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AERODYNAMIC COEFFICIENTS APPROXIMATIONS FOR ARBITRARY ANGLES

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This paper addresses the whole angle approximations of aerodynamic coefficients. Several nonlinear approximations are proposed to describe lift, drag and side force coefficients in the range from 0 to 360 degrees based on accuracy and simplicity.

Keywords: flight simulations; aerodynamic coefficients; nonlinear approximations.

Introduction. New improvements in modern aircraft designs have resulted in the need for highly sophisticated mathematical models of aircraft dynamics. Such a model could not be developed without corresponding lift, drag and side force representations that will be both accurate and relatively simple. At the same time, simple and accurate models of aircraft dynamics are highly necessary for flight control and navigation system research and development, and for flight simulations in both civil aviation and military trainings. The aerodynamic model is possibly the most critical element of a flight simulator. Although conventional and widely used currently linear approximations of lift and drag coefficients deliver good results for the small incidence angle, they expectedly fail long before the incidence angle even reaches its stall value [1].

In [1] several harmonic approximations of lift and drag coefficients are compared and analyzed. The best representation in terms of accuracy and simplicity is found and proposed, and the problem of its parameters estimation is solved. In this paper several nonlinear approximations of side force coefficients are compared.

Aerodynamic Forces and Coefficients. Let us consider two reference systems with the same origin O at the center of gravity. Fixed-body system $X_bY_bZ_b$ has the X_b axis pointing forward out of the nose of the aircraft, the Y_b axis pointing out the starboard wing and the Z_b axis pointing down. And velocity reference system $X_wY_wZ_w$, where $\vec{V}$ – the total velocity vector, as shown in fig. 1. The aerodynamic forces depend only upon the angles α (incidence angle) and β (sideslip angle), which orient the total velocity vector $\vec{V}$ in relation to the axis X_b.

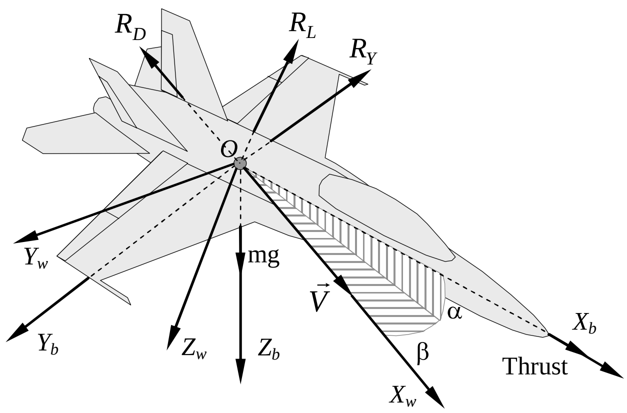

Fig. 1. Aerodynamic forces

Viewed from above (in plan form), most aircrafts are symmetric. However, if the fuselage is not aligned with the direction of flight, it will make an incident angle with the wind, known as the angle of sideslip, β. Although the side force generated will be considerably less than the lift force on an aircraft, it is still significant.

Under these conditions, the fuselage acts as a large airfoil, with an angle of incidence β to the wind. Consequently, lift is generated by the fuselage but in the direction along the wing, that causes a lateral force, known as side force.

For the essentially subsonic velocities, aerodynamic lift, drag and side forces are usually calculated as [2]:

$$R_L = \frac{\rho V^2 S}{2} C_L(\alpha), \quad R_D = \frac{\rho V^2 S}{2} C_D(\alpha), \quad R_Y = \frac{\rho V^2 S}{2} C_Y(\beta), \tag{1}$$

where $C_Y(\beta)$ is the side force coefficient, $C_L(\alpha)$ is the lift coefficient, $C_D(\alpha)$ is the drag coefficient, $C_Y(\beta)$ is the side force coefficient, ρ is the air density at a given flight altitude, and S is the characteristic wing area.

Using the conventional approach and assuming small incidence angles, the lift coefficient is usually linearly approximated as:

$$C_L(\alpha) = C_{L_0} + C_{L_\alpha}\alpha, \tag{2}$$

where C_{L_0} is the value of C_L for zero incidence angle, $C_{L_\alpha} = \partial C_L / \partial\alpha$ is the change in airplane lift force coefficient due to a change in incidence angle.

For the same small incidence angles, the aerodynamic drag coefficient is represented in the following form:

$$C_D(\alpha) = C_{D_0} + C_{D_\alpha}\alpha, \tag{3}$$

where C_{D_0} is the value of to C_D for zero incidence angle, $C_{D_\alpha} = \partial C_D / \partial\alpha$ is the change in airplane drag force coefficient due to the change in incidence angle.

The side force coefficient is usually linearly approximated as:

$$C_Y(\beta) = C_{Y_0} + C_{Y_\beta}\beta, \tag{4}$$

where C_{Y_0} is the value of C_Y for: $\alpha = \beta = 0$, $C_{Y_\beta} = \partial C_Y / \partial\beta$ is the change in airplane side force coefficient due to a change in angle of sideslip (at constant angle of attack) [2].

In reality the aerodynamic drag coefficient is not only dependent on the incidence angle, but also on the sideslip angle. Similarly to the longitudinal motion harmonic approximation of the drag coefficient in [1], in case of both incidence and side-slip angles present the following approximation can be suggested:

$$C_D(\alpha,\beta) = d_0 + d_1\cos(2\alpha) + d_2\cos(4\alpha) + d_3\cos(2\beta) + d_4\cos(4\beta). \tag{5}$$

Coefficients d_0, d_1, d_2, d_3, d_4 could be easily calculated using the same approach as for the side force coefficient, for this purpose wind tunnel data or data obtained could be used using CFD tools.

Because of all of the preceding approximations (2–4) are true for the small incidence angles, it is not suitable for the whole incidence angle (360-deg) modeling. In this case, other approximations are need to be proposed and investigated, that was done in [1] for the lift and drag coefficients and in this research nonlinear approximations were proposed for the side force coefficient.

Different Nonlinear Approximations. If we consider the fact that all of the aerodynamic coefficients are naturally 2π periodic, approximations that use sine and/or cosine functions are the first and obvious choice [1].

For the investigation of different approximation approaches for side force coefficient data from [3] is used. How side force coefficient depends on angle of attack and sideslip angle is shown in fig. 2.

As one can see from the fig. 2 the surface of the side force coefficient under different α and β has complex shape. So, the approximations for this coefficient investigated in this paper are as follows:

$$C_Y = \sin((k_1\beta)(s_1 \sin(k_2\beta + c_2) + s_2(\cos(\alpha + c_1))), \tag{6}$$

$$C_Y = \sin((k_1\beta)(s_2(\sin(\alpha + c_1))^2), \tag{7}$$

$$C_Y = \sin((k_1\beta)(s_1 \sin(k_2\beta + c_2) + s_2(\sin(\alpha + c_1))^5), \tag{8}$$

$$C_Y = \sin((k_1\beta)(s_2(\sin(\alpha + c_1))^3), \tag{9}$$

where α is the incidence angle in radians, β is sideslip angle in radians $k_1, k_2, s_1, s_2, c_1, c_2$ are independent parameters for the side force coefficient, that are to be determined based on the best of experimental data.

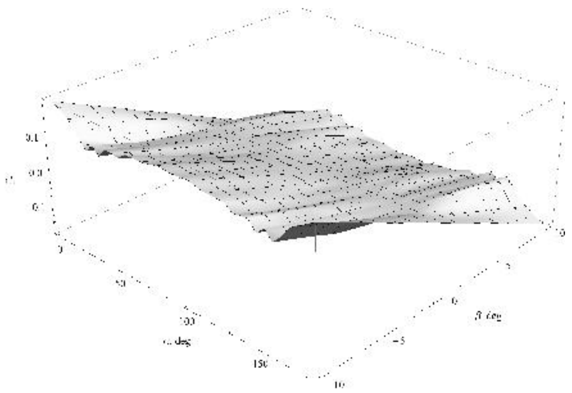

Fig. 2. Side force under different α and β

Approximation Parameters Calculation. The set of approximation functions was considered during the research (fig. 3).

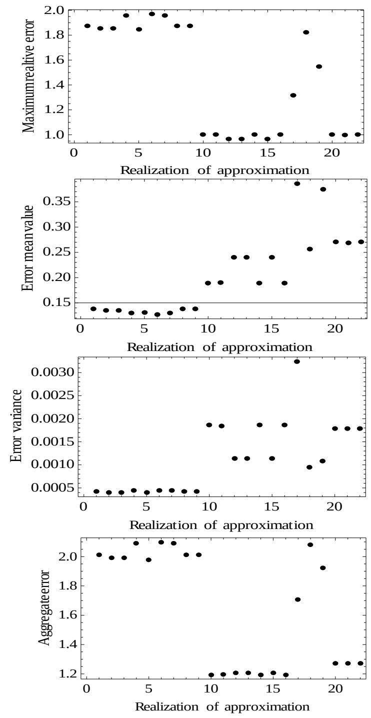

Fig. 3. Characteristics of errors for different approximations

For each function parameters were calculated, which were best fitted into the experimental data using the nonlinear regression method. For every function the mean relative error, maximum relative error and variance were evaluated and it is an aggregate characteristic according to the experimental data, which were used from [3].

Let's define non-dimensional characteristics presented in fig. 4.

Maximum relative error is defined as follows:

$$MRE = \frac{\max\sqrt{(C_{Ya} - C_{Ye})^2}}{C_{Ym}},$$

where C_{Ya} is a vector of approximated values, C_{Ye} is a vector of experimental data and C_{Ym} is a value of C_{Ye} with the highest absolute error.

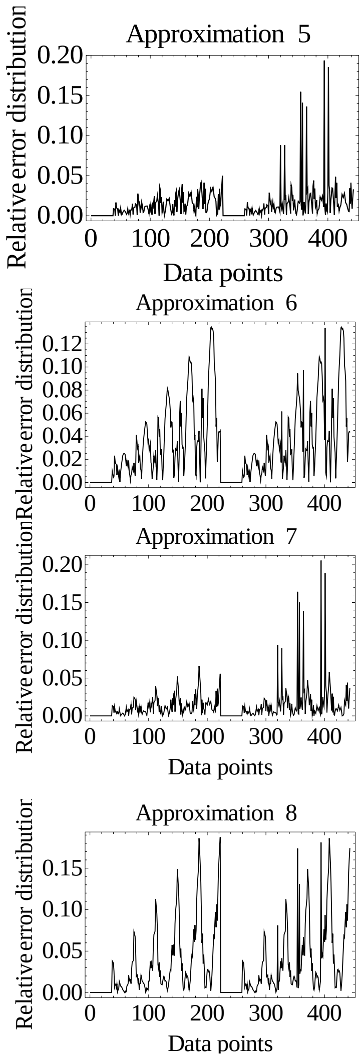

Fig. 4. Characteristics of errors for different approximations

Mean error value is defined by the following formula:

$$RME = \frac{mean\sqrt{(C_{Ya} - C_{Ye})^2}}{C_{Ym}}.$$

Variance is defined in the following way:

$$RVE = mean(C_{Ya} - C_{Ye})^2.$$

For this research an aggregate error is defined as:

$$A = MRE + RME + RVE.$$

Functions (6) – (9) have distribution of errors for the given experimental data as shown in fig. 4; relative error distribution is deviation of the approximated values from the experimental values normalized by the experimental values.

Conclusions. Empirical aerodynamic side force coefficient representations (6) – (9) allow expanding the existing linear representation (4) to the whole range of possible incidence and sideslip angles. Such improved harmonic representations could easily replace the conventional linear representations and deliver better results. Also, it's necessary to mention that these approximations were made only for the one set of experimental data and as future research obtained formulas should be checked on the different sets of experimental data or, as alternative, on the data obtained from CFD tools.

References

1. *Apostolyuk V.*, "Harmonic representation of aerodynamic lift and drag coefficients," AIAA Journal of Aircraft, Vol. 44, July-August 2007. – P. 1402–1404.
2. *Cook M.*, Flight Dynamics Principles, Second Edition: A Linear Systems Approach to Aircraft Stability and Control, 2nd ed. Butterworth-Heinemann, 2007.
3. *Hoffler K.*, *Fears S.*, *Carzoo S.*, "Generic airplane model concept and four specific models developed for use in piloted simulation studies," NASA CR-201651, February 1997. – P. 30–31.

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Апроксимація аеродинамічних коефіцієнтів для довільних кутів

Розглянуто представлення основних аеродинамічних коефіцієнтів для довільних кутів атаки та ковзання. Декілька нелінійних апроксимацій запропоновано для опису коефіцієнтів підйомної та бокової сили, а також сили опору в діапазоні від 0 до 360 градусів з точки зору точності та простоти.

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Аппроксимация аэродинамических коэффициентов для произвольных углов

Рассмотрено представление основных аэродинамических коэффициентов для произвольных углов атаки и скольжения. Несколько нелинейных аппроксимаций предложено для описания коэффициентов подъемной и боковой силы, а также силы сопротивления в диапазоне от 0 до 360 градусов с точки зрения точности и простоты.