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SUBJECTIVELY PREFERRED OPTIMALLY CONTROLLED MODES OF OPERATION FOR AN AIRCRAFT MAXIMAL DURATION HORIZONTAL FLIGHT

It is researched the operation of an aircraft on the modes of horizontal flight with the maximal duration. Extremals of the functional on the basis of the traditional form of the integral acquired with respect to the equations of motion of a material point with the changeable mass by I.V. Mescherskiy (1893-1897), T. Levi-Civita (1928) with the addition of the entropy of the controlling element's of the active system individual subjective preferences has been obtained as a solutions of the variational problem on the necessary conditions for the extremums existence in the view of the system of the Euler's-Lagrange's equations. Canonical distributions of the preferences and the optimal mode of the horizontal flight for the maximal duration have been got also in the case when the extremal speed cannot be reached or realized for one reason or another. The control in such a case implies preferences of the greater derivatives or differentials as the functions of effectiveness. The optimal modes of operation are being obtained on the basis of the subjective entropy extremum only and no other restrictions, except normalizing condition, has been imposed in the problem formulation. The necessary calculations are conducted. Plotted corresponding diagrams.

Key words: *maximal duration, horizontal flight, subjective entropy, individual preferences, multi-alternativeness, active systems control, active element, variational problem, operational functional.*

Introduction

It is generally known and accepted that the maximal duration of a horizontal flight for an aircraft equipped with any type of aircraft powerplants exists objectively [1]. The solution of the simplest problem about the maximal duration of a horizontal flight shown in [2, 3] related an aircraft equipped with a sky rocket engine.

Problem formulation in the general view. The problem of control in an active system should make allowance for a choice made by the system's active element. Moreover, there should be some scientifically reasonable explanation of the active element's rational behavior, for example, when choosing the appropriate operational mode.

The problem relation to important scientific and practical tasks. It is always important to control and operate a system in some optimal way. An effective process of an operational control of an aircraft flight is not an exception. That implies holding the flight safety and economical parameters within their accepted intervals.

Scientific substantiation of the optimality with respect to fuel consumption effectiveness results in practically up to 15%, or even sometimes more, savings compared to operational modes neglecting extremal principles.

Analysis of the latest researches and publications. There are a lot of scientific monographs, training books, guidances, and text books dedicated to the problem issues of flight modeling and optimization. In our researches we are guided mostly by [2 – 11] with elaboration the still unsolved part of the general optimization problem with respect to the postulated in subjective analysis [6 – 11] the entropy extremal principle.

The task setting. The purpose of this paper is to show the usefulness of the entropy extremal principle concept in application to the solution of the problem of the optimal control mode of operation choice when in the variational problem setting for an aircraft maximal duration horizontal flight we formulate the functional including subjectively preferred alternatives.

The main content of the researches

Variational problems of the changeable mass material point dynamics allow correct researching a series of non-stationary problems of the aircraft flight dynamics problem not only with the sky rocket engine [3, P. 198-218] but also with a propeller or any other propulsive complex.

1. Objectively existing optima

1.1. Maximal duration horizontal flight of the aircraft with the sky rocket engine

Concerning the maximal duration horizontal flight of the aircraft with the sky rocket engine, the achieved solution [3, § 5, P. 203, (22)]:

$$f = \frac{v^2}{v_0^2}, \qquad (1)$$

where f – function, which is being variated, or the free/loosened function, – the law of the airplane mass change [3, § 5, P. 198];

v – speed of the flying object center of masses, it is assumed, that when the fuel is being burnt, the center of masses of the aircraft has no displacement relatively to its fuselage hull, hence, the vector differential equation of the center of masses motion will not be different from the equation of the material point with the changeable mass motion, that is from the equation by I.V. Mescherskiy (1893-1897) [3, P. 7], [3, § 5, P. 199], T. Levi-Civita (1928) [3, P. 9, 11, 12], [3, § 1, P. 19, (13)];

v_0 – value of the initial speed of the flying object horizontal flight [3, § 5, P. 202, 208, 210];

is the equation of the extremal of the corresponding functional [3, § 5, P. 202, (16)]:

$$T = \int_{v_E}^{v_0} \frac{(f + f' V_r) v^{2n-2}}{A v^{2n} + B f^n} dv, \qquad (2)$$

where T – time (duration) of the horizontal flight at the engine running;

v_E – speed of the airplane flight at the end of the active segment of the horizontal flight, that is at the end of the engine run [3, § 5, P. 202];

f' – derivative of the flying object mass change function with respect to the speed of the horizontal flight, that is $f' = \frac{df}{dv}$ [3, § 5, P. 201];

V_r – effective relative speed of the burning products flowing out from the nozzle of the reactive (jet) engine, being $V_r = const$ [3, § 5, P. 199];

n – some certain constant, being determined within the given diapason of speeds from the blowings in wind tunnels [3, § 5, P. 199];

A та B – constants, being determined by the expressions [3, § 5, P. 202]:

$$A = \frac{C_{x_0} \rho S}{2 M_0}, \qquad B = \frac{b g^n (2 M_0)^{n-1}}{(\rho S)^{n-1}}, \qquad (3)$$

where C_{x_0} – value of the head resistance force coefficient at the value of the lifting force when it equals zero;

ρ – density of the air at the given altitude;

S – character square area of the flying object [3, § 5, P. 199];

M_0 – mass of the flying apparatus at the initial moment in time (at the point of the airplane coming up to the straight line horizontal trajectory) [3, § 5, P. 201, 202];

b – some stable value which is being determined within the given diapason of speeds from the blowings in wind tunnels;

g – acceleration, stipulated by the gravitational force, which is considered being constant and equaled to $g = 9.81$ m/s^2;

written in the view of integral (2) on the basis of [3, § 5, P. 202, (14)]:

$$dt = -\frac{(f + f' V_r) v^{2n-2}}{A v^{2n} + B f^n} dv, \qquad (4)$$

where t – time.

1.2. Without jet force flight

For the case of an airplane equipped with the propeller propulsive complex flight for the maximal duration of the horizontal flight, instead of the reactive force of the engine, written in the view of [3, § 5, P. 199, (1)]:

$$\Phi = -\frac{dM}{dt} \cdot V_r = m V_r, \qquad (5)$$

where $M = M(t)$ – mass of the flying object at the given moment of time t;

m – fuel mass consumption per second;

in the equations of the aircraft center of masses motion in projection upon the tangent line to its trajectory we obtain the thrust of the propellers:

$$P = -\eta_н \frac{dm}{dt}, \qquad (6)$$

where $\eta_н$ – coefficient of proportionality of the propellers thrust to the secondly fuel mass consumption;

m – mass of the flying apparatus.

Neglecting the forces of inertia will be the next characteristics of the main forces and suppositions at the considered problem formulation. It is justified because of insignificant changes of the flight speeds at the rather prolonged duration (time) of the flight, that is tangent accelerations are negligibly small.

Thus, at the given case, for the rough, simplified problem setting, we obtain the differential equations of the airplane center of masses motion in projections upon the tangent and normal lines of the flight trajectory in the view of:

$$0 = P - R = -\eta_н \frac{dm}{dt} - C_x \frac{\rho v^2}{2} S, \qquad (7)$$

$$0 = -G + Y = -mg + C_y \frac{\rho v^2}{2} S, \qquad (8)$$

where R – aerodynamic force of the head resistance;

C_x – coefficient of the aerodynamic force of the head resistance; G – force of gravity;

Y – aerodynamic lifting force;

C_y – coefficient of the aerodynamic lifting force.

We evaluate the coefficient of $\eta_н$ through the expression:

$$\eta_н = \eta \frac{Q}{v}, \qquad (9)$$

where η – efficiency of the propulsive complex, a constant for the rough problem formulation;

Q – low calorific value of the fuel by its working mass.

We approximately estimate the coefficient of C_x through the traditional equation of the parabolic polara [3, § 5, P. 199]:

$$C_x = C_{x_0} + bC_y^n = C_{x_0} + \frac{C_y^2}{\pi\lambda}, \qquad (10)$$

where λ – coefficient of the effective extension of the wing.

Then, from the differential equations of the motion (7), (8), we acquire an expression to the differential of the flight duration (time):

$$dt = -\frac{2\eta Q\rho v S}{C_{x_0}\left(\rho v^2 S\right)^2 + b(2mg)^2} dm. \qquad (11)$$

The corresponding integral of the duration of the flight will be

$$T = \int_{M_0}^{M_E} -\frac{2\eta Q\rho v S}{C_{x_0}\left(\rho v^2 S\right)^2 + b(2mg)^2} dm. \qquad (12)$$

where M_E – mass of the flying apparatus at the end of the active segment of the horizontal flight, that is at the end of the engine run.

Determination of $v = v(m)$ from the functional (12), from the mathematical point of view is the simplest problem of the calculus of variations, which has the optimal speed for the flight with the maximal duration as the problem corresponding solution:

$$v_T(m) = \sqrt[4]{\frac{4}{3}\frac{bm^2g^2}{C_{x_0}\rho^2 S^2}}. \qquad (13)$$

From the expression for the derivative:

$$\frac{dm}{dt} = -\frac{C_{x_0}\left(\rho v^2 S\right)^2 + b(2mg)^2}{2\eta Q\rho v S}, \qquad (14)$$

substituting into (14) the corresponding solution (13), by integration, we find the optimal laws of the mass change as a function of time for the corresponding functional (12) in the view of

$$m_T(t) = \frac{M_0}{\left(\left(\frac{4}{3}\right)^{3/4} \frac{\sqrt[4]{C_{x_0}}\left(bg^2\right)^{3/4}}{\eta Q\sqrt{\rho S}} t\sqrt{M_0} + 1\right)^2}. \qquad (15)$$

Substituting (15) into (13) we get the corresponding laws of the flight speed change as a function of time:

$$v_T[m_T(t)] = \sqrt[4]{\frac{4}{3}\frac{bm_T(t)^2 g^2}{C_{x_0}\rho^2 S^2}} = v_T(t). \qquad (16)$$

Finally, substituting the extremals, the optimal speeds (13), as the functions of masses, into the integrals of (12), we determine the maximal duration of the horizontal flight.

2. Subjective entropy approach

There is a postulated principle of the human being behavioral optimality in subjective analysis [6-11]. For an active system it means that the system's active element (individual, responsible subject, making controlling decisions person) distributes his own preferences in some certain optimal way extremizing some value (generally speaking a functional) which includes a measure of the individual's subjective preferences uncertainty in the view of entropy.

2.1. Operational control functional with respect to subjective preferences entropy

Regarding to operational multi-alternativeness we compile the so-called operational functional:

$$\Phi_\pi = -\sum_{i=1}^{N} \pi_i \ln \pi_i - \beta\sum_{i=1}^{N} \pi_i F_i + \gamma\left[\sum_{i=1}^{N} \pi_i - 1\right], \qquad (17)$$

where π_i – function of the individual's subjective preferences of the i^{th} achievable alternative;

N – number of the achievable alternatives;

β – structural parameter;

F_i – function, related to the i^{th} achievable alternative;

γ – structural parameter.

The prototype functional of the general view for the functional (17) is described, for instance, in [7, P. 119, (3.38)].

The structural parameters β and γ can be considered in different situations as Lagrange coefficients, weight coefficients or endogenous parameters that represent some certain properties of the individual's psych.

Without any additionally imposed preconditions, except for the necessary conditions of the extremums existence, the functional of the view (17) ensures, if the extremums exist or not, the optimal choice of operational control modes.

We imply the maxima of the first member, subjective entropy, of (17) and either correspondingly minima in the related to the achievable alternatives functions if β >0 follows the negative sign, or vice versa maxima of the functions if β >0 follows the positive sign.

2.2. Operational control functional with respect to subjective preferences entropy for the maximal duration horizontal flight

The functional compiled for such a case is analogous to [9, P. 57, (1)]:

$$\Phi_\pi = \int_{t_0}^{t_1} \left\{ -\sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t) + \beta \sum_{i=1}^{N} \pi_i(t) F_i + \right.$$

$$\left. + \gamma \left[\sum_{i=1}^{N} \pi_i(t) - 1 \right] \right\} dt \, . \qquad (18)$$

For the problem of the maximal flight duration control, for the simplest case on condition of two achievable alternatives, it will be

$$\Phi_\pi = \int_{M_0}^{M_E} \left\{ H_\pi - \beta \left[\pi_1 \frac{2\eta Q \rho v_{opt} S}{C_{x_0} \left(\rho v_{opt}^2 S\right)^2 + b(2mg)^2} + \right. \right.$$

$$\left. \left. + \pi_2 \frac{2\eta Q \rho v S}{C_{x_0} \left(\rho v^2 S\right)^2 + b(2mg)^2} \right] + \gamma \left[\sum_{i=1}^{2} \pi_i - 1 \right] \right\} dm \, , \quad (19)$$

where H_π – active element's (individual's) subjective preferences entropy,

$$H_\pi = -\sum_{i=1}^{2} \pi_i \ln \pi_i \, ; \qquad (20)$$

v_{opt} – unknown optimal speed of the horizontal flight with regards to the time (duration) of the flight;

v – arbitrary chosen function of speed.

From the necessary conditions for an extremal to exist, written in the view of the system of the equations by Euler-Lagrange:

$$\frac{\partial R^*}{\partial \pi_i} - \frac{d}{dm}\left(\frac{\partial R^*}{\partial \pi_i'}\right) = 0 \, , \quad \frac{\partial R^*}{\partial v_{opt}} - \frac{d}{dm}\left(\frac{\partial R^*}{\partial v_{opt}'}\right) = 0 \, , \quad (21)$$

where R^* – under-integral function of the corresponding integral (19);

π_i' – derivatives of the preferences functions with respect to the mass, that is $\pi_i' = \frac{d\pi_i}{dm}$;

v_{opt}' – derivative of the speed with respect to the mass, that is $v_{opt}' = \frac{dv_{opt}}{dm}$;

for this special case considered:

$$\frac{\partial R^*}{\partial \pi_i'} \equiv 0 \, , \qquad \frac{\partial R^*}{\partial v_{opt}'} \equiv 0 \, ,$$

hence, this transforms the system (21) to

$$\frac{\partial R^*}{\partial \pi_i} = 0 \, , \qquad \frac{\partial R^*}{\partial v_{opt}} = 0 \, , \qquad (22)$$

therefore, we get the corresponding expressions of the canonical distributions of the preferences, likewise in [9, P. 58, (4)], [7, P. 115-135], for (19) in the traditional form, and for the optimal speed we get the expression absolutely identical to (13).

3. Practical application of the problem solution

Let us assume an airplane that has the flight parameters as follows: $M_0 = 10{,}000$ kg; $M_E = 8{,}000$ kg; $\eta = 0.3$; $Q = 42{,}700 \cdot 10^3$ J/kg; $\rho = 1$ kg/m^3; $S = 50$ m^2; $C_{x_0} = 0.02$; $b = 0.045$.

Calculation experiments by the methods (1)-(22) demonstrate the optimal speed, fig. 1, prevalence (optimal operational speed of the horizontal flight control dominating) preferences comparatively to any other modes of flight (operation).

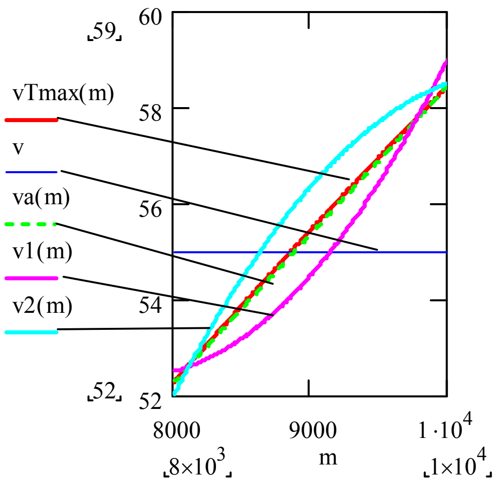

Fig. 1. Variants of the horizontal flight speeds

3.1. Models with unextremal speeds of the horizontal flight

A practically interesting case, when there is not the variated function of the flight speed in the functional (19), is illustrated in fig. 1, 2.

It may relate with the situation when the extremal (13) of the functional (19) definitely exists, however, it cannot be reached or realized for one reason or another.

In fig. 1 it is depicted: $vT\max(m)$ – the extremal (optimal) speed of the horizontal flight; v – a constant flight speed from the optimal diapason; $va(m)$ – the approximated optimal speed (it is the closest to the extremal, but not optimal although); $v1(m)$, $v2(m)$ – the changeable unextremal speeds.

It is important to emphasize that we do not impose which operational mode is better. That is, in no way and never in this problem formulation we have asserted conditions of the kind of inequalities.

3.2. Comparison of two unextremal speeds of the horizontal flight

If (19) contains the two unextremal speeds only.

In fig. 2 it is shown: the preferences functions – $\pi 1(m)$, $\pi 2(m)$; the entropies – $H(m)$ and $\ln(2)$, in corresponding scales f and n; the function $FGo(m)$ – represents the relations between π_1 – in the canonical view of the equation, and the functions of the derivatives from (19), in fig. 2 it is similar with π_1; the function $\Delta v1(m)$ – it takes into consideration the difference between the extremal speed of the flight and the one not extremal from (19); the function $\Delta v2(m)$ – it takes into account the difference between the extremal speed of the flight and the other not extremal from (19); the function $\Delta v12(m)$ – it takes into account the difference between these not extremal speeds of the flight from (19).

4. The researches results

The knot points of $FGo(m)$, in fig. 2 with the coordinates of $(8110, 0.5)$, $(9090, 0.5)$, $(9890, 0.5)$, which is a function of the difference $F_1 - F_2$, signify the fact that the active element at the control of the alternative operational modes is guided by the principle of the choice of the prevailing derivative or differential, the first difference of the first derivatives or differentials.

Conclusions

We specially need to emphasize that there was not any preconditions of the kind of which mode is better at the problem formulation. Nevertheless, the principle of the entropy extremization allows making a choice of the prevailing alternative.

Prospects of further researches. It is important to investigate other types of functionals of the kind of (19), as well as with different functions of effectiveness of the sort of F_1 and F_2, also research operational modes of control for horizontal flights of maximal distance.

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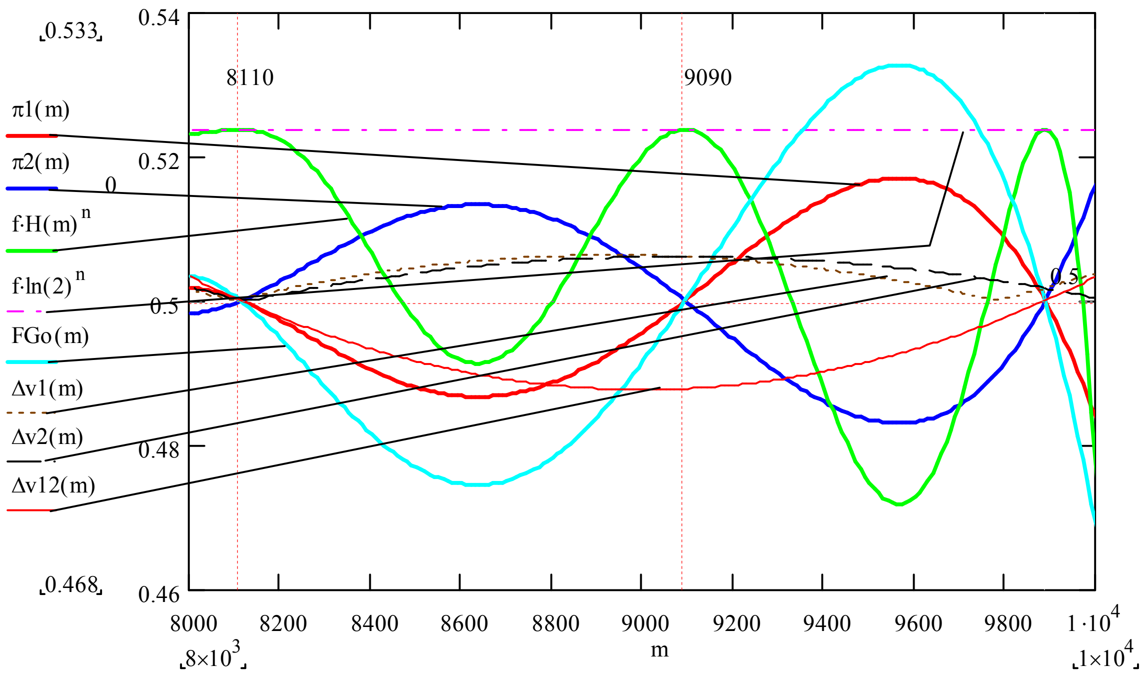

Fig. 2. Controlling operational modes preferences and subjective entropy formed by the effectiveness functions

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ОПТИМАЛЬНО УПРАВЛЯЕМЫЕ СУБЪЕКТИВНО ПРЕДПОЧИТАЕМЫЕ РЕЖИМЫ ЭКСПЛУАТАЦИИ ГОРИЗОНТАЛЬНОГО ПОЛЕТА САМОЛЕТА МАКСИМАЛЬНОЙ ПРОДОЛЖИТЕЛЬНОСТИ

В.А. Касьянов, А.В. Гончаренко

Исследована эксплуатация самолета на режимах горизонтального полета максимальной продолжительности. Экстремали функционала на основе интеграла традиционной формы, полученного с учетом уравнений движения материальной точки переменной массы И.В. Мещерского (1893-1897), Т. Леви-Чивита (1928), с дополнением энтропии индивидуальных субъективных предпочтений управляющего элемента активной системы получены как решения вариационной задачи при необходимых условиях существования экстремумов в виде системы уравнений Эйлера-Лагранжа. Канонические распределения предпочтений и оптимальный режим горизонтального полета максимальной продолжительности получены также в случае, когда экстремальная скорость не может быть достигнута либо реализована по тем или иным причинам. Управление в таком случае подразумевает предпочтения больших производных либо дифференциалов в качестве функций эффективности. Оптимальные режимы эксплуатации получаются на основе экстремума субъективной энтропии только, и никаких других ограничений, кроме условий нормировки, не накладывается в постановке проблемы. Проведены необходимые расчеты. Построены соответствующие диаграммы.

Ключевые слова: максимальная продолжительность, горизонтальный полет, субъективная энтропия, индивидуальные предпочтения, многоальтернативность, управление активными системами, активный элемент, вариационная задача, эксплуатационный функционал.

ОПТИМАЛЬНО КЕРОВАНІ СУБ'ЄКТИВНО ПЕРЕВАЖНІ РЕЖИМИ ЕКСПЛУАТАЦІЇ ГОРИЗОНТАЛЬНОГО ПОЛЬОТУ ЛІТАКА МАКСИМАЛЬНОЇ ТРИВАЛОСТІ

В.О. Касьянов, А.В. Гончаренко

Досліджено експлуатацію літака на режимах горизонтального польоту максимальної тривалості. Екстремалі функціоналу на основі інтеграла традиційної форми, отриманого із урахуванням рівнянь руху матеріальної точки змінної маси І.В. Мещерського (1893-1897), Т. Леві-Чівіта (1928), із доповненням ентропії індивідуальних суб'єктивних переваг керуючого елемента активної отримано як розв'язки варіаційної задачі при необхідних умовах існування екстремуму у вигляді системи рівнянь Ейлера-Лагранжа. Канонічні розподіли переваг та оптимальний режим горизонтального польоту максимальної тривалості отримано також у випадку, коли екстремальна швидкість не може бути досягнута або реалізована з тих чи інших причин. Керування у такому випадку передбачає переваги більших похідних або диференціалів у якості функцій ефективності. Оптимальні режими експлуатації отримуються на основі екстремуму суб'єктивної ентропії лише, і ніяких інших обмежень, за винятком умови нормування, не накладено у постановці проблеми. Проведено необхідні розрахунки. Побудовано відповідні діаграми.

Ключові слова: максимальна тривалість, горизонтальний політ, суб'єктивна ентропія, індивідуальні переваги, багатоальтернативність, керування активними системами, активний елемент, варіаційна задача, експлуатаційний функціонал.

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