the object factor varies if the set of the second player optimal strategies is rendered out by the game (2) with (3) continual.

4. HITTING THE TARGET

Above all, it ought to be mentioned that although the second player optimal strategies in the game (2) with (3) have not been substantiated to constitute the continuum set, for $N = 3$ and $N = 4$ it was [9, 10] proved that, speaking generally, in the game (2) with (3) from (1) the second player possesses the continuum $\breve{\mathcal{Q}}$ of its optimal strategies. Meanwhile, it is easy to see that for $N = 2$ in the game (2) with (3) from (1) the second player possesses the single optimal strategy $\breve{\mathbf{Q}} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Nevertheless, assumption about that in the game (2) with (3) from (1) the second player possesses the continuum $\breve{\mathcal{Q}}$ of its optimal strategies is most likely to be substantiated soon.

And now, for to help in hitting the target, there is a useful lemma, which can come in handy for a lot of practical situations.

Lemma 1. In the support of an optimal strategy of the first player of the matrix game there are only pure strategies, on which the optimal game value is reached.

Proof. May in the matrix game, where the first player has its N pure strategies, be the optimal game value w_{opt}. An optimal strategy of the first player as an N-dimensional point of the $(N-1)$-dimensional fundamental simplex is presented with its coordinates $\{\widehat{p}_i\}_{i=1}^{N}$ by $\widehat{p}_i \in [0; 1]$ at $\sum_{i=1}^{N} \widehat{p}_i = 1$. Let the support of the first player optimal strategy consist of its pure strategies with their numbers within the set $\widehat{I} \subseteq \left\{\overline{1, N}\right\}$. Suppose that $\exists \ \widehat{I}_1 \subseteq \widehat{I}$ such that $\forall \ i_1 \in \widehat{I}_1 \subseteq \widehat{I}$ the first player payoff $w_1^{\langle i_1 \rangle}$, resulting from that, that the first player selects its i_1-th pure strategy, while the second player holds to the optimality principle, is such:

$$w_1^{\langle i_1 \rangle} > w_{\text{opt}}. \tag{4}$$

But then (4) violates the optimality principle for second player, who cannot lose greater than w_{opt}. So, the set $\widehat{I}_1 = \varnothing$. Now suppose that $\exists \ \widehat{I}_0 \subseteq \widehat{I}$ such that $\forall \ i_0 \in \widehat{I}_0 \subseteq \widehat{I}$ there is

$$w_1^{\langle i_0 \rangle} < w_{\text{opt}}, \tag{5}$$

where $w_1^{\langle i_0 \rangle}$ is the first player payoff when it selects its i_0-th pure strategy. Then for denoting

$$w_1^{\langle i_0 \rangle} = w_{\text{opt}} - s_{i_0} \quad \text{at} \quad s_{i_0} > 0 \quad \forall\, i_0 \in \widehat{I}_0 \subseteq \widehat{I} \tag{6}$$

from (5) the first player payoff can be assigned as

$$\sum_{i_0 \in \widehat{I}_0} \widehat{p}_{i_0} w_1^{\langle i_0 \rangle} + \sum_{i \in \widehat{I} \setminus \widehat{I}_0} \widehat{p}_i w_{\text{opt}} = \sum_{i_0 \in \widehat{I}_0} \widehat{p}_{i_0} \left(w_{\text{opt}} - s_{i_0} \right) + \sum_{i \in \widehat{I} \setminus \widehat{I}_0} \widehat{p}_i w_{\text{opt}} =$$

$$= w_{\text{opt}} \left(\sum_{i_0 \in \widehat{I}_0} \widehat{p}_{i_0} + \sum_{i \in \widehat{I} \setminus \widehat{I}_0} \widehat{p}_i \right) - \sum_{i_0 \in \widehat{I}_0} \widehat{p}_{i_0} s_{i_0} = w_{\text{opt}} - \sum_{i_0 \in \widehat{I}_0} \widehat{p}_{i_0} s_{i_0} \geq w_{\text{opt}} . \tag{7}$$

But the assignment statement (7) with (6) is possible only if $\widehat{I}_0 = \varnothing$. This means that the first player, selecting its i-th pure strategy, gets the payoff $w_1^{\langle i \rangle} = w_{\text{opt}}$, $\forall\, i \in \widehat{I} \subseteq \left\{ \overline{1, N} \right\}$. The lemma has been proved.

Obviously, this lemma may be proved in many other ways [4]. Besides, the assertion of Lemma 1 can be stated also for the support of an optimal strategy of the second player by symmetric reasoning. Moreover, the assertion of Lemma 1 can be broadened from the matrix game to the antagonistic game, having its solution [4].

Actually, further there should be examined the set $\breve{\mathcal{Q}}$ of all the second player optimal strategies in the game (2) with (3) from the aggregate (1), that it is better to consider as an $\mathbb{R}^N$ point

$$\mathbf{V} = \begin{bmatrix} v_1 & v_2 & \cdots & v_{N-1} & v_N \end{bmatrix} \text{ at } v_i \in \mathbb{R}, \ \forall\, i = \overline{1, N} \text{ and } v_j < v_{j+1}, \ \forall\, j = \overline{1, N-1} . \tag{8}$$

Having

$$\breve{\mathcal{Q}} = \arg \min_{\mathbf{Q} \in \mathcal{Q}} \max_{\mathbf{P} \in \mathcal{P}} \left(\mathbf{P} \cdot \mathbf{U} \cdot \mathbf{Q}^{\mathrm{T}} \right) = \arg \min_{\mathbf{Q} \in \mathcal{Q}} \left(\widehat{\mathbf{P}} \cdot \mathbf{U} \cdot \mathbf{Q}^{\mathrm{T}} \right) \tag{9}$$

by

$$\mathbf{Q} = \begin{bmatrix} q_1 & q_2 & \cdots & q_N \end{bmatrix} \in \mathcal{Q} = \left\{ \mathbf{Q} \in \mathbb{R}^N : \sum_{j=1}^{N} q_j = 1,\ q_j \in [0;\, 1] \ \forall\, j = \overline{1, N} \right\}, \tag{10}$$

$$\breve{\mathbf{Q}} = \begin{bmatrix} \breve{q}_1 & \breve{q}_2 & \cdots & \breve{q}_N \end{bmatrix} \in \arg \min_{\mathbf{Q} \in \mathcal{Q}} \max_{\mathbf{P} \in \mathcal{P}} \left(\mathbf{P} \cdot \mathbf{U} \cdot \mathbf{Q}^{\mathrm{T}} \right) = \breve{\mathcal{Q}} \subset \mathcal{Q} , \tag{11}$$

and

$$\mathbf{P} = \begin{bmatrix} p_1 & p_2 & \cdots & p_N \end{bmatrix} \in \mathcal{P} = \left\{ \mathbf{P} \in \mathbb{R}^N : \sum_{i=1}^{N} p_i = 1,\ p_i \in [0;\, 1] \ \forall\, i = \overline{1, N} \right\}, \tag{12}$$

$$\widehat{\mathbf{P}} = \begin{bmatrix} \widehat{p}_1 & \widehat{p}_2 & \cdots & \widehat{p}_N \end{bmatrix} \in \arg \max_{\mathbf{P} \in \mathcal{P}} \min_{\mathbf{Q} \in \mathcal{Q}} \left(\mathbf{P} \cdot \mathbf{U} \cdot \mathbf{Q}^{\mathrm{T}} \right) = \widehat{\mathcal{P}} \subset \mathcal{P} , \tag{13}$$

there is the expected value

$$\widetilde{v}\left(\breve{\mathbf{Q}} \right) = \sum_{j=1}^{N} \breve{q}_j v_j = \breve{\mathbf{Q}} \cdot \mathbf{V}^{\mathrm{T}} \tag{14}$$

over the data in the set (1), found by the probability distribution or the convex combination from the element (11) of the set (9). The following assertion is effective for those sets in the form of (9), where $\left|\breve{\mathcal{Q}}\right| > 1$.

Theorem 1. The expected value (14) as scalar product of the point (8) and the optimal strategy (11) by the game (2) with (3) is an invariant of this strategy on the convex compact (9).

Proof. The just declared assertion means that the expected value (14) does not depend upon the optimal strategy (11) from the set (9), when this set contains more than the single element, that is the value (14) stays constant $\forall\ \breve{\mathbf{Q}} \in \breve{\mathcal{Q}}$. May this theorem assertion be false. Then there $\exists\ \breve{\mathbf{Q}}_1 \in \breve{\mathcal{Q}}$ and $\exists\ \breve{\mathbf{Q}}_2 \in \breve{\mathcal{Q}}$ such that

$$\tilde{v}\left(\breve{\mathbf{Q}}_1\right) > \tilde{v}\left(\breve{\mathbf{Q}}_2\right) \tag{15}$$

by $\breve{\mathbf{Q}}_1 \neq \breve{\mathbf{Q}}_2$. In the scalar product term, the inequality (15) is

$$\breve{\mathbf{Q}}_1 \cdot \mathbf{V}^{\mathrm{T}} > \breve{\mathbf{Q}}_2 \cdot \mathbf{V}^{\mathrm{T}}. \tag{16}$$

Now in stating the product

$$\mathbf{U} \cdot \breve{\mathbf{Q}}^{\mathrm{T}} = \mathbf{H} = \left[h_{i1}\right]_{N\times 1} \tag{17}$$

from the product $\widehat{\mathbf{P}} \cdot \mathbf{U} \cdot \breve{\mathbf{Q}}^{\mathrm{T}}$ have:

$$\begin{aligned} h_{i1} &= \sum_{j=1}^{N} u_{ij}\breve{q}_j = \sum_{j=1}^{N}\left|v_i - v_j\right| \cdot \breve{q}_j = \sum_{j=1}^{i-1}\breve{q}_j\left(v_i - v_j\right) + \sum_{j=i+1}^{N}\breve{q}_j\left(v_j - v_i\right) = \\ &= \sum_{j=1}^{i-1}\breve{q}_j v_i - \sum_{j=1}^{i-1}\breve{q}_j v_j + \sum_{j=i+1}^{N}\breve{q}_j v_j - \sum_{j=i+1}^{N}\breve{q}_j v_i = \\ &= v_i\sum_{j=1}^{i-1}\breve{q}_j - \sum_{j=1}^{i-1}\breve{q}_j v_j + \sum_{j=i+1}^{N}\breve{q}_j v_j - v_i\sum_{j=i+1}^{N}\breve{q}_j = \\ &= v_i\left(\sum_{j=1}^{i-1}\breve{q}_j - \sum_{j=i+1}^{N}\breve{q}_j\right) - \sum_{j=1}^{i-1}\breve{q}_j v_j + \sum_{j=i+1}^{N}\breve{q}_j v_j \quad \text{at} \quad i = \overline{1, N}, \end{aligned} \tag{18}$$

whereas

$$\breve{\mathbf{Q}} \cdot \mathbf{V}^{\mathrm{T}} = \sum_{j=1}^{N}\breve{q}_j v_j = \sum_{j=1}^{i-1}\breve{q}_j v_j + \breve{q}_i v_i + \sum_{j=i+1}^{N}\breve{q}_j v_j\ . \tag{19}$$

With the expansion (19) the i-th element (18) of the matrix vector (17) is

$$\begin{aligned} h_{i1} &= \sum_{j=1}^{N} u_{ij}\breve{q}_j = \breve{\mathbf{Q}} \cdot \mathbf{V}^{\mathrm{T}} - 2\sum_{j=1}^{i-1}\breve{q}_j v_j - \breve{q}_i v_i + v_i\left(\sum_{j=1}^{i-1}\breve{q}_j - \sum_{j=i+1}^{N}\breve{q}_j\right) = \\ &= \breve{\mathbf{Q}} \cdot \mathbf{V}^{\mathrm{T}} - 2\sum_{j=1}^{i-1}\breve{q}_j v_j + v_i\left(\sum_{j=1}^{i-1}\breve{q}_j - \sum_{j=i}^{N}\breve{q}_j\right) = \breve{\mathbf{Q}} \cdot \mathbf{V}^{\mathrm{T}} + g_i\left(\breve{\mathbf{Q}}\right) \quad \text{at} \quad i = \overline{1, N}. \end{aligned} \tag{20}$$

Letting the support of the first player optimal strategy consist of its pure strategies with their numbers within the set $\widehat{I} \subseteq \left\{\overline{1, N}\right\}$ by Lemma 1, from (20) have

$$\breve{\mathbf{Q}} \cdot \mathbf{V}^{\mathrm{T}} + g_i\left(\breve{\mathbf{Q}}\right) = w_{\mathrm{opt}} \quad \forall\ i \in \widehat{I} \subseteq \left\{\overline{1, N}\right\}. \tag{21}$$

But consider an affine equivalent game

$$\left\langle \{m_k\}_{k=1}^{N},\ \{c_j\}_{j=1}^{N},\ \mathbf{U}-\breve{\mathbf{Q}}\cdot\mathbf{V}^{\mathrm{T}} \right\rangle \tag{22}$$

to the game (2) with the matrix $\mathbf{U}-\breve{\mathbf{Q}}\cdot\mathbf{V}^{\mathrm{T}}$. In the game (22) both the players have the same sets of optimal strategies [4] as in the game (2). The optimal value of the game (22) is [4, 11] expressed via the optimal value (21) of the game (2):

$$w_{\mathrm{opt}}^{\langle 1\rangle} = w_{\mathrm{opt}} - \breve{\mathbf{Q}}\cdot\mathbf{V}^{\mathrm{T}} = \breve{\mathbf{Q}}\cdot\mathbf{V}^{\mathrm{T}} + g_i\left(\breve{\mathbf{Q}}\right) - \breve{\mathbf{Q}}\cdot\mathbf{V}^{\mathrm{T}} = g_i\left(\breve{\mathbf{Q}}\right),\ \ \forall\, i \in \widehat{I} \subseteq \left\{\overline{1,\, N}\right\}. \tag{23}$$

As the strategies $\breve{\mathbf{Q}}_1 \in \breve{\mathcal{Q}}$ and $\breve{\mathbf{Q}}_2 \in \breve{\mathcal{Q}}$ are optimal for the second player in the game (22), then

$$w_{\mathrm{opt}}^{(1)} = g_i\left(\breve{\mathbf{Q}}_1\right) = g_i\left(\breve{\mathbf{Q}}_2\right),\ \ \forall\, \breve{\mathbf{Q}}_1 \in \breve{\mathcal{Q}} \ \text{ and } \ \forall\, \breve{\mathbf{Q}}_2 \in \breve{\mathcal{Q}} \tag{24}$$

from (23), what shows that on the convex compact (9)

$$\frac{d}{d\breve{\mathbf{Q}}} g_i\left(\breve{\mathbf{Q}}\right) = 0 \ \ \forall\, \breve{\mathbf{Q}} \in \breve{\mathcal{Q}} \ \text{ at } i \in \widehat{I} \subseteq \left\{\overline{1,\, N}\right\}$$

and the value $g_i\left(\breve{\mathbf{Q}}\right)$ is constant $\forall\, \breve{\mathbf{Q}} \in \breve{\mathcal{Q}}$ at $i \in \widehat{I} \subseteq \left\{\overline{1,\, N}\right\}$. Then from (16) and (24) there goes

$$\breve{\mathbf{Q}}_1\cdot\mathbf{V}^{\mathrm{T}} + g_i\left(\breve{\mathbf{Q}}\right) > \breve{\mathbf{Q}}_2\cdot\mathbf{V}^{\mathrm{T}} + g_i\left(\breve{\mathbf{Q}}\right),\ \ \forall\, i \in \widehat{I} \subseteq \left\{\overline{1,\, N}\right\}$$

what is false because of

$$w_{\mathrm{opt}} = \breve{\mathbf{Q}}_1\cdot\mathbf{V}^{\mathrm{T}} + g_i\left(\breve{\mathbf{Q}}\right) = \breve{\mathbf{Q}}_2\cdot\mathbf{V}^{\mathrm{T}} + g_i\left(\breve{\mathbf{Q}}\right),\ \ \forall\, i \in \widehat{I} \subseteq \left\{\overline{1,\, N}\right\},$$

taken from (21). The assumption about the varying expected value (14) over the set (9) is contradictory. The theorem has been proved.

Once again, this theorem, bearing theoretical and practical purport, may be proved in a good few other ways [4]. But it would be precautiously fine to prevent broadening the assertion of Theorem 1 from the matrix game to the antagonistic game, having its solution [4, 12], as then the sequence with its values from analogue of the sorted set (1) must be a continuous increasing function, owning, very likely, some supplementary properties.

5. DEDUCTION

The proved Theorem 1 assists in finding the expected value (14) by the probability distribution, determined with minimizing assuredly the absolute deviations (3) through any second player optimal strategy in the game (2). Also it helps to evaluate the object factor, although the invariant (14) brings the investigator to the choice problem over the set (9), when the specific probability distribution is needed. Having (9) by (10) – (13), the expected value (14) may be found conveniently, in particular, by the element of the set (9) with the maximal number of zero probabilities, that is with minimum of the support cardinality of the second player optimal strategy. Initially for $N \in \{2,\, 3,\, 4\}$ it is [9, 10] learned that such a strategy support contains two pure strategies c_1 and c_N with probabilities $\breve{q}_1 = \breve{q}_N = \frac{1}{2}$. For greater integer N substantiating the minimum of the support cardinality of the second player optimal strategy for finding the expected value (14) conveniently is the further job, though.

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Стаття: надійшла до редколегії 03.03.2014
доопрацьована 09.04.2014
прийнята до друку 16.04.2014

ІНВАРІАНТ ОЧІКУВАНОГО ЗНАЧЕННЯ ЗА РОЗПОДІЛОМ ЙМОВІРНОСТЕЙ ЯК ОПТИМАЛЬНОЮ СТРАТЕГІЄЮ ДРУГОГО ГРАВЦЯ У ГРІ З ГАРАНТОВАНОЮ МІНІМІЗАЦІЄЮ АБСОЛЮТНИХ ВІДХИЛЕНЬ

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Досліджуєтмо задачу про те, чи залишається незмінним очікуване значення за розподілом імовірностей, який визначається гарантованою мінімізацією абсолютних відхилень серед зафіксованих N значень деякого фактора об'єкта. Для цього доведено, що таке очікуване значення як скалярний добуток N-вимірної точки й оптимальної стратегії другого гравця за відповідною матричною $N \times N$-грою є інваріантом цієї стратегії на опуклому компакті, який є множиною всіх оптимальних стратегій другого гравця. Доведення допомагає в оцінюванні фактора об'єкта, хоча й цей інваріант приводить дослідника до задачі вибору на тому опуклому компакті, коли потрібно виділити особливий розподіл імовірностей.

Ключові слова: очікуване значення, розподіл імовірностей, гарантовано мінімізовані абсолютні відхилення, оптимальні стратегії другого гравця, спектр оптимальної стратегії, опуклий компакт, інваріант.

ИНВАРИАНТ ОЖИДАЕМОГО ЗНАЧЕНИЯ ПО РАСПРЕДЕЛЕНИЮ ВЕРОЯТНОСТЕЙ КАК ОПТИМАЛЬНОЙ СТРАТЕГИЕЙ ВТОРОГО ИГРОКА В ИГРЕ С ГАРАНТИРОВАННОЙ МИНИМИЗАЦИЕЙ АБСОЛЮТНЫХ ОТКЛОНЕНИЙ

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Исследовано задачу о том, остаётся ли неизменным ожидаемое значение по распределению вероятностей, определяемое гарантированной минимизацией абсолютных отклонений среди зафиксированных N значений некоторого фактора объекта. Для этого доказано, что такое ожидаемое значение как скалярное произведение N-мерной точки и оптимальной стратегии второго игрока по соответствующей матричной $N \times N$-игре является инвариантом этой стратегии на выпуклом компакте, являющимся множеством всех оптимальных стратегий второго игрока. Доказательство помогает в оценивании фактора объекта, хотя и этот инвариант приводит исследователя к задаче выбора на том выпуклом компакте, когда требуется выделить особое распределение вероятностей.

Ключевые слова: ожидаемое значение, распределение вероятностей, гарантировано минимизированные абсолютные отклонения, оптимальные стратегии второго игрока, спектр оптимальной стратегии, выпуклый компакт, инвариант.