Advances in Astronomy and Space Physics, **6**, 73-78 (2016) doi: 10.17721/2227-1481.6.73-78

The structure of the test function for the phenomenological modelling of eclipsing binaries

M. G. Tkachenko[*]

Department "Mathematics, Physics and Astronomy", Odessa National Maritime University, Mechnikova Str., 34, 65029, Odessa, Ukraine

The dependence of the test function on the phenomenological parameters used in the "NAV" ("New Algol Variable") algorithm (Andronov, 2012) is studied in the present work. Due to a presence of local minima, the method of minimisation contains two steps: the "brute force" minimisation at a grid in the 4D parameter space, and further iterations using the differential corrections. This method represents an effective approximation of the lightcurve using the special pattern (shape) for the primary and secondary minima separately. The application of the method to real star systems is briefly reviewed.

Key words: stars: variable stars, binary stars

INTRODUCTION

Currently nearly 400 000 variable stars are listed in the Variable Stars Index (VSX[1]). In the General Catalogue of Variable Stars (GCVS, Samus, et al., 2007–2016 [27]), the online version of which is available at SAI Web-page[2], there are currently 52011 objects with official GCVS names, including 10845 objects classified as eclipsing ones. These objects distributed among the subtypes as 5294 (EA), 3018 (EB), 1434 (EW), 1099 (E). Only few dozens of them were studied in details, using not only photometric, but also spectral and (rarely) polarimetrical observations. For such rare objects, the "physical" modelling is possible with a determination of radii, masses, and temperatures. The "standard" approach is the so-called "Wilson – Devinney" model [33, 34], which was realised in some famous programs: "Binary Maker" [16], Phoebe [25], and in the set of programs elaborated by S. Zoła et al. [35, 36]. Different problems of the physical modelling were described e. ,g. in [18, 19, 20].

For the majority of stars, there are only photometric observations, often obtained with one (or no) filter, thus the "physical" modelling is not possible because of unknown temperatures of the components and their mass ratio. In this case, only "phenomenological" modelling is available, which is characterised by a smaller number of parameters describing the lightcurve, namely, the period P, the initial epoch T_0, brightness in primary maximum $m_{\max}$ and primary minimum $m_{\min}$, the duration of the eclipse D. Additionally, the brightness at the secondary minimum and (if different from $m_{\max}$) at the secondary maximum, and the phase shift of the secondary minimum in respect to the phase 0.5 (significant in a case of elliptical orbits) are listed in the section "remarks" of the GCVS [27, 30].

Typically the values of brightness and phases are determined using local approximations of observations in intervals, which include the extrema (either maximum, or minimum) (e. g. [4]).

Some methods use the trigonometrical polynomial approximation of the complete lightcurve [26]. Also there is a set of studies based on the "simplified physical" model, which suggests spherically symmetrical components with uniform brightness distribution [12, 22, 28].

More accurate methods, based on special shapes (patterns) of the minima, were actively used recently [5, 6, 23].

Another algorithm for statistically optimal degree s of the trigonometrical polynomial (sometimes called the "restricted Fourier series") determination is based on the minimisation of the r.m.s. estimate of the accuracy of the smoothing function at the arguments of observations [1, 2]. Another algorithm for statistically optimal degree s of the trigonometrical polynomial (sometimes called the "restricted Fourier series") determination is based on the minimisation of the smoothing function accuracy r.m.s. at the observational arguments [1, 2]. This method was effectively applied also for pulsating Mira-type variables [21].

In this paper, we compared previously used approximations with approximations that use the spe-

[*] masha.vodn@yandex.ua

[1] http://aavso.org/vsx

[2] http://www.sai.msu.su/gcvs/gcvs/

cial shape (pattern) and studied behaviour of the test function in the parameter space. For illustration, we have used $n = 1000$ values of the phenomenological "NAV" function [6] with fixed parameters, which models the lightcurve of an EA-type eclipsing binary.

THE METHODS OF CALCULATIONS: TRIGONOMETRIC POLYNOMIAL

There were oversimplified models, which could be effective for automatic classification of numerous newly discovered variable stars using the surveys, e. g. the "EA" catcher with a parabolic shape of minima of equal width and different depth [10].

More recently, Papageorgiou et al. [24] proposed a model of parabolic shape either for the out-of-the eclipse parts (phases (0.1..0.4) and (0.6 − 0.9) of the lightcurve), or for the eclipses (fixed phases (−0.2..0.2), (0.3..0.7)). The corresponding curve is shown in Fig. 1. One may note a reasonably good approximation out of eclipses, but a bad approximation at the phases of minima because of an overestimated eclipse width.

Moreover, the lightcurve is not continuous. In the much earlier "EA" catcher [10] the smoothing function was continuous, and the width (as well as the phase shift) was determined using non-linear least squares fitting (see examples of this and other functions in [9, 13, 14]). The approximation of the variable stars extrema using the algebraic polynomial of the statistically optimal degree was realised in the software [15, 17]. The separate case of abrupt decline and inclined parts of the lightcurve, when the analytical function gives obviously bad approximations, was discussed in [8]. The fixed width in the method from [24] leads to a systematic differences between the observations and the approximation.

Next approximation is a trigonometrical polynomial:

$$x_c(\phi) = C_1 + \sum_{j=1}^{s} [C_{2j}\cos(2\pi j\phi) + C_{2j+1}\sin(2\pi j\phi)] = C_1 + \sum_{j=1}^{s} R_j \cos\left[2\pi j(\phi - \phi_{0j})\right],$$

where ϕ is the phase, ϕ_{0j} are initial phases corresponding to the maximum of the wave with the j^{th} term of the sum, and R_j are corresponding semi-amplitudes. The coefficients C_α ($\alpha = 1..m = 1 + 2s$) are determined using the least squares method.

The approximations, which use the trigonometric polynomial of different degrees s, are shown in Fig. 2. One may note an expected refinement of the approximation with an increasing of s. The coefficients C_{2j} are shown in Fig. 3. They describe terms with a cosine function, thereby the "symmetrical" part of the lightcurve. For even j the absolute values are typically larger, which is explained by a similarity in depth of the primary and secondary minima, as the coefficients with even j approximate a mean lightcurve with a double frequency, and the coefficients with odd j approximate the difference:

$$\frac{x_c(\phi) + x_c(\phi + 0.5)}{2} = C_1 + \sum_{k=1}^{s/2} [C_{4k}\cos(4\pi k\phi) + C_{4k+1}\sin(4\pi k\phi)],$$

$$\frac{x_c(\phi) - x_c(\phi + 0.5)}{2} = \sum_{k=1}^{s/2} [C_{4k-2}\cos(2\pi(2k-1)\phi) + C_{4k-1}\sin(2\pi(2k-1)\phi].$$

Additionally, if the O'Connell effect is practically absent (that is the case for the majority of objects) [24], the terms with sine vanish, and one gets only sums of terms with cosines.

At this dependence, the coefficients tend to zero, but too slowly. E. g. the last coefficient exceeding an arbitrary limiting value of 0.001 occurs at $j = 64$, so the corresponding number of parameters $m = 1 + 2 \cdot 64 = 129$ is extremely large.

For determination of the statistically optimal value of s different criteria may be used (e. g. [1, 2]). The first is based on the Fischer's criterion, which assumes uncorrelated observational errors obeying the normal distribution. For our data set (which contains computed values without any noise), this criterion is not applicable, as the deviations between the data and the approximation are systematic and not random. For real stars, we used this criterion as well (e. g. [2, 9]).

The second criterion is based on minimisation of the r.m.s. accuracy estimate $\sigma[x_c]$ of the approximation $x_c(\phi)$ at the arguments of observations ϕ_k:

$$\sigma^2[x_c] = \frac{m}{n}\sigma^2_{0m}, \quad \sigma^2_{0m} = \frac{\Phi_m}{n-m}, \quad \Phi_m = \sum_{k=1}^{n} w_k \cdot (x_k - x_c(\phi_k))^2.$$

Here σ_{0m} is a "unit weight error", Φ_m as a "test" ("target") function to be minimised in the parameter space.

The dependence of $\sigma[x_c]$ on the number of parameters m is shown in Fig. 4. In fact, it may be split into two almost monotonic sequences for even and odd degrees of the trigonometric polynomial (as a consequence of the separate dependencies of the coefficients described above). The bottom sequence shows a systematic decrease with s, so formally the degree of the trigonometric polynomial should be extremely large, close to $n/2$, i. e. the approximation tends to an interpolating function. This is because the data are precisely described by a function.

In a real situation, the present statistical errors lead to qualitative and quantitative changes. For an illustration, we have suggested an additional observational noise with a standard error of (arbitrarily) 0.001 and 0.01. The resulting dependencies show a broad, but distinct minimum in Fig. 4 at $s = 132$ and $s = 34$, respectively.

The additional noise shifts the position of the minimum of the dependence of the r.m.s. value of the accuracy of the approximation towards smaller values, leading to the systematic shifts. Anyway, the degree is very large, which leads to a considerable number of statistically insignificant coefficients.

THE METHODS OF CALCULATIONS: THE NAV ALGORITHM

To decrease the number of the parameters, Andronov [5, 6] proposed the following approximation called "the NAV" ("New Algol Variable") algorithm:

$$\begin{aligned} x_c(\phi) = G_1 + G_2\cos(2\pi\phi) + G_3\sin(2\pi\phi)+ \\ + G_4\cos(4\pi\phi) + G_5\sin(4\pi\phi)+ \\ + G_6 H(\phi - C_4, C_1, C_2)+ \\ + G_7 H(\phi - C_4 - 0.5, C_1, C_3), \end{aligned}$$

where the shape (pattern) is localised to the phase interval

$$H(\zeta, C_1, \beta) = \begin{cases} V(z) = (1-|z|^{\beta})^{3/2}, & \text{if} |z| < 1, \\ 0, & \text{if} |z| \geq 1, \end{cases}$$

where $z = \zeta/C_1$, and C_1 is the eclipse *half*-width. In the GCVS [27], the eclipse *full* width in per cent D is required for the classiïňĄcation. Thus $D = 200 \cdot C_1$, which is rounded to integer number (of per cent), $C_1 = D/200$ is expressed in parts of the orbital periods, as phases.

The second-order trigonometrical polynomial is typically sufficient to describe the effects of reflection, ellipticity and asymmetry (O'Connell effect). The $H-$ functions describe the shapes of the minima, with a parameter β, which is generally different for the primary ($\beta_1 = C_2$) and secondary ($\beta_2 = C_3$) minima. Generally, there may be a shift $\phi_0 = C_4$. Phenomenological modelling of multi-color observations of a newly discovered eclipsing binary 2MASS J18024395 + 4003309 = VSX J180243.9+400331 is presented in [11].

In Fig. 5, we show dependencies of the best fit approximation with one parameter changing in a some range, while other "non-linear" parameters ($C_1..C_4$) are set to the best fit values, whereas the "linear" parameters ($G_1..G_7$) are determined using the least squares subroutine.

The central thick line coincides with our artificial data, which were used for an illustration. It is clearly seen that the change of one of the "non-linear" parameters leads to changes in the "linear" parameters and, thus, the approximation.

In Fig. 6, we show the "levels" – lines of equal values of Φ_m at the two-parameter diagrams. They resemble deformed ellipses and show only a slight inclination close to the best fit point (marked by an arrow). The most drastic changes of the lightcurve are due to variations of the phase shift $C_4 = \phi_0$. There are only one global minimum of the function, except for the dependence with a phase shift C_4. Such structure of the test function leads to the following algorithm of determination of the global minimum – at first, "brute force" determination of the minimum at a grid of values of $C_1..C_4$ with a further iterations using the differential corrections. However, the usage of some middle point as a starting point may lead to iterations convergence to a local minimum instead of the global one.

CONCLUSIONS

The approximations with special pattern (also called "shape" or "profile") to fit the minima have much better quality of convergence of the smoothing curve with the data points. In this paper, we studied the dependence of the test function on four "non-linear parameters" $C_1..C_4$, whereas the "linear" parameters $G_1..G_7$ are determined using the method of the least squares. The "NAV" ("New Algol Variable") algorithm is an effective tool presenting a good pattern for the minima, which may be improved by using an additional parameter, which describes its shape.

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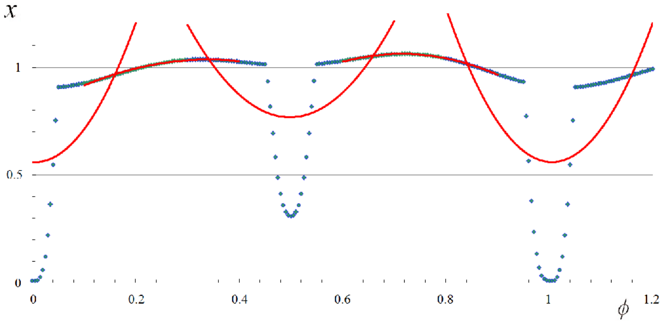

Fig. 1: The model lightcurve and its approximation by parabola at the intervals of phases centred on minima and maxima, as proposed by (Papageorgiou et al., 2014)

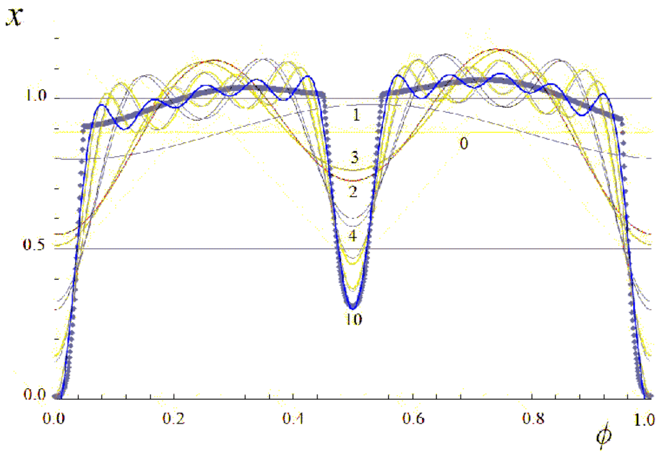

Fig. 2: Trigonometrical polynomial approximations of the phenomenological lightcurve. The degree s is shown by numbers near corresponding curves.

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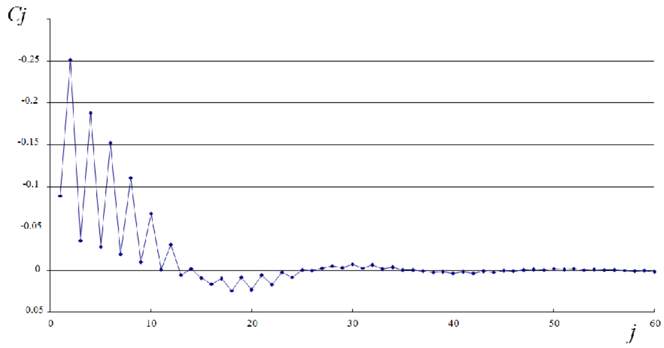

Fig. 3: Dependence of coefficients C_j on j.

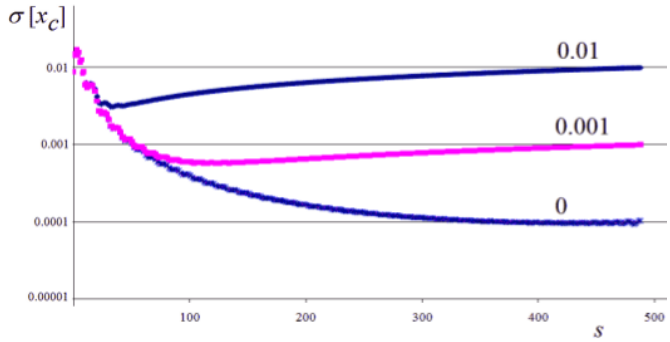

Fig. 4: Dependence of the mean squared error estimate $\sigma[x_c]$ of the approximation for an additional noise with r.m.s value of 0 (bottom), 0.01 (up) , 0.001 (middle).

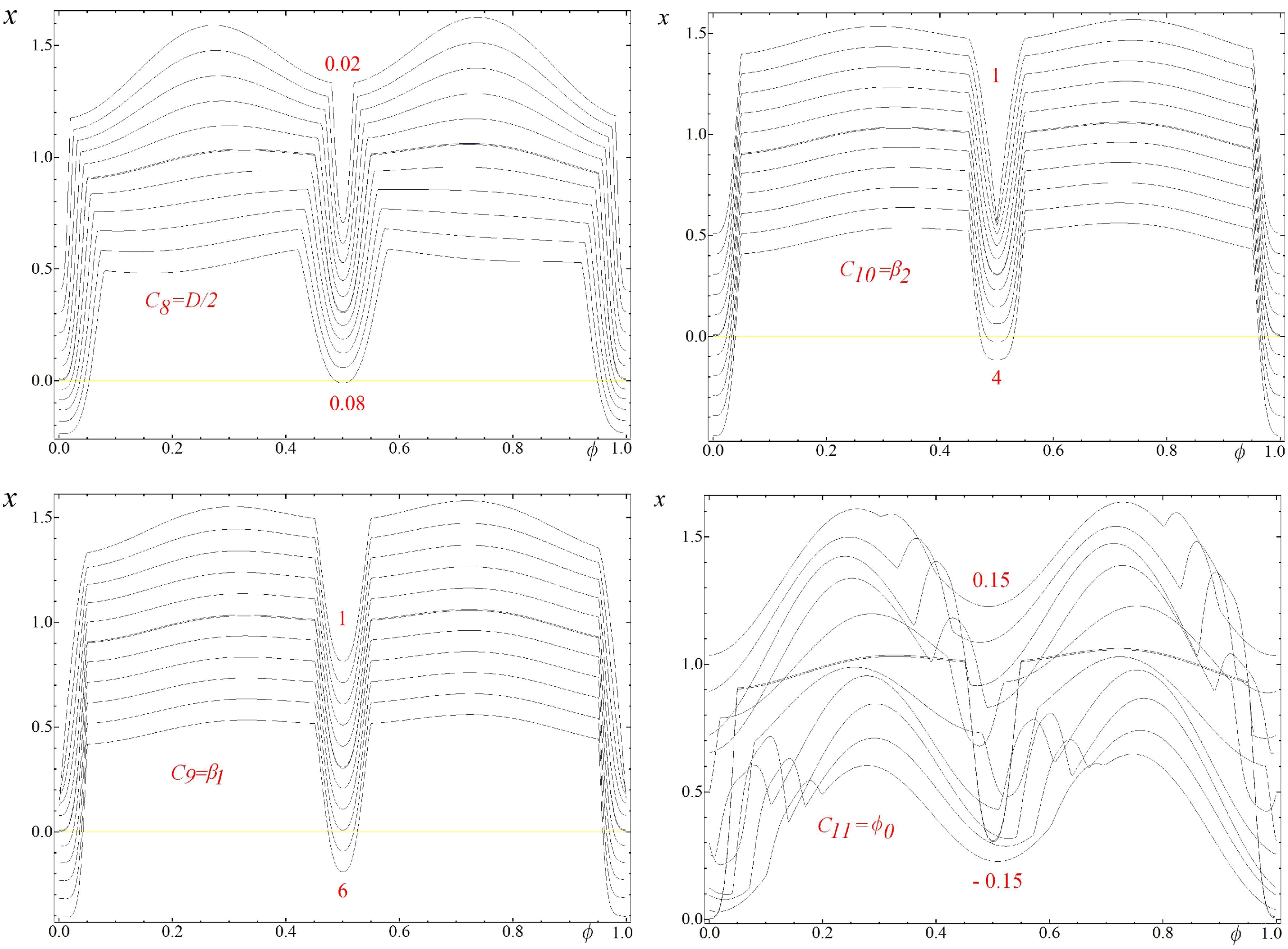

Fig. 5: Dependencies of the lightcurves (intensity vs. phase) on the parameters $C_8 = D/200$ (left) and $C_9 = \beta_1$ (right). The relative shift in intensity between subsequent curves is 0.1. The thick line shows a best fit curve.

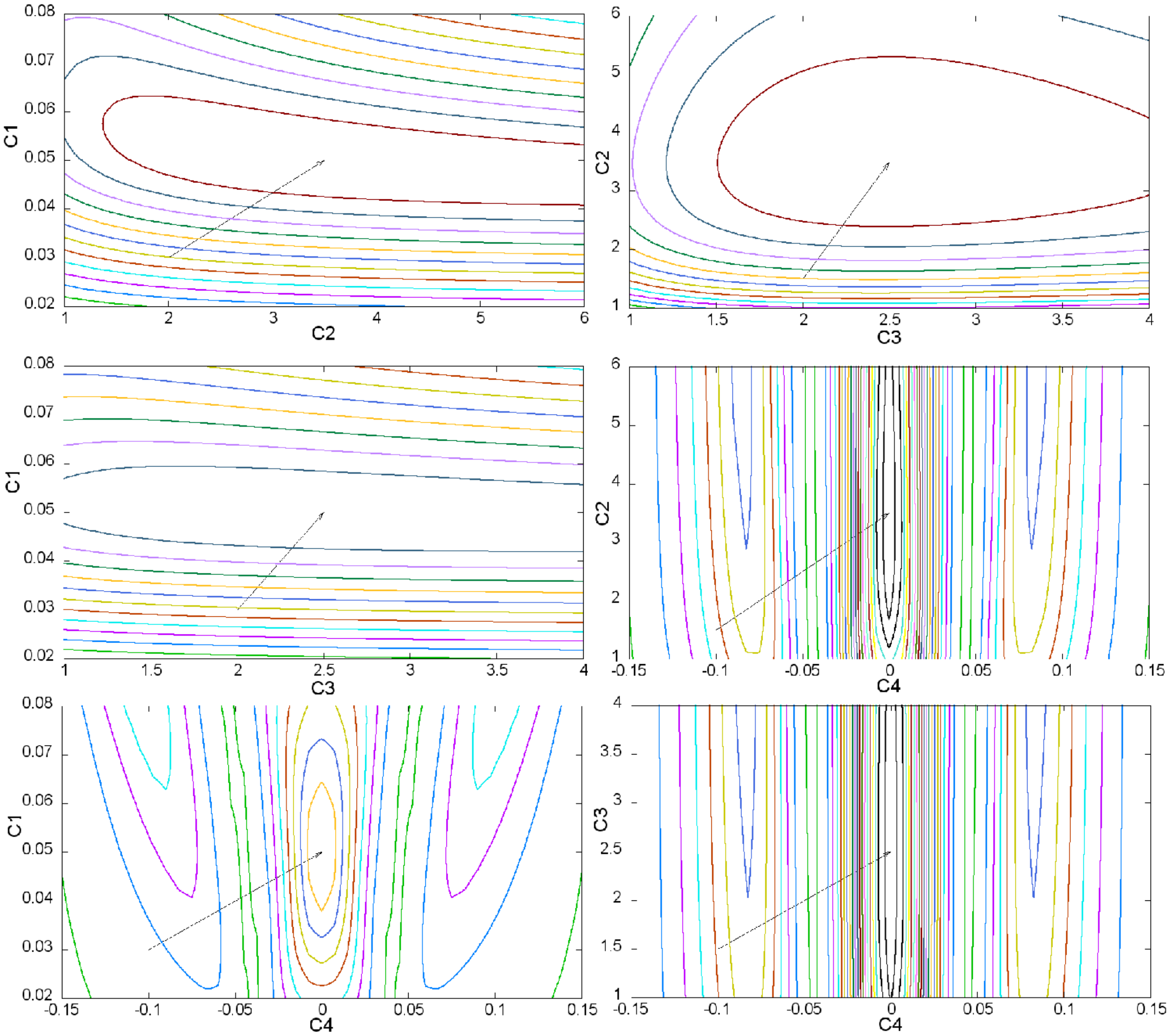

Fig. 6: Lines of equal levels of the test-function for different pairs of the parameters. Arrows show the best fit point.