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Improve the yen carry trade with economic fundamentals

Abstract

In this paper, economic fundamental variables are used to predict the exchange rates of major currencies against the Japanese yen in a factor augmented regression, where the factor is constructed from the risk premium of each currency. The yen carry trade is then simulated based on these forecasts. Carry trades based on these forecasts out-perform the naive carry trade based on the random walk forecast in terms of risk-adjusted returns and return skewness. The better performance is robust for different time periods and after controlling for the transaction cost. The result suggests that fundamentals are useful in practice although the academia generally consider them ineffective in predicting exchange rates.

Keywords: yen carry trade, exchange rate models, economic fundamentals, factor augmented regression, Kalman filtering, currency risk premium.
JEL Classification: F31, F37, G15.

Introduction

This study investigates whether the economic fundamentals of exchange rate models can improve the performance of currency trading. Particularly, we focus on the economic fundamentals used in the Taylor rule. Unlike the classic macroeconomics, where interest rate is the equilibrium result of monetary variables, the Taylor rule specifies how the interest rate responds to economic fundamentals. For example, when the inflation rate rises, the interest rate will increase in the Taylor rule while it will decrease in the monetary model. According to Engel and West (2005), the Taylor rule models are gaining momentum recently because they appear to be the potential candidate to "beat the random walk model". Many studies have found improvement in the forecasting ability of the exchange rate models when they include the economic fundamentals related to the Taylor rule (see Chinn and Pascual, 2005; Choi, Mark and Sul, 2006; Engel and West 2005; 2006; Engel, Mark and West, 2007; Molodtsova and Papell, 2008; Murray and Papell, 2002; Taylor, Peel, and Sarno, 2001). Given this encouraging development in the exchange rate modeling, one would wonder how useful the economic fundamentals are in the decision-making of currency investments.

We focus on a particular type of currency trading – the yen carry trade. Carry trade is a simple currency trading strategy of borrowing low-interest-rate currency and investing in high-interest-rate currency. The Japanese yen has become the major funding currency of the carry trade since the mid-1990s because of its unusually low interest rates[1]. Profit from the yen carry trade is the sum of the interest rate differential between the yen and a target currency, and the change in the exchange rate of the target currency. According to uncovered interest rate parity (UIRP), carry trade is unprofitable on average because the interest rate differential would be offset by the relative depreciation of the target currency. However, almost all empirical studies point to the opposite conclusion (for instance, see Cheung, Chinn, and Pascual, 2002; Engel, 1996; Lewis, 1995; Mark and Sul, 2001; Meese and Rogoff, 1983a; 1983b). This implies that the carry trader can pocket both the interest rate differential and the appreciation of the target currency, with zero capital. It is precisely for this reason that the cohort of hedge funds engaging in carry trade is growing.

The use of economic fundamentals in currency trading presents a mixed picture. According to Cheung and Chinn (2001), 90% of short-term foreign exchange (FX) traders in the UK trade currencies with technical analysis. For example, PowerShares DB G10 Currency Harvest (Ticker: DBV), an exchange traded fund (ETF), trades simply by borrowing low-interest-rate currency and investing in high-interest-rate currency. On the other hand, Moore Capital's Global Fund, a hedge fund, uses solid macro fundamentals to guide its carry trade. But one question remains unanswered. Are the economic fundamentals useful in improving the profitability of the carry trade at all?

This study is motivated by Engel and West (2005), who show that economic fundamentals and exchange rates are closely linked, even though the consensus in academia is that for prediction, "exchange rate models cannot beat the random walk". Furthermore, Engel, Mark, and West (2007) indicate that the unobservable factor in the exchange rates themselves may contain useful information for prediction. Naturally we would guess that the economic fundamentals should be useful for the carry trade. To fully uncover the usefulness of the economic fundamentals, a dynamic factor model is employed to extract information from the risk premium in exchange rates. This method is appealing in that the

[1] Galati, Heath, and McGuire (2007) claim that the yen accounted for about 80% of funding currency for carry trades in 2007.

factor is derived from the interest rate parity and has economic meaning[1]. The factor is then combined with other economic fundamental variables in the Taylor rule to forecast exchange rates in a factor augmented regression (FAR) model. Specifically, the factor is estimated in a dynamic factor model with the Kalman filtering technique. To accommodate possible nonlinearity in the exchange rate models, we also experiment with adding nonlinear forms of interest rate differentials as explanatory variables in the FAR. We recognize that some success has been reported in nonlinear modeling of foreign exchange rates, but do not find support for this success.

The yen carry trade is simulated in each of the following major target currencies: Australian dollar, New Zealand dollar, British pound, Canadian dollar, euro, and U.S. dollar. For each currency, the carry trade follows a go or no-go binary process at a monthly frequency. When the expected return is positive, the carry trade is executed; otherwise, the trade is skipped or a short position in the target currency is taken to enhance profit. The expected return is calculated based on the forecast exchange rates against the yen. Several specifications of FAR-based forecasting models are tested against the random walk model. Under the random walk theory, a naive carry trade will always occur as long as the interest rate is higher for the target currency than for the yen. Average return, Sharpe ratio, and skewness of returns are the main performance statistics reported.

The performance of yen carry trade with fundamentals are compared to models without fundamentals – random walk or AR(1). The results show a better performance of carry trades when the fundamentals are added. In particular, if the Taylor rule fundamentals are included under the FAR framework, carry trade generates better risk-adjusted returns across the six target currencies as well as the equally-weighted portfolio. To compare the tail risk, we compute the skewness of returns too[2]. The skewness is sharply improved in the FAR framework. The conclusion remains the same, even after accounting for transaction costs and simulation in different periods.

This paper stands out with the use of a dynamic factor framework[3]. It is well known that time-varying risk premiums in equity may be able to explain and predict returns (see, for example, Ferson and Harvey, 1991). The most recent effort by Engel, Mark, and West (2007) has generated a lot interest in this aspect in predicting exchange rates, followed by Christiansen, Ranaldo, and Soderllind (2010). We find a similar pattern, that a macro-fundamental derived factor plays a decisive role in boosting the returns to carry trade. We attribute this success to the factor's high persistence and strong correlation with the exchange rate. The Diebold-Mariano test and a simple count of directional forecasts also confirm the superior forecasting ability of the FAR model.

The paper is organized as follows. Section 1 presents the exchange rate models. Then we present the empirical resultsan analysis of factors and forecast evaluation and reality cheak. The last Section concludes.

1. Foreign exchange rate model and estimation method

The predictive regression model of exchange rate is:

$$\Delta s_{t+1} = \alpha + \beta_F F_t + z_t + \varepsilon_{t+1}, \tag{1}$$

where $\varepsilon_{t+1} \sim NID\left(0, \sigma^2\right)$,

where s_t is the log exchange rate expressed as units of yen per unit of target currency and $\Delta s_{t+1} = s_{t+1} - s_t$. An increase in s_t indicates appreciation of the target currency and depreciation of the yen, and vice versa.

The regression model is parsimonious. When we set $\beta_F = 0$, $\beta_z = 0$, the regression is the random walk (RW) model. This is the standard benchmark in the exchange rate prediction literature and practice. Another specification without fundamentals is obtained by setting $\beta_F = 0$, and $z_t = \beta_s \Delta st$. This is an AR(1) model that has been used extensively.

The regression equation has a generated regressor F_t, which is called the factor. The factor is estimated from the UIRP:

$$\Delta s_{t+1} = F_t + \left(i_t - i_t^*\right) + \varepsilon_{t+1}, \tag{2}$$

where i_t is the interest rate of the yen and i_t* is the interest rate of the target (foreign) currency. An asterisk denotes variables in the non-Japan (target) country. The unobservable F_t is regarded as the risk premium of exchange rates. The interest rate parity simply states that the current spot rate is expected to depreciate/appreciate by the amount of the interest rate differential ex ante. If the interest parity holds, a linear regression of the change in exchange rate onto the interest rate differential should yield a coefficient of one. Unfortunately, most empirical studies

[1] One criticism of the factor analysis is its lack of economic or financial meaning.

[2] Leland (1997) pointed out that the standard Sharpe ratio is not applicable in non-normal or dynamic settings.

[3] For application of FAR model, see Ludvisgson and Ng (2009) on analysis of bond risk premia, Bernanke and Boivin (2003) and Giannone et al. (2005) on monetary policy analysis, and Stock and Watson (2005) on business cycle forecasting. Bai and Ng (2008) provide a comprehensive survey on this topic.

have found the coefficient to be near zero or negative. Engel and West (2005) argued that the failure of the UIRP has to do with the unobservable component F_t and this component contains useful information for predicting exchange rates. For this reason, we will apply the interest rate parity to estimating the factor.

Engel and West (2005) argue that the exchange rate itself contains information that is hard to extract from observable fundamentals. This information might include risk premium or data about fundamentals, which is usually persistent and time-varying. By this notion, the factor follows the dynamic process:

$$\Delta s_{t+1} - (i_t - i_t^*) = F_t + v_t, \qquad F_t = aF_{t-1} + w_t, \tag{3}$$

where w_t and v_t are identically independently distributed (i.i.d.) white noises. The parameter a measures the persistence of the factor and is between 0 and 1. The procedure to estimate the unobservable factor F_t involves the Kalman filtering technique and maximum likelihood method[1]. The estimation procedure is presented in the Appendix. Readers are referred to Hamilton (1994) and Green (2003) for standard treatment of equation (3) on estimating the dynamic factor. See Bai (2003), Bai and Ng (2008), and Stock and Watson (2005) for more discussion on statistical inference of the dynamic factor model.

The specification of z_t is inspired by interest rate parity and the recent success of the Taylor rule,

$$z_t = i_t - i_t^* + O(i_t - i_t^*), \tag{4}$$

where $O(i_t - i_t^*)$ is the polynomial of higher orders of the interest rate differentials. When $O(i_t - i_t^*)$ is zero, the regression equation (1) is the UIRP. Furthermore, given the recent empirical evidence on the possibility of beating the random walk by using the Taylor rule model, the choice of fundamental variables is narrowed according to the Taylor rule[2]. Specifically, interest rates are determined by key macroeconomic variables: economic growth rate, inflation rates and past interest rates[3]. The interest rate differential is constructed simply as[4]:

$$i_t - i_t^* = \beta_0 + \beta_y (y_t - y_t^*) + \beta_\pi (\pi_t - \pi_t^*) + \beta_{i1}(i_{t-1} - i_{t-1}^*). \tag{5}$$

Since many studies have shown that the exchange rate is possibly a nonlinear function of fundamentals (see Chinn, 1991; 2008; Taylor, Peel, and Sarno, 2001; Kilian and Taylor, 2003; Rossi, 2005), a type of nonlinearity for $O(i_t - i_t^*)$ in our FAR framework is considered as follows:

$$O(i_t - i_t^*) = \beta_{i2}(i_t - i_t^*)^2 + \beta_{i3}(i_t - i_t^*)^3. \tag{6}$$

In summary, the following is a list of various specifications of equation (1) that are tested in this paper.

Model 1 (Random walk): $\alpha = 0,\ \beta_F = 0,\ z_t = \Delta s_{t-1}$.

Model 2 (AR(1)): $\beta_F = 0,\ z_t = \rho \Delta s_{t-1}$.

Model 3 (AR(1) + Taylor rule): $\beta_F = 0,\ z_t = \beta_0 + \beta_y (y_t - y_t^*) + \beta_\pi (\pi_t - \pi_t^*) + \beta_{i1}(i_{t-1} - i_{t-1}^*) + \rho \Delta s_{t-1}$.

Model 4 (Taylor rule): $\beta_F = 0,\ z_t = \beta_0 + \beta_y (y_t - y_t^*) + \beta_\pi (\pi_t - \pi_t^*) + \beta_{i1}(i_{t-1} - i_{t-1}^*)$.

Model 5 (Taylor rule + Nonlinear):

$$\beta_F = 0,\ z_t = \beta_0 + \beta_y (y_t - y_t^*) + \beta_\pi (\pi_t - \pi_t^*) + \beta_{i1}(i_{t-1} - i_{t-1}^*) + \beta_{i2}(i_t - i_t^*)^2 + \beta_{i3}(i_t - i_t^*)^3.$$

Model 6 (Factor + Taylor rule): $z_t = \beta_0 + \beta_y (y_t - y_t^*) + \beta_\pi (\pi_t - \pi_t^*) + \beta_{i1}(i_{t-1} - i_{t-1}^*)$.

Model 7 (Factor + Taylor rule + Nonlinear):

$$z_t = \beta_0 + \beta_y (y_t - y_t^*) + \beta_\pi (\pi_t - \pi_t^*) + \beta_{i1}(i_{t-1} - i_{t-1}^*) + \beta_{i2}(i_t - i_t^*)^2 + \beta_{i3}(i_t - i_t^*)^3.$$

[1] The Kalman filtering and maximum likelihood estimation (MLE) method is more convenient than the principal component analysis for two reasons: (1) principal component analysis (PCA) is more fit for static factors; and (2) non-stationary data can be used to estimate the factor consistently under the Kalman filtering and MLE method.

[2] Models incorporating Taylor rule fundamentals, as propounded by Engel and West (2006), Mark (2007), Molodtsova and Papell (2008), and Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008) have recently gained prominence as a means of explaining movements in the exchange rate.

[3] Much of the Taylor rule literature uses expected inflation in the monetary policy rule. Since we don't have data on expected inflation in all countries we study, we leave out this variable in our Taylor rule model.

[4] See Engel, Mark, and West (2007) for detailed discussion on equation (5).

2. Carry trade

In this paper, a carry trade is a binary trading strategy in spot markets[1] that is based on projected return. The trading rule is that if $i_t^* - i_t > 0$ and the expected return is positive as predicted by the model, there is carry trade between a target foreign currency and the yen. We use $c = 1$ to denote a decision of carry trade:

$$c_t = \begin{cases} 1 & i_t^* - i_t + E(\Delta s_{t+1}) > 0 \text{ and } i_t^* - i_t > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Note that under the random walk theory, the change of expected exchange rate is zero, i.e. $E_t(\Delta s_{t+1}) = 0$, so the carry trade decision depends solely on the interest rate differential. Because the yen has been on the lowest interest rate among the major currencies since the mid-1990s, the yen carry trade will almost always occur, under the random walk forecast.

The size of the carry trade is the borrowed amount of yen. The profit in carry trade can be scaled by its size. For the size of ¥1, the return to carry trade is calculated as:

$$r_t = \begin{cases} i_t^* - i_t + \Delta s_{t+1}, & \text{if } c_t = 1, \\ 0, & \text{if } c_t = 0. \end{cases}$$

In periods without carry trade, the factor model predicts relatively large depreciation of the target currency. This implies that shorting the target currency would generate additional profit. We enhance the carry trade by reversing the currency trade in the spot market during these periods. We term this strategy the enhanced carry trade (ECT)[2]. The return to ECT is calculated as

$$r_t = \begin{cases} i_t^* - i_t + \Delta s_{t+1}, & \text{if } c_t = 1, \\ -\Delta s_{t+1}, & \text{if } c_t = 0. \end{cases}$$

3. Empirical results

3.1. Data. The data consist of monthly series of all variables, 1973.1-2010.1 (with exceptions noted below). The sample size is 444, due to the loss of one observation to differencing. We study bilateral Japanese yen exchange rates versus those of the six other countries: Australia, Canada, the Eurozone, New Zealand, the United Kingdom, and the U.S. The Federal Reserve's (FRED) is the source for the end-of-month exchange rates. We sampled end-of-month exchange rates from daily exchange rates. The international financial statistics (IFS) CD-ROM is the source for all the fundamental economic variables: money supply, industrial production, consumer prices, and interest rates. Since consumer price index (CPI) and industrial production data for Canada and New Zealand are missing for earlier years[3], data for 1975.1-2010.1 and 1978.5-2010.1 are used, respectively. German exchange rates and fundamentals are substituted for those of Eurozone before 1999.1[4].

The out-of-sample performance testing starts from January 1999, when the euro became official. At each month, we use data only up to the prior month for forecasting. Any missing data will be imputed with the "cubic spline" method in Matlab. We then estimate the unobservable factor. After obtaining the sequence of factors, we estimate coefficients of equation (1) using the OLS method and forecast exchange rates for that month. The out-of-sample forecast is then used to construct carry trade strategy. Performance is computed using the realized exchange rates. The process is performed for each of the six nations. The Figure 1 illustrates how data are used at January 1999. The same process is repeated as we move on to the next period, until January 2010. Therefore, 145 months of trade decision take place in total. Since volatility is not estimated, we can't construct a mean-variance optimal portfolio. Therefore, we present performance of an equally-weighted portfolio of the six carry trades.

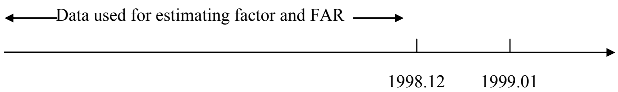

Fig. 1. Data used in Jenuary 1999

Table 1 presents some basic statistics of the whole sample. Variables are all very volatile because their relatively large sample deviations are compared to their sample means. All countries have higher interest rates on average than Japan. Except for the UK, all countries experienced lower industrial growth rates than Japan.

Panel B in Table 1 reports the augmented Dickey-Fuller tests of unit-root for three time series: change in exchange rates (Δs), interest rate differentials ($i^* - i$), and the risk premium ($\Delta s - (i^* - i)$). Both Δs and $\Delta s - (i^* - i)$ seem to be non-stationary time series because we are unable to reject the null of unit roots in them. This makes our choice of method to estimate the dynamic factor to be the Kalman filtering because generally the Bayesian method is relatively robust to non-stationarity.

[1] Note that buying target currency in forward markets is an equivalent carry trade strategy.
[2] An ECT is where a short sale of target currency is executed when carry trade is predicted to lose.
[3] Imputed data would be very unreliable because of a large block of missing data.
[4] The exchange rate of the euro is converted from the German mark at the rate of mark 1.955828/euro.

Table 1. Basic statistics

	Australia	Canada	Eurozone	New Zealand	United Kingdom	U.S.
Panel A. Average and standard deviations of six target currencies						
Δs	-0.003 (0.034)	-0.003 (0.031)	-0.001 (0.027)	-0.003 (0.034)	-0.004 (0.029)	-0.003 (0.033)
$i^* - i$	0.044 (0.035)	0.036 (0.026)	0.012 (0.026)	0.073 (0.040)	0.035 (0.038)	0.025 (0.030)
$\Delta(y^* - y)$	0.002 (0.018)	0.000 (0.018)	0.000 (0.021)		-0.001 (0.020)	0.000 (0.016)
$\pi^* - \pi$		0.002 (0.006)	-0.001 (0.008)		0.003 (0.007)	0.001 (0.006)
Panel B. Augmented Dickey-Fuller test, H0 = Unit root, 1 = Accept, 0 = Reject						
Δs	1	1	1	1	1	1
$i^* - i$	0	1	1	0	1	0
3. $\Delta s - (i^* - i)$	1	1	1	1	1	1

Note: The number in parentheses under each variable is the standard deviation of the indicated variable. An asterisk indicates a non-Japan value, and the absence of an asterisk indicates a Japan value; Δs is the percentage change in the yen exchange rate (a higher value indicates appreciation against the yen); i is the money market rate or government bond yield; Δy is the growth rate of the industrial production index; π is the rate of inflation. Data are monthly, mostly 1973.1-2010.1. Exceptions include a beginning date of 1975.1 for Canada and 1978.5 for New Zealand. Monthly data of CPI for Australia and New Zealand are not available.

3.2. Performance of carry trades. Table 2 reports the annualized return, Sharpe ratio, and skewness of returns for the period of 1999.1-2010.1. Let's first examine the risk-adjusted returns. We notice that models with fundamentals (models 3-7) generally outperform the RW model or AR(1) model, which do not include fundamentals. More important, models with factors generate better returns than those without. We find that Taylor rule fundamentals have a large improvement only after the factor is included (models 6 or 7). For example, for all of the six target currencies, carry trades generate higher Sharpe ratios than models without fundamentals. The naive carry (RW) trade generates positive returns for all six currencies against the yen, with the returns ranging from 0 to 9%. The AR(1) model also has positive returns, but performs somewhat worse than even the naive strategy. But either of the carry trades in the non-fundamental models has poor Sharpe ratios compared to the average Sharpe ratio of 0.3 in the S&P 500. In contrast, carry trade based on FAR (models 6 or 7) improves the Sharpe ratio for each one of the six currencies. For instance, the Sharpe ratio rises to 0.65 (model 6) or 0.71 (model 7) from 0.51 (RW) or 0.18 (AR(1)) for the portfolio.

Skewness is an equally important performance statistic. Large negative skewness implies the high probability of large losses such as market crashes. This is a big concern for investors because it may mean short-term insolvency, even bankruptcy. The two biggest currency corrections during the testing period, November 2000 and October 2008, occurred when the yen appreciated suddenly against most of the target currencies. For instance, the yen appreciated against the dollar by 8% in October 2008. The huge negative skewness, for instance -2.02 in the Australian dollar naive carry trade, indicates great tail risk in the yen carry trade. The FAR model improves the skewness. Carry trades in six currencies have better skewness than naive carry trades, except for the U.S. dollar. The Australian dollar has a -0.48 skewness, which is a great improvement over the skewness of its counterparts (-2.02 (RW)) or -2.77 (AR(1)). The improvement in skewness also clearly shows up on the portfolio.

Fundamentals are helpful in detecting the direction of currency fluctuations. This can be seen from the change of performance in the AR(1) model. After including the Taylor fundamentals, AR(1) performs slightly better. But the fundamentals alone are not enough. Only after they are put into a FAR framework do they drastically increase the Sharpe ratios and skewness. The Sharpe ratio of the equally-weighted portfolio rises from 0.38 to 0.65 from 0.38 (RW) or 0.28 (AR(1)) and the skewness increases to -0.3 from -1.60 (RW) or -0.9 (AR(1)).

ECTs have even better skewness from returns, with little sacrifice in return. The results are shown in Table 3. The results tell a similar story as in Table 2, except with much better performance for the FAR models in terms of skewness. The FAR models generate a Sharpe ratio of 0.54 (model 6) or 0.6 (model 7), but a greatly improved skewness of 1.7 for the portfolio.

We examined several episodes of large correction in yen exchange rates. We found that FAR models have avoided these periods for carry trades with 70-80 percent chances (not shown). This contributes to the great performance of the FAR-based carry trades.

The nonlinear terms in models 5 or 7 do not outperform their counterparts. A couple of reasons may cause this. First, the factor has already absorbed all relevant information that otherwise is contained in the nonlinear terms. We did try adding nonlinear terms in a standard regression without the factor, and found relatively better performance. Second, the specification of nonlinearity may not hold up well simply because of lack of knowledge. We did not try other forms of nonlinearity, since we fear the arbitrary nature of such attempts[1].

Table 2. Performance statistics of carry trade

	Australia	Canada	Eurozone	New Zealand	United Kingdom	U.S.	Portfolio
Model 1: Random walk							
Mean return	0.08	0.04	0.04	0.09	0.02	0.00	0.04
Sharpe ratio	0.51	0.27	0.37	0.53	0.18	-0.04	0.38
Skewness	-2.02	-1.42	-1.80	-1.19	-1.16	0.37	-1.60
Model 2: AR(1)							
Mean return	0.02	0.01	0.03	0.01	0.03	0.01	0.02
Sharpe ratio	0.18	0.11	0.39	0.09	0.37	0.15	0.28
Skewness	-2.77	-0.92	-0.98	-2.05	-0.54	0.21	-0.92
Model 3: AR(1) + Taylor							
Mean return	0.08	0.01	0.03	0.07	0.02	0.03	0.04
Sharpe ratio	0.54	0.06	0.27	0.47	0.33	0.43	0.45
Skewness	-2.05	-2.02	-1.99	-1.27	0.45	0.55	-1.81
Model 4: Taylor rule							
Mean return	0.08	0.01	0.04	0.06	0.03	0.02	0.04
Sharpe ratio	0.51	0.11	0.37	0.38	0.47	0.31	0.45
Skewness	-2.02	-2.03	-1.80	-1.14	0.69	0.56	-1.77
Model 5: Taylor Rule + nonlinear model							
Mean return	0.05	0.01	0.04	0.07	0.00	0.01	0.03
Sharpe ratio	0.34	0.11	0.37	0.45	0.09	0.24	0.38
Skewness	-2.31	-1.63	-2.07	-1.26	0.05	0.82	-2.13
Model 6: Factor + Taylor Rule							
Mean return	0.07	0.03	0.03	0.09	0.04	0.01	0.04
Sharpe ratio	0.65	0.33	0.38	0.80	0.47	0.19	0.65
Skewness	-0.48	-0.17	-0.75	0.06	-0.14	0.39	-0.31
Model 7: Factor + Taylor rule + nonlinear model							
Mean return	0.07	0.03	0.04	0.10	0.03	0.01	0.04
Sharpe ratio	0.71	0.31	0.46	0.84	0.36	0.12	0.65
Skewness	-0.46	-0.18	-0.60	0.05	-0.16	0.43	-0.34

Note: The mean return is annualized. The Sharpe ratio is defined as the ratio of mean return to the standard deviation.

Table 3. Performance statistics of enhanced carry trade

	Australia	Canada	Eurozone	New Zealand	United Kingdom	U.S.	Portfolio
Model 1: Random walk							
Mean return	0.08	0.04	0.04	0.09	0.02	0.00	0.04
Sharpe ratio	0.51	0.27	0.37	0.53	0.18	-0.04	0.38
Skewness	-2.02	-1.42	-1.80	-1.19	-1.16	0.37	-1.60
Model 2: AR(1)							
Mean return	0.01	-0.01	0.03	-0.05	0.05	0.03	0.01
Sharpe ratio	-0.07	-0.04	0.25	-0.30	0.38	0.30	0.10
Skewness	-1.60	1.14	0.80	-1.24	0.74	-0.41	-0.89
Model 3: AR(1) + Taylor							
Mean return	0.09	-0.02	0.02	0.07	0.04	0.07	0.04
Sharpe ratio	0.56	-0.12	0.15	0.40	0.30	0.67	0.56
Skewness	-2.06	-1.43	-1.82	-1.13	1.12	-0.38	-1.18
Model 4: Taylor rule							
Mean return	0.08	0.00	0.04	0.04	0.06	0.05	0.04
Sharpe ratio	0.51	-0.03	0.37	0.24	0.45	0.52	0.60
Skewness	-2.02	-1.45	-1.80	-1.01	1.07	-0.28	-1.36

[1] Regime switch may be another way to model nonlinearity.

Table 3 (cont.). Performance statistics of enhanced carry trade

	Australia	Canada	Eurozone	New Zealand	United Kingdom	U.S.	Portfolio
Model 5: Taylor rule + nonlinear model							
Mean return	0.06	-0.05	0.04	0.03	-0.03	0.03	0.01
Sharpe ratio	0.39	-0.34	0.37	0.18	-0.28	0.34	0.18
Skewness	-1.94	-1.34	-1.80	-0.96	-0.43	-0.29	-3.22
Model 6: Factor + Taylor rule							
Mean return	0.07	0.03	0.02	0.11	0.07	0.04	0.06
Sharpe ratio	0.41	0.23	0.17	0.72	0.54	0.38	0.54
Skewness	1.42	1.13	1.23	0.87	0.86	-0.29	1.72
Model 7: Factor + Taylor rule + nonlinear model							
Mean return	0.08	0.03	0.03	0.12	0.05	0.03	0.06
Sharpe ratio	0.48	0.20	0.30	0.77	0.40	0.28	0.54
Skewness	1.41	1.13	1.19	0.84	0.90	-0.22	1.66

See notes in previous tables.

3.3. Transaction cost. The transaction cost may change the decision making of carry trades that rely on fundamentals, so it will be helpful to see what is its effect. A typical transaction cost per foreign exchange trade is between 2-40 base points (see Burnside, 2007). So we simulate our carry trades for a transaction cost ranging from 2bps to 40bps per carry trade. We show the return to the portfolio of six carry trades. The result in Table 4 indicates that transaction cost does not change the notion that FAR-based carry trade outperforms that based on RW or AR(1) models.

Table 4. Returns to carry trade with transaction cost

Transaction cost (bps)	2	5	10	20	30	40
Model 1: Random walk						
Mean return	0.04	0.04	0.03	0.02	0.01	0.00
Sharpe ratio	0.36	0.33	0.28	0.17	0.07	-0.03
Skewness	-1.60	-1.60	-1.60	-1.60	-1.60	-1.60
Model 6: Carry trade portfolio						
Mean return	0.04	0.04	0.03	0.02	0.01	0.01
Sharpe ratio	0.64	0.61	0.49	0.36	0.21	0.16
Skewness	-0.31	-0.32	-0.29	-0.21	-0.36	-0.50
Model 6: Enhanced carry trade portfolio						
Mean return	0.05	0.05	0.04	0.03	0.02	0.01
Sharpe ratio	0.52	0.49	0.40	0.31	0.21	0.16
Skewness	1.69	1.70	1.72	1.76	1.58	1.81

3.4. Alternative periods of carry trade. FAR-based carry trades are also tested in two other periods, 1996.1-2010.1 and 2006.1-2010. Table 5 reports the result for both periods. Panel A reports the performance statistics for 1991.1-2010.1 and Panel B reports those for 2006.1-2010. Again, performance statistics in Table 6 tell a similar story to those in Tables 2 and 3: FAR model-based carry trade with fundamentals outperforms naive or AR(1) carry trade.

Table 5. Returns to carry trade in different periods

	Australia	Canada	Eurozone	New Zealand	United Kingdom	U.S.	Portfolio
Panel A. 1996.1-2010.1							
Model 1: Random walk, naive carry trade							
Mean return	0.05	0.04	0.02	0.06	0.04	0.02	0.04
Sharpe ratio	0.34	0.28	0.16	0.36	0.29	0.21	0.32
Skewness	-1.58	-1.41	-1.31	-0.91	-1.18	-0.65	-1.45
Model 3: Taylor rule							
Mean return	0.05	0.01	0.02	0.05	0.04	0.04	0.03
Sharpe ratio	0.33	0.09	0.16	0.35	0.41	0.41	0.35
Skewness	-1.60	-1.92	-1.38	-1.17	-1.08	-1.21	-1.74
Model 6: Factor + Taylor rule							
Mean return	0.04	0.03	0.02	0.06	0.04	0.04	0.04
Sharpe ratio	0.34	0.28	0.20	0.56	0.39	0.52	0.49
Skewness	-0.74	-0.22	-1.16	-0.01	-1.08	-0.08	-0.50
Model 6: Factor + Taylor rule, enhanced carry trade							
Mean return	0.04	0.02	0.03	0.09	0.05	0.07	0.05
Sharpe ratio	0.24	0.17	0.20	0.57	0.40	0.59	0.49
Skewness	0.93	1.14	0.26	0.66	0.03	0.33	1.13

Table 5 (cont.). Returns to carry trade in different periods

	Australia	Canada	Eurozone	New Zealand	United Kingdom	U.S.	Portfolio
Panel B. 2006.1-2010.1							
Model 1: Random walk, naive carry trade							
Mean return	0.03	-0.02	0.00	0.01	-0.05	-0.03	-0.01
Sharpe ratio	0.16	-0.12	0.00	0.06	-0.32	-0.31	-0.06
Skewness	-1.74	-1.39	-1.85	-0.82	-1.09	0.44	-1.43
Model 3: taylor rule							
Mean return	0.03	-0.01	0.00	0.01	0.01	-0.01	0.00
Sharpe ratio	0.16	-0.07	0.00	0.06	0.13	-0.24	0.04
Skewness	-1.74	-1.97	-1.85	-0.82	-0.54	-0.50	-1.56
Model 6: Factor + Taylor rule							
Mean return	0.03	-0.03	0.03	0.07	0.02	0.01	0.02
Sharpe ratio	0.25	-0.26	0.44	0.47	0.22	0.16	0.27
Skewness	-0.64	-0.34	-0.40	0.45	-0.76	-0.31	-0.18
Model 6: Factor + Taylor rule, enhanced carry trade							
Mean return	0.05	-0.03	0.07	0.14	0.10	0.06	0.06
Sharpe ratio	0.22	-0.17	0.51	0.64	0.65	0.57	0.47
Skewness	1.34	1.41	1.55	0.86	0.76	-0.64	1.65

4. An analysis of factors and forecast evaluation

Why does factor matter even though it is unobservable? We may find a little clue in the statistics about the factor. Table 6 lists some of the statistics related to the estimated dynamic factor for each target currency. Panel A reports the contemporaneous correlation between the factor and relevant variables. Panel B reports the correlation between time $t-1$ factor and time t variables. One thing stands out: the factor that is correlated to the exchange rate changes in both ways with a high degree of statistical significance. This is probably the reason that the factor has strong predictive power in carry trades. Another fact is that the factor is very persistent, since its autocorrelation does not die to zero, even at a time lag of 10. Bartholomew and Knott (1999), Diebold and Nerlove (1989), and Rossi (2005) show that a persistent factor has a strong predictive power if it is highly correlated with other variables.

Table 6. Statistics about factors

Panel A. Correlation with factor						
	Australia	Canada	Eurozone	New Zealand	United Kingdom	U.S.
Δs	0.39 (0.00)	0.42 (0.00)	0.46 (0.00)	0.38 (0.00)	0.48 (0.00)	0.44 (0.00)
i^*-i	0.11 (0.02)	0.39 (0.00)	0.20 (0.00)	0.24 (0.00)	0.31 (0.00)	0.41 (0.00)
$\Delta(y^*-y)$		-0.17 (0.00)	-0.03 (0.54)		-0.12 (0.01)	-0.04 (0.43)
$\pi^*-\pi$		0.03 (0.59)			0.11 (0.02)	0.10 (0.03)
Panel B. Correlation with one-lag factor (t–1)						
Δs	0.21 (0.00)	0.34 (0.00)	0.31 (0.00)	0.31 (0.00)	0.30 (0.00)	0.27 (0.00)
i^*-i	0.10 (0.04)	0.38 (0.00)	0.18 (0.00)	0.24 (0.00)	0.32 (0.00)	0.38 (0.00)
$\Delta(y^*-y)$		-0.19 (0.00)	-0.03 (0.58)		-0.17 (0.00)	-0.06 (0.22)
$\pi^*-\pi$		0.00 (0.94)			0.11 (0.02)	0.07 (0.12)
Lags	Panel C. Autocorrelation of factor					
1	0.74	0.97	0.90	0.98	0.87	0.87
2	0.46	0.90	0.73	0.93	0.67	0.70
3	0.22	0.82	0.55	0.87	0.47	0.54
4	0.06	0.73	0.38	0.79	0.32	0.40
5	-0.03	0.64	0.26	0.72	0.21	0.29
6	-0.04	0.56	0.18	0.65	0.14	0.22
7	0.02	0.49	0.14	0.58	0.14	0.18

Table 6 (cont.). Statistics about factors

Lags	Panel C. Autocorrelation of factor					
	Australia	Canada	Eurozone	New Zealand	United Kingdom	U.S.
8	0.05	0.43	0.10	0.52	0.16	0.17
9	0.08	0.38	0.07	0.46	0.17	0.15
10	0.09	0.32	0.02	0.40	0.16	0.14

Note: Numbers in parentheses are the statistical p-value. A lower p-value indicates the correlation coefficient is more significantly different from zero.

Underlying the better profits of carry trades from the FAR models, there is also better forecasting performance. We believe that the factors contribute to this. Panel A in Table 7 shows the Diebold-Mariano test of the one-month forecasting performance of fundamental models against the random walk model. The tests overwhelmingly indicate that FAR models could outperform the random walk. In Panel B, a simple non-parametric analysis of the directional forecasts also points to the same conclusion: models with the factors perform better.

Table 7. Forecasting performance for month ahead

Panel A. Diebold-Mariano test							
Model	Model	Australia	Canada	Eurozone	New Zealand	United Kingdom	U.S.
2	AR(1)	1	1	0	0	0	1
3	AR(1) + TR	1	1	0	1	1	0
5	TR + NL	1	1	0	1	1	0
6	FAR + TR	1	1	1	1	1	1
7	FAR + TR + NL	1	1	1	1	1	1
Panel A. Percentage of correct directional forecasts, Total = 131							
Model	Model	Australia	Canada	Euro Zone	New Zealand	United Kingdom	U.S.
2	AR(1)	42.0%	48.9%	44.3%	38.9%	43.5%	51.9%
3	AR(1) + TR	51.1%	50.4%	51.9%	42.7%	45.0%	55.7%
5	TR + NL	46.6%	45.0%	52.7%	53.4%	45.8%	52.7%
6	FAR + TR	55.0%	41.2%	54.2%	59.5%	52.7%	59.6%
7	FAR + TR + NL	54.2%	42.0%	59.6%	61.8%	61.2%	51.1%

Note. The forecast starts at February 1999 until January 2010. There are totally 131 monthly forecasts for this period. On panel A, the performance of forecasting from each model is tested against the random walk model using the Diebold-Mariano test. The null hypothesis is that the specific model performs equally as the random walk model. The test statistics is then calculated under the absolute error and uniform kernel for the long-run variance. 1 indicates the acceptance of the null hypothesis while 0 indicates the reject of the null. TR = Taylor rule, NL = Nonlinear, and FAR = Factor augmented regression.

5. Reality check

PowerShares DB G10 Currency Harvest (Ticker: DBV) is an ETF that has a naive carry trade strategy of longing currencies with high interest rates and shorting the three countries with the lowest interest rates. Since its inception in October 2006, PowerShares' average return is -2.12% and its Sharpe ratio is -15.6%, with an unpleasant skewness of -170%. The performance is similar to that of yen carry trade based on the RW model (see Table 3 or Table 5). The FAR-based (model 6) carry trade would have a better average return of 0.11% and a positive Sharpe ratio of 1.36%. Skewness in return is significantly reduced to -0.25%. The enhanced carry trade based on the FAR Model 6 generates an impressive 4.84% return and a 33.18% Sharpe ratio. Considering the negative 20% Sharpe ratio of the S&P 500 during the same period, this is a stunning result. The skewness of 157.07% suggests the ECT has avoided many major losses in foreign exchange rates. The cumulative return from the comparison group is also plotted in Figure 1. Note that the S&P 500 (not shown) has a *worse* return than any of the trading strategies for the same period.

Table 8 Performance compared to G10 ETF since October 2006

	G10	RW	CT (Model 6)	Enhanced CT (Model 6)
Mean return	-2.10%	-3.50%	0.11%	4.94%
Sharpe ratio	-15.60%	-21.63%	1.36%	33.18%
Skewness	-170.00%	-125.08%	-25.40%	157.07%

Note: Monthly returns are annualized. G10's return data is obtained from finance.yahoo.com.

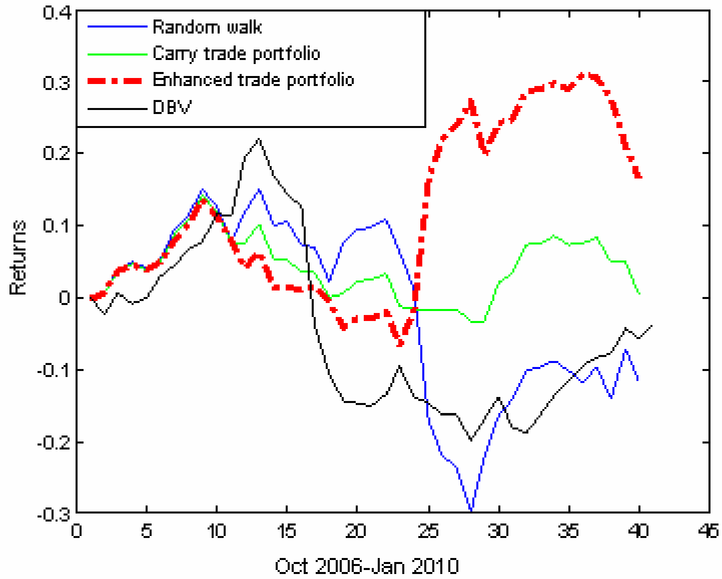

Fig. 1. Cumulative returns to carry trades in recent financial crisis

Conclusion

In this paper, exchange rate models with Taylor rule fundamentals improve the profitability of yen carry trades. It is so especially when the fundamentals are used under the factor-augmented regression framework. The virtue of the fundamentals is mainly in the form of the derived factors. A brief examination of factors shows that the unobservable factor contains very useful information for forecasting future exchange rates. It is highly persistent and correlated with future exchange rates. Given that we don't have a fully working model of exchange rate for prediction, these attributes make the FAR model the attractive alternative for exchange rate predictability. We hope this study will contribute in this direction.

This paper contributes to the literature by investigating the usefulness of economic fundamentals in the yen carry trade. Until recently, there has been relatively little attention to carry trade in the literature (see, for example, Burnside, 2006; Jordà and Taylor, 2009). The existing literature tends to study the implication of carry trade for the exchange rate models (see for example, Corte, Sarno, and Tsiakas, 2008)[1]. This paper provides a practical view of exchange rate modeling for the currency trading community.

Another contribution of this study is its novel method to evaluate the exchange rate models. It presents another angle to "beating" the random walk model. In recent years there have been several studies in the Taylor rule models in the literature of "beating" the random walk model (notably Engel and West, 2006, and Molodtsova, Tanya, and David Papell, 2008). Many of them have found improvement in the forecasting ability of the exchange rate models when they incorporate the Taylor rule (see Choi, Mark, and Sul, 2006; Clarida, Gali, and Gertler, 1998; Engel and West, 2006; Engel, Mark and West, 2007; Gali and Monacelli, 2005; MacDonald and Taylor, 1994; Mark, 1995; Murray and Papell, 2002; Taylor, Peel, and Sarno, 2001). But the improvement mostly shows up in the long-term forecasting in the mean root squared predition errers (MRSPE). In this paper, fundamentals in the Taylor rule appear to boost the profits of carry trade in a monthly frequency over the naive carry trade based on non-fundamentals forecasts. It suggests that fundamentals do improve the forecasting ability of exchange rate models in terms of carry trades. This is an important result because it implies that we cannot simply dismiss fundamental models. It is so especially for the investment community. For practitioners, what matters ultimately is the investment performance after all.

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[1] Corte, Sarno, and Tsiakas (2008) evaluated the importance of fundamentals in the currency portfolio and the relative performance of different exchange rate models. More recent studies, such as Burnside (2006) showed profit from carry trades being relatively high even after controlling the transaction cost and hedging the downside risk, which adds another challenge to the UIPR.

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Appendix. Algorithm for estimating the dynamic factor

Kalman filtering and maximum likelihood method can estimate the time-varying factor through two steps: (1) construct an approximate estimate of the factor using the Kalman filtering; (2) the approximate factor is plugged into the likelihood function to estimate the unknown parameters using the ML. These two steps are repeated until the estimates of parameters converge. This algorithm is also called the expectation-maximization (EM) algorithm. Because the algorithm utilizes the Kalman filtering, we briefly introduce the Kalman filtering first.

1. Kalman filtering. The Kalman filtering is a set of mathematical equations that provides an efficient computational (recursive) solution of the least-squares method. Detailed derivation of the filter is provided here. Readers are referred to Hamilton (1994). Hamilton (1994) for extensive discussion on the Kalman filtering for time series.

The Kalman filtering addresses the general problem of estimating the hidden state $F \in R$ of a discrete-time process that is governed by the linear stochastic difference equation

$$F_t = aF_{t-1} + w_t,$$

where *a* belongs to the interval of (0,1). The observation $y = \Delta s - (i^* - i)$ is $y_t = F_t + v_t$.

The random variables w_t and v_t represent the process and observation noise respectively. They are assumed to be independent of each other, Gaussian and with probability distribution:

$$w \sim N(0,Q),\ v \sim N(0,R).$$

The factor *F* is assumed to start with the initial value $F_0 \sim N(\pi_0, V_0)$.

Suppose we have already observed a sequence of y at time t, denoted by $y^t = \{y_1,\dots,y_t\}$. The best estimate of the factor is its conditional expectation on y^t, i.e. $\hat{F}_t = E(F_t|y_t)$. Because noises are Gaussian, the conditional expectation is the same as the generalized least-squares estimate. Calculating $\hat{F}_t$ for every time period is tedious if we use apply the conditional expectation. The Kalman filtering provides a very efficient way to calculate $\hat{F}_t$ by a set of recursive equations. The recursive formula is shown below.

$$\hat{F}_t = a\hat{F}_{t-1} + K_t\left(y_t - \hat{F}_{t-1}\right),$$

$$K_t = \left(a^2 P_{t-1} + Q\right)\cdot\left[\left(a^2 \cdot P_{t-1} + Q\right) + R\right]^{-1},$$

$$P_t = \left(I - K_t\right)\left(a^2 P_{t-1} + Q\right).$$

With the initial values $P_0 = V_0$ and $\hat{F}_t = \pi_0$.

2. Maximum likelihood estimation. Here we explain how the parameters are estimated. Assume we have observed a sample y^T. Let $f(y,F|F_0)$ denote the joint density of the observable y^{-T} and unobservable factors $F^T = \{F_1, F_2, ..., F_T\}$ so that:

$$f(y,F|F_0) = f(F_0)\prod_{t=1}^{T}\{f(y_t|F_t)f(F_t|F_{t-1})\},$$

where $f(y_t \mid F_t) = \exp\left\{-\frac{1}{2}(y_t - F_t)^2 R^{-1}\right\}(2\pi R)^{-1/2}$ and

$f(F_t \mid F_{t-1}) = \exp\left\{-\frac{1}{2}(F_t - aF_{t-1})^2 Q^{-1}\right\}(2\pi Q)^{-1/2}$. Then the log-likelihood function is given by:

$$\ln L = \log(f(y,F|F_0)) = -\sum_{t=1}^{T}\left\{\frac{1}{2}(y_t - F)^2 R^{-1}\right\} - \frac{T}{2}\log|R| - \sum_{t=1}^{T}\left\{-\frac{1}{2}(F_t - aF_{t-1})^2 Q^{-1}\right\} - \frac{T}{2}\log|Q| -$$

$$-\frac{1}{2}(F_0 - \pi_0)^2 V_0^{-1} - \frac{1}{2}log|V_0| - T\log 2\pi.$$

The parameters needs to be estimated are $\xi = \{a, Q, R, \pi_0, V_0\}$. Since F^T is not observable, the maximum likelihood method is practically impossible. A way to get around this problem is to replace the factor with the Kalman filtering estimate $\hat{F}_t$.

The maximum likelihood estimation of ξ is carried recursively. At iteration l, an estimate of $\xi^{(l)}$ is obtained from the previous estimate $\xi^{(l-1)}$. The iterative process will stop if the new estimate cannot improve the log-likelihood. The following steps illustrate the iteration process.

Step 1: Set $l = 0$ and choose $\xi^{(0)}$ with a good guess.

Step 2: Set $\xi = \xi^{(1)}$. Calculate the conditional expectation of the log-likelihood $\ln L$ on y^T, $E(ln\,L|y^T)$. It involves calculating $E(F_t|y^T)$, $E(F_t^2|y^T)$ and $E(F_tF_{t-1}|y^T)$. They are computed using the factor estimate $\hat{F}_t$, which is conveniently computed by the Kalman filtering.

Step 3: Maximize the log-likelihood function $E(\ln L|y^T)$ to obtain a new estimate $\xi^{(l+1)}$. In this step, use the first order necessary condition or the generalized least squared to estimate the parameter.

Step 4: Repeat steps 2 and 3 until a stopping criterion is satisfied.