AUTOMATIC CONTROL SYSTEMS

UDC 629.3.025(045)

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INFLUENCE OF SHOCK ON THE VIBRATION AMPLITUDE STABILIZATION SYSTEM OF CORIOLIS VIBRATORY GYROSCOPE RESONATOR

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The problem of parameters choice of Coriolis vibratory gyroscope resonator oscillation amplitude stabilization system which provide small prescribed contribution to angle and angle rate errors under specified external shocks is solved in this work.

Key words: stabilization system, gyroscope, external shock.

Introduction. Coriolis vibratory gyroscope (CVG) is important inertial technology, not only because these gyroscopes showed high quality as, for example, quartz hemispheric resonator gyroscope [1], but also because two other solid-state gyroscopes, ring laser and fiber optic ones, does not yield to such natural miniaturization as CVG. A microminiature variant of CVG based on microelectromechanical system (MEMS gyroscope), is already today practical reality and a subject of proceeding intensive researches in all high technology countries.

CVG can have two modes of operation: angle rate mode and rate integrated mode. Angle rate mode of operation is that radial standing wave in a cylindrical resonator (other geometries of resonators are possible) is excited on the second vibration mode, which is characterized by four antinodes and four nodes of vibration. At the rotation of resonator about axis of its symmetry Coriolis force appears that is equal to $F_c = 2mV\Omega$, where $V = A_0\omega_r\sin(\omega_r t)$ is linear velocity of the elementary masses of resonator in the process of vibration, A_0 is amplitude of vibrations, ω_r is resonant frequency of the cylinder, Ω is measured angular rate. This Coriolis force is measured and provided to CVG vibratory wave control system, that produces the feedback signal so, that the vibrations caused by Coriolis force are damped, at this amplitude of feedback signal is proportional to angular rate.

As can be seen from the relationships given above Coriolis force is amplitude modulated by angular rate, and carrier is a resonant frequency. Coriolis force is proportional to the angular rate provided that vibration amplitude A_0 is constant.

The problem of amplitude stabilization system parameters choice, which give small prescribed contribution to the angular rate and angular measurement error under action of external shock is solved in this work. The problem is complicated by that to form a control signal it is necessary at first to demodulate the measured signal, and to re-modulate it by resonant frequency before sending it to resonator, but modulation and demodulation procedures make the system nonlinear one.

Plant description. The plant is a CVG sensing element (SE), consisting of metallic resonator in form a cylinder fixed on the basis with stem, as shown in fig. 1. The bottom of cylinder has 8 openings between which there are spokes. Flat piezoceramic electrodes are glued on spokes symmetrically on a circle by angular step in 45^o [2].

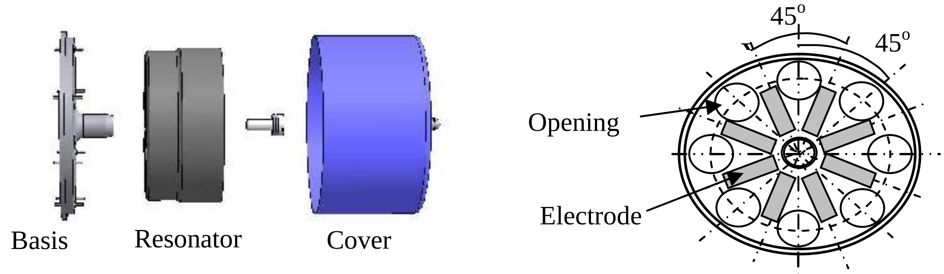

Fig. 1. Coriolis vibratory gyroscope sensing element as a plant

The rim of resonator has a thickness greater, than its bottom, and the vibration of the rim is excited by applying variable voltage at the resonant frequency on the piezoceramic electrodes. Diametrically opposite electrodes are connected for symmetry of applying forces, and also to increase response signal. Four electrodes located through 90^o participate in the excitation of SE vibration with constant amplitude using stabilization system, and four other electrodes, also located through 90^o participate in the system of measuring and compensation for the Coriolis force.

Transfer function linearization. One of the difficulties to analyze CVG SE amplitude stabilization system is that the presence of modulator and demodulator which make the system nonlinear. The system has to measure vibration amplitude by demodulating of SE output signal, to form the control signal, then to modulate this control signal by resonant frequency and to provide it to resonator to retain vibration amplitude at the desired level of A_0 under action of external impulse shock. One of the possible ways to realize such control system is to use automatic gain control (AGC) which structure in application to CVG is shown in fig. 2.

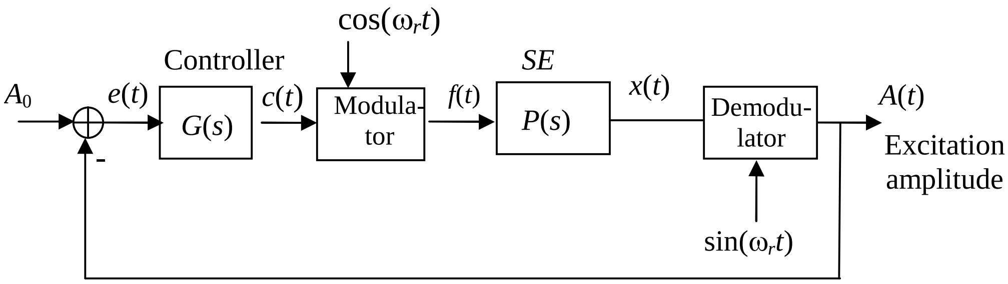

Fig. 2. Block diagram of automatic gain control

Control signal is formed by the comptroller with transfer function $G(s)$ and by means of feedback (see fig. 2) stabilizes vibration amplitude. For determination of transfer function of controller, i. e. control law, it is necessary to know how changes in the signal provided to excitation will influence on vibration amplitude of the resonator. In other words, it is needed to replace the demodulator and modulator for one block, as it is shown in fig. 3 and to define its transfer function. This transfer function that is designated as $H(s)$ will give an opportunity to build the transfer function of controller $G(s)$, that will stabilize vibration amplitude.

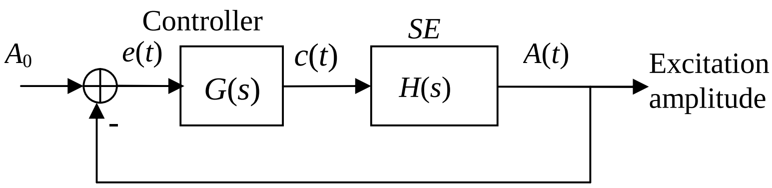

Fig. 3. The reduced AGC block diagram

Before the determination of SE transfer function $H(s)$ it should be noted that modulation and demodulation are nonlinear operation and the linear transfer function of *H(s)* does not exist. However, during gyro operation at the resonant frequency, this nonlinear function can be approximated by a linear transfer function that will describe nonlinear SE plus modulator and demodulator exactly. One of such methods of approximation was presented in [3]. Another method is based on SE dynamic equations in the so-called “slow variables” [4; 5], i. e. demodulated variables. From the point of view of the automatic control systems, it is expedient to use the method presented in [3].

Let’s present a signal $c(t)$ in the input of $H(s)$ (see fig. 3) as a sine wave with frequency ω

$$c(t) = A\sin(\omega t). \tag{1}$$

After modulation by the resonant frequency ω_r, this signal as a function $f(t)$ (see. fig. 2) is applied to SE:

$$f(t) = c(t)v(t) = A\sin(\omega t)\cos(\omega_r t). \tag{2}$$

Sensing element is an oscillatory system that can be represented by the linear link of the second order:

$$P(s) = \frac{P_0}{s^2 + \frac{\omega_r}{Q}s + \omega_r^2}, \tag{3}$$

where Q is the resonator quality factor.

Sensing element output signal, $x(t)$, when there is a signal $f(t)$ on its input, is equal to:

$$x(t) = \frac{1}{2}\left|P(\omega+\omega_r)\right|\sin\left[(\omega_r+\omega)t+\varphi_1\right] + \frac{1}{2}\left|P(\omega-\omega_r)\right|\sin\left[(\omega_r-\omega)t+\varphi_2\right], \tag{4}$$

where $\left|P(\omega \pm \omega_r)\right|$ is the resonator amplitude response, φ_1, φ_2 are phases, which acquire signal when passing through the resonator at the frequencies $\omega + \omega_r$ and $\omega - \omega_r$, that is resonator phase response.

This expression can be considerably simplified, if $\omega << \omega_r$, that can be valid when frequencies of external mechanical shocks are considerably less than resonant frequency (4–5 kHz). In this case amplitude and phase responses of the resonator are equal to:

$$P_1 = \left|P(\omega+\omega_r)\right| = \left|P(\omega-\omega_r)\right| \approx \frac{QP_0}{\omega_r}\frac{1}{\sqrt{(2Q\omega)^2\omega_r^2}}, \tag{5}$$

$$\varphi_1 = -\frac{\pi}{2} - \alpha;\ \varphi_2 = -\frac{\pi}{2} + \alpha;\ \alpha = \operatorname{arctg} - \frac{2Q\omega}{\omega_r}; \tag{6}$$

Sensing element output signal, $A(t)$, after demodulation by reference signal $\sin(\omega_r t)$, can be represented as follows:

$$A(t) = P_1\sin(\omega t + \alpha). \tag{7}$$

So, when $H(s)$ input signal is $c(t)$, there is $A(t)$ signal on its output provided that $\omega << \omega_r$ with amplitude and phase responses which are presented by the expressions (6) and (7), respectively. This means that single block $H(s)$: SE, modulator and demodulator, is linear and time invariant block with the following transfer function:

$$H(s) = \frac{QP_0}{\omega_r^2}\frac{1}{\frac{2Q}{\omega_r}s + 1}. \tag{8}$$

The demodulator dynamics does not take into account in the transfer function (8) that is the demodulator is considered to be an ideal one. Real demodulator has errors, for example, due to time delay caused by low pass filter. Demodulator dynamics can be introduced into AGC structure as it is shown in fig. 4.

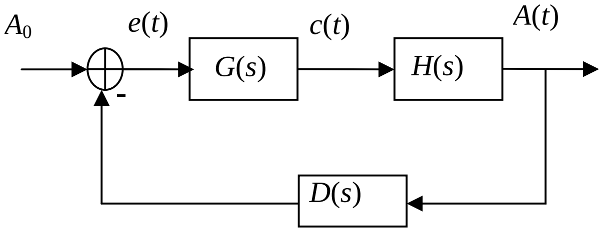

Fig. 4. Automatic gain control block diagram with demodulator time delay $D(s)$

The transfer function $D(s)$ depends on the method of demodulation and for the standard method of amplitude demodulation, this transfer function can be represented as a time delay T_d with an amplification coefficient K_d:

$$D(s) = K_d e^{-sT_d}. \tag{9}$$

The amplification coefficient K_d depends on algorithm which is used for the demodulation operation, and time of delay T_d is determined mainly by the order of low pass filter being used. In case of digital realization of AGC time delay T_d will be also dependant on processor speed.

Because implementation of control system is mainly carried out in a digital form, then in order that the results of analog design could be expanded to the digital systems it is necessary to choose sampling frequency far exceeding Nyquist frequency. For resonators with resonant frequency in the range 4–6 kHz, the Nyquist minimum sampling frequency should be 8–12 kHz. So that, if to choose sampling frequency of about 100 kHz or higher (i. e. an order of magnitude more), then the results obtained for the analog system will be close to digital one.

Automatic gain control block diagram with proportional and integral (PI) controller is presented in fig. 5.

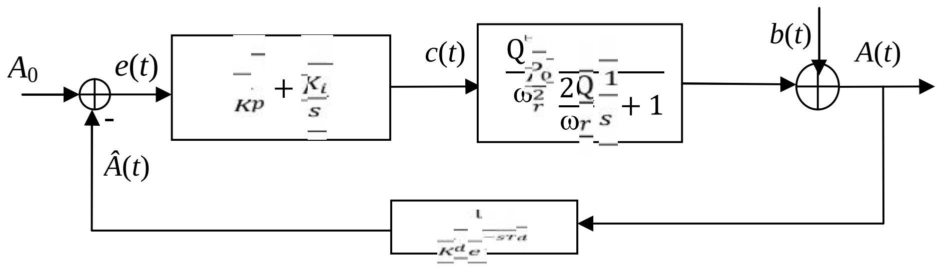

Fig. 5. Linearized AGC block diagram with PI controller

Parameter values of transfer functions presented in fig. 5 have been taken close to that of real CVG: $Q = 3 \cdot 10^4$ (metallic resonator), $\omega_r = 2{,}5 \cdot 10^4$ rad/s (resonator diameter 25 mm), ratio of output and input signal amplitudes is approximately equal to 0,5–0,6. This means that $P_0 = 10^4$. In addition we will accept $K_d = 1$, $T_d = 10^{-4}$s.

Shock disturbance suppression. The main task of controller is to suppress vibration amplitude disturbances caused by external shock to an acceptable prescribed level. Amplitude disturbances can be presented as an additive component of $b(t)$ entering the system on the output of SE as it is shown in fig. 5. A transfer function on the introduced shock disturbance is determined by the following expression:

$$\frac{A(s)}{B(s)} = \frac{1}{1 + D(s)H(s)G(s)} = \frac{\omega_r^2 s\left(\dfrac{2Q}{\omega_r}s + 1\right)}{\omega_r^2 s\left(\dfrac{2Q}{\omega_r}s + 1\right) + QP_0 K_d K_i \left(\dfrac{K_p}{K_i}s + 1\right)e^{-sT_d}}. \tag{10}$$

It is required to choose K_i and K_p parameters so that the angular rate integral measurement error e_α (i. e. angular measurement error) due to vibration amplitude changes, caused by shock, would be less than any small prescribed value. Because CVG described above is the rate gyro, then the angular error e_α can be written down as follows:

$$e_\alpha = v\int_0^t \left|A(t) - A_0\right| dt, \tag{11}$$

where ν is a coefficient of vibration amplitude instability influence on the angular measurement error, $A(t)$ is the measured vibration amplitude. If $t \to \infty$, then e_α is total angular measurement error and it can be estimated provided that $b(t) = 0$ for $t \le 0$ on the basis of Laplace transform property, saying that integral in time domain is numerically equal to the value of its Laplace transform at $s = 0$. Let step occurs in vibration amplitude by the value of β, as a result of external shock. Laplace transform of such step is equal to $B(s) = \beta/s$, and the angular error, taking into account the expression (11) at $s = 0$, is estimated by the expression:

$$\lim_{n\to\infty} e_\alpha = vA(0) = \frac{v\omega_r^2\beta}{K_d K_i P_0 Q}. \tag{12}$$

It should be noted that total angular error does not depend on amplification coefficient K_p of proportional part of the PI controller, but depends only on amplification coefficient K_i of integral part of the PI controller and SE parameters. It should be also noted, that the more the resonator quality factor (Q-factor), the less the angular measurement error, and the less the resonant frequency ω_r (i. e. the more the cylinder diameter), the more accurate CVG is.

If not to change position of transfer function zeros, i. e. to keep unchanged the ratio of K_i/K_p, then at the increase of K_i it will increase the bandwidth of vibration amplitude stabilization channel. However noise of this channel will increase, too. A compromise is therefore needed between the bandwidth and the noise. If it is required to choose K_i so that a contribution to the angular measurement error due to this channel would not exceed the level of E_s, then it is necessary to choose K_i from the following condition:

$$K_i \ge \frac{v\omega_r^2\beta}{E_s K_d P_0 Q}, \tag{13}$$

¬ for CVG can be equal to¬ = 1.

For the estimation of K_i magnitude, it is necessary to estimate β. Suppose that in the process of CVG operation the external shock occurred of 5 ms duration and 100 g amplitude along the CVG sensing axis. Suppose, also, that an impulse has a rectangular form (the worst case). Under the action of such shock the bottom of resonator together with piezoelectrodes are deformed and voltage is picked off from the piezoelectrodes in the form of shock impulse. The resonator bottom deformation is due to the force acting from the upper massive ring (see fig. 1). Mass of the ring is determined as product of its volume by material density:

$$m = 2\pi r h t_r \rho, \tag{14}$$

where r is the ring radius, h is the ring height, t_r is the ring thickness, ρ is the ring material density. Under action of the 100 g shock the ring's weight increases 100 times and bottom experiences total force equal to:

$$F = 100mg = 200\pi r h t_r \rho g. \tag{15}$$

Each of eight electrodes glued at the spokes (see fig. 1) is subjected to the action of the force equal to $F_p = F/8 = 25\pi r h t_r \rho g$. At this, spokes are deformed under action of moment of force $M = F_p L_s$, where L_s is spokes length. As result of this deformation elongation (compression) of spokes together with piezoelectrode is occurred. Tensile strain ε under this deformation is determined as:

$$\varepsilon = \frac{\Delta L_s}{L_s} = \frac{t_s}{2E_r I} M = \frac{t_s L_s}{2E_r I} F_p \to \Delta L_s = \frac{t_s L_s^2}{2E_r I} F_p; \qquad I = \frac{L_s\left(t_s + t_p\right)^3}{12}. \tag{16}$$

Here t_s is spoke thickness, t_p is piezoelectrode thickness, I is spoke with glued piezoelectrode cross section moment of inertia, E_r is Young modulus material, ΔL_s is spokes elongation.

Substitution of expression for I into expression for ΔL_s yields:

$$\Delta L_s = \frac{6t_s L_s}{E_r (t_s + t_p)^3} F_p. \quad (17)$$

Tensile force acting on the spoke with electrode can be determined as follows:

$$F_t = k_s \Delta L_s, \quad (18)$$

k_s is the spoke rigidity. Spoke rigidity is on the order of magnitude higher, than that of piezoelectrode, therefore in (18) piezoelectrode rigidity can be neglected. Spoke rigidity can be calculated using Young modulus E_r and its size as follows:

$$k_s = \frac{E_r t_s^3}{12 L_s^2}. \quad (19)$$

Electrical voltage, which is induced under action of force F_t on piezoelectrode, taking into account (17) and (19), can be obtained by the following expression:

$$V = \frac{g_{31} F_t}{w_p} = \frac{g_{31} k_s \Delta L_s}{w_p} = \frac{25 g_{31} t_c^4 \pi r h t_k \rho g}{2(t_c + t_p)^3 L_c w_c} =$$
$$= \frac{25 \cdot 11,2 \cdot 10^{-3} (3 \cdot 10^{-4})^4 \cdot 3,14 \cdot 12,5 \cdot 10^{-3} \cdot 10^{-2} \cdot 10^{-3} \cdot 7,8 \cdot 10^3 \cdot 9,8}{2(5 \cdot 10^{-4} + 3 \cdot 10^{-4})^3 5 \cdot 10^{-3} \cdot 3 \cdot 10^{-34}} \approx 0,005\ V, \quad (20)$$

where w_p is piezoelectrode width (it is equal to spoke width) the step.

Thus at 100 g shock amplitude along CVG sensing axis jump of voltage on the output of SE is equal to 5 mV. Because signals of two diametrically opposite electrodes in the resonator are connected to each other, then to the input of the vibration amplitude stabilization system the step of 10 mV voltage will be entered in view of rectangular impulse of 5ms duration. The signal component concentrated in the frequency range of $\omega_r \pm 100$ Hz can pass through the system because the system is a pass band filter. For real CVG $\omega_r = 4000$ Hz, that is the frequency range is [3,9–4,1] kHz.

To determine amplitude of the indicated component, the 0,01V amplitude impulse of 5 ms duration is passed through the pass band filter of [3,9–4,1] kHz. Fig. 6 shows the output signal of the pass band filter.

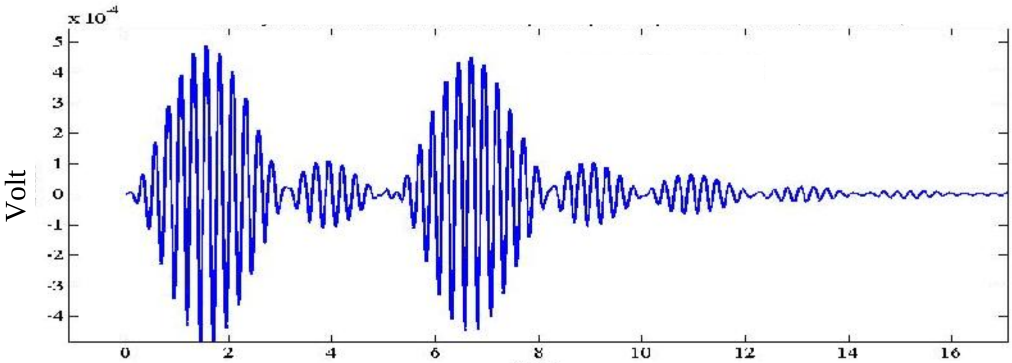

Fig. 6. Band pass filter response on the 10 mV impulse of 5 ms duration

As can be seen from the graph the maximum of the filter response amplitude is equal to $4,86 \cdot 10^{-4}$ V. So, it can be stated that 100 g external shock amplitude of 5 ms duration appears on the output of the stabilization system as 0,485 mV amplitude step which induces the angular error.

Returning to the estimation of amplification coefficient K_i of integrating part of PI controller according to expression (13), let's take $\beta = 4,86 \cdot 10^{-4}$V, $P_0 = 10^4$, $K_d = 1$, $\omega_r = 2,5 \cdot 10^4$ rad/s, $Q = 3 \cdot 10^4$, $\nu = 1$, and total angular measurement error $E_s = 0,1$ arc. min ($E_s = 2,9 \cdot 10^{-5}$ rad), obtain:

$$K_i \geq \frac{v\omega_r^2\beta}{E_s K_d P_0 Q} = \frac{(25\cdot 10^{-4})^2 \cdot 4{,}86\cdot 10^{-4}}{2{,}9\cdot 10^{-5}\cdot 10^4 \cdot 3\cdot 10^4} \approx 33. \tag{21}$$

Automatic gain control amplitude-frequency characteristic on external disturbance for K_i = 50 and K_p = 3000 is shown in fig. 7. As can be seen from the figure, suppression band of external disturbances (shocks, vibration acceleration during motion etc.) is extended to 150–200 Hz inside the SE pass band.

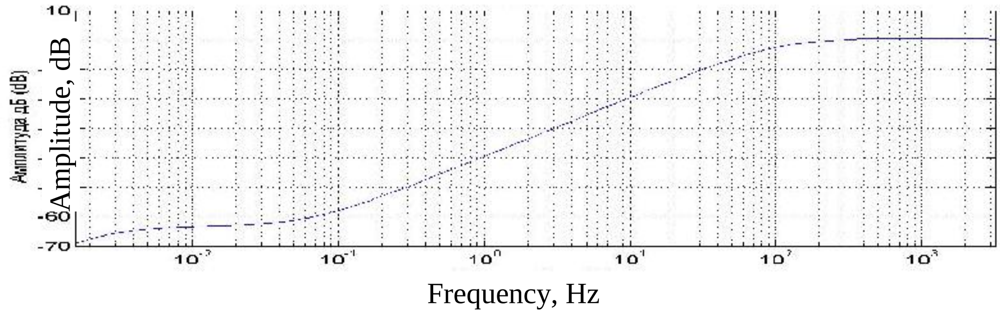

Fig. 7. Amplitude –frequency characteristic of vibration amplitude stabilization system

Fig. 8 shows step response of the stabilization system. As can be seen settling time is equal to 3,53 ms that assures accuracy under action of external shocks.

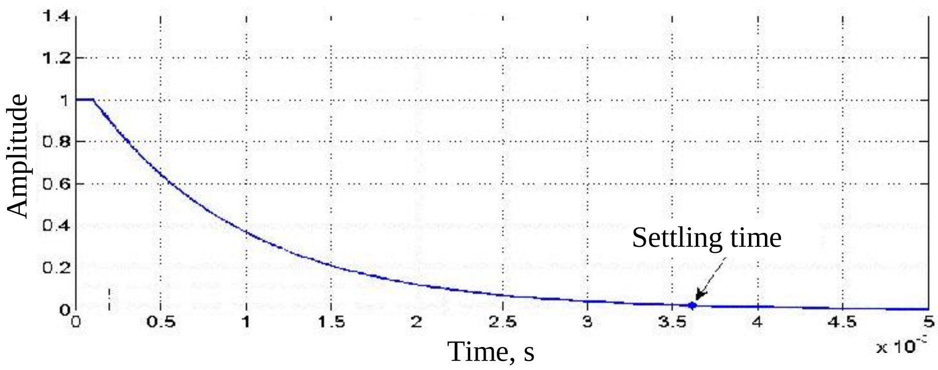

Fig. 8. Step response of amplitude stabilization system

It should be noted that the wider the suppression band of external disturbances, the higher the noise in the stabilization system. Noise in the stabilization system, if it has zero average, does not influence on the CVG main performance parameter that is bias. When integrating CVG output signal to calculate angular of rotation, noise contribution is proportional to $\sqrt{t}$, where t is integration time, if t >>1, then angle measurement error due to noise (even large one) is significantly less, than that of bias (even not large), since the latter is accumulated proportional to the first order of t. Nevertheless, as a rule, from practical point of view it is expedient to choose compromise value of the external disturbance suppression band.

Conclusion. Metallic resonator CVG is a promising gyro, which is able to provide high accuracy when measuring rotation angle and angular rate in severe environment. The measured angular rate modulation by high resonant frequency (4–6 kHz) significantly restricts the influence of external disturbances on the CVG output signal. In application where external disturbances can reach the resonant frequency conventional methods to protect CVG from the external disturbances should be used, for example, dampers. Protection from high frequency mechanical disturbance can be effectively provided by the passive dampers on the basis of rubber, metallic rubber and others. Digital implementation of AGC for CVG is undoubtedly more expedient, than analog one.

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В. В. Чіковані

Вплив удару на систему стабілізації амплітуди коливань резонатора коріолісового вібраційного гіроскопа

Розв’язано задачу вибору параметрів системи стабілізації амплітуди коливань коріолісового вібраційного гіроскопа, яка дає малий, наперед заданий, внесок у похибку вимірювання кутової швидкості та кута повороту, при заданих зовнішніх ударних впливах.

В. В. Чиковани

Влияние удара на систему стабилизации амплитуды колебаний резонатора кориолисового вибрационного гироскопа

Решена задача выбора параметров системы стабилизации амплитуды колебаний кориолисового вибрационного гироскопа, которая дает малый наперед заданный вклад в погрешность измерения угловой скорости и угла поворота, при заданных внешних ударных воздействиях.