Вісник Тернопільського національного технічного університету
Scientific Journal of the Ternopil National Technical University
2017, № 3 (87)
ISSN 1727-7108. Web: visnyk.tntu.edu.ua

MECHANICS AND MATERIALS SCIENCE

МЕХАНІКА ТА МАТЕРІАЛОЗНАВСТВО

UDC 539.3

INVESTIGATION OF THE SHOCK WAVES IMPACT ON THE DYNAMIC STRESS STATE OF MEDIUM WITH THE SYSTEM OF TUNNEL CAVITIES

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Summary. *The method to study distribution of dynamic stresses in elastic bodies with tunnel cavities according to integrated and discrete Fourier transform over time has been developed in the paper. In the field of Fourier transforms application of the boundary integral equation method and the complex variable theory made it possible to develop an efficient algorithm for determining dynamic stress state of bodies with cavities for almost arbitrary cross-section. Basing on the proposed method the numerical calculations of dynamic stresses concentration at the boundary of tunnel cavities of circular and elliptical cross-section have been performed and temporal distributions of dynamic stresses under impact loads have been built. The effect of reflected waves on the dynamic stress state of tunnel cavities has been studied.*

Key words: *non-stationary problem, tunnel cavity.*

Received 18.08.2017

Statement of the problem. The problem of investigation of dynamic processes in solid fracture bodies is one of the most actual in mechanics, which comprises both theoretical and practical problems. Its solving, beside sufficient theoretical importance, is of practical use, because taking advantage of the developed methods for design and investigation in engineering will provide the possibility to apply materials more economically.

Application of such methods to calculate dynamic impacts on the objects located in the areas of high seismic activity is of paramount importance. Power dynamic loads cannot be investigated without studying the whole space-time picture of the stress state of the object, which is resulted from the propagation of the plastic waves in solid bodies. That is why investigations of non-stationary processes in solid body mechanics are of paramount importance today.

Analysis of available investigation results. Available powerful computing technologies, efficient software in particular, have sufficiently contributed lately to the growth of investigations of the dynamic tasks of the elastoplastic theory for fractured bodies.

To solve these problems Chen Y.M., Shahani A.R., Zhang J.Y., etc. took advantage of direct numerical methods of finite differences and finite elements, basing on which numerical investigation of the dynamically stressed bodies under the load variable in time. The advantage of these methods is that of the possibility to apply them in bodies with arbitrary boundary and arbitrary shape fractures, anisotropic bodies, non-linear tasks under finite and plastic deformations. But their main disadvantages are a need of quantification of the movement equations in all body region. Under fast-changing loads numerical differentiation requires quantification scale narrowing to provide calculation accuracy. It results in problems while

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solving tasks by direct numerical methods for the regions of arbitrary cross-section fractures under impact load.

Guz O.M., Zozulya V.V., Kubenko V.D. [1] have greatly contributed to the dynamic fracture mechanics. For investigation of non-stationary and constant oscillations the method of series has been used in their papers. In the case of axis-symmetric non-stationary load the integral Laplace transform was used, which made possible to obtain the analytical solution for the non-finite elastic plate with the circular hole. To investigate stale oscillations of fractured bodies with non-circular cross-section the method of the boundary shape diffusion is used combined with the method of series. But under dynamic load such approach makes numerical calculations more difficult.

The boundary integral equations method (BIEM) and boundary elements method (BEM) were found to be efficient to investigate dynamic stress state of fractured bodies under constant oscillations in the papers by Simons D.A., Jain D.L., Srivastava K.N., etc. according to BIEM the solutions for limited bodies with the strip cracks, thin inclusions and crack-like holes [2] under oscillation load, have been obtained. Under dynamic load integral transformations have been used, which made it possible to bring the tasks to the integral equations solved by the numerical methods.

In the papers by Yemets V.F., Kunets I.Y., Matus V.V., Pasternak Y.M. [3] diffraction antiplate wave tasks for unlimited bodies with thin elastic inclusions have been investigated taking advantage of the potential theory methods.

Basing on the available methods of investigation of propagation and diffraction of non-stationary waves analytical solutions not only for the cases of circular non-section holes have been obtained, as the stress distribution along the boundaries is stable. For other cross-section shapes stress distribution along the boundary is of complex nature, which makes the calculation of their dynamic stress state more difficult. Besides, most available methods cannot investigate the impact of the reflected from the boundaries wave cavities on the stress state of bodies.

The Objective of the work is to develop analytical-numerical method of investigation of the dynamic stress state of bodies with tunnel cavities or systems of tunnel cavities under impact loading applied to the cavity boundary, which would be able to make possible to study stress distribution along the boundary, and to investigate the effect of reflected wave cavities from the boundaries.

Statement of the task. Let us analyse the uniform isotropic medium with the Young's modulus E and Poisson's ratio ν with tunnel cavity or a system of tunnel cavities of constant cross-section. Let us sign their cross-section outline as $L_1, L_2, \ldots, L_K$. Let us treat the examined body in the Cartesian coordinates $Ox_1x_2x_3$. Let us investigate the dynamic stress distribution on the cavity boundary under impact load.

Analysis of the investigations. For the body under plane deformation, which is parallel to the plane Ox_1x_2, the movement equation in displacement looks like [4]:

$$\left(c_1^2 - c_2^2\right)u_{i,ij} - c_2^2 u_{j,ii} + b_j = \partial^2 u_j / \partial t^2, \tag{1}$$

where $\mathbf{u}(\mathbf{x},t) = \left\{u_j(\mathbf{x},t)\right\},\ j = 1, 2$ – displacement vector of arbitrary point, $c_1 = \sqrt{(\lambda + 2\mu)/\rho},\ c_2 = \sqrt{\mu/\rho}$ – rate of wave expansion and shearing, λ, μ – Lamé constants, t – time, $\mathbf{b} = \{b_j\}$ – mass forces vector is treated as a differentiation along x_j.

Similarly to [3, 5], using Fourier transform

$$\tilde{f}(x, \omega) = \int_{-\infty}^{\infty} f(x, t) e^{-i\omega t} dt,$$

to the movement equation (1), we will obtain the equation:

$$\left(c_1^2 - c_2^2\right) \tilde{u}_{i},_{ij} - c_2^2 \tilde{u}_{j,ii} + \tilde{b}_j + \omega^2 \tilde{u}_j = 0$$

which are equivalent to the equations of the stable oscillations with the cyclic frequency ω [4]. Here $\tilde{u}_j$, $\tilde{b}_j$ are the Fourier images of the displacement vectors and mass forces, which are found basing on the dependencies (2). Further investigations of the dynamic stress state will be carried out in the field of the Fourier images.

To reconstruct the dynamic stress fields let us model the impact load, which is applied to the cross-section of the tunnel cavity boundary, as follows:

$$\phi(t) = \begin{cases} 0, & t < 0; \\ p_1 t^{n_1} e^{-\alpha_1 t} H(t), & 0 \le t \le \alpha_1; \\ H(t), & \alpha_1 \le t \le \alpha_2; \\ p_2 t^{n_2} e^{-\alpha_2 t} H(t), & t > \alpha_2, \end{cases} \tag{2}$$

where $p_1, p_2, \alpha_1, n_2, \alpha_2, n_2$ – constants; $H(t)$ – Heaviside function.

Boundary conditions of the task in the field of the Fourier images are written as follows:

$$\tilde{\sigma}_n\big|_L = \tilde{\phi}(\omega), \ \tilde{\tau}_{sn}\big|_L = 0, \tag{3}$$

where $\tilde{\phi}(\omega)$ – the function image (2).

In the case of the first principle task potential image of the general solution in the displacement looks like [6]:

$$\tilde{u}_i(\mathbf{x}, \omega) = \int_L p_j(\mathbf{x}^0, \omega) \cdot U_{ji}^*(\mathbf{x}, \mathbf{x}^0, \omega)\, ds, \tag{4}$$

where p_1, p_2 – unknown complex potential functions; $L = L_1 \cup L_2 \cup ... \cup L_K$ – area boundary. Integration along the boundary is performed according to variables $\mathbf{x}^0 = \left\{x_1^0, \ x_2^0\right\}$. Expressions for the function images U_{ij}^* is chosen taking into account the вибираються з врахуванням Sommerfeld conditions [4] similarly to [5] as follows:

$$U_{ij}^* = \left(K_0\left(k_2 r\right)\delta_{ij} + \partial_i \partial_j \left(K_0\left(k_1 r\right) - K_0\left(k_2 r\right)\right) / k_2^2\right) / 2\pi\mu, \tag{5}$$

where $\partial_j = \partial / x_j$, $k_j = i\omega / c_j$, $j = 1, 2$ – wave numbers, $K_0(r)$ – modified 3-d level or zero order Bessel function (or Macdonald function); $r = \sqrt{(x_1 - x_1^0)^2 + (x_2 - x_2^0)^2}$ – distance.

To find unknown values on the function boundary p_1, p_2, let us provide the boundary conditions (4), finding the stress according to the formula [4]:

$$\tilde{\sigma}_n = (\sigma_{11}+\sigma_{22})/2 + \left(e^{-2i\alpha}\left(\sigma_{11}-\sigma_{22}+2i\sigma_{12}\right)+e^{2i\alpha}\left(\sigma_{11}-\sigma_{22}-2i\sigma_{12}\right)\right)/4;$$
$$\tilde{\tau}_{sn} = i\left(e^{2i\alpha}\left(\sigma_{11}-\sigma_{22}-2i\sigma_{12}\right)-e^{-2i\alpha}\left(\sigma_{11}-\sigma_{22}+2i\sigma_{12}\right)\right)/4; \qquad (6)$$

here α – the angle between the normal $\vec{n}$ to the plane and the axis Ox_1.

Having substituted (5) in formula (6), we will obtain integral dependencies of the type:

$$\tilde{\sigma}_n = \int_L f_j\left(\mathbf{x},\mathbf{x}^0\right)p_j ds;\ \tilde{\tau}_{sn} = \int_L g_j\left(\mathbf{x},\mathbf{x}^0\right)p_j ds,$$

where $f_j,\ g_j,\ j=1,\ 2$ – the known functions, which have 3-d level Bessel function, which in the operation form is written as follows:

$$f_j = \left(\alpha_1^{(j)}K_0(k_2 r)+\alpha_2^{(j)}\left(K_0(k_1 r)-K_0(k_2 r)\right)/k_2^2\right)/2\pi\mu;$$
$$g_i = i\left(\beta_1^{(j)}K_0(k_2 r)+\beta_2^{(j)}\left(K_0(k_1 r)-K_0(k_2 r)\right)/k_2^2\right)/2\pi\mu;$$
$$\alpha_1^{(1)} = (1+e^{2i\alpha})\partial_z + (1+e^{-2i\alpha})\partial_{\bar{z}};\ \alpha_1^{(2)} = i\left((1-e^{2i\alpha})\partial_z - (1-e^{-2i\alpha})\partial_{\bar{z}}\right);$$
$$\alpha_2^{(1)} = \partial_1\left(\Delta + 2\left(e^{2i\alpha}\partial_z^2 + e^{-2i\alpha}\partial_{\bar{z}}^2\right)\right);\ \alpha_2^{(2)} = \partial_2\left(\Delta + 2\left(e^{2i\alpha}\partial_z^2 + e^{-2i\alpha}\partial_{\bar{z}}^2\right)\right);$$
$$\beta_1^{(1)} = e^{2i\alpha}\partial_z - e^{-2i\alpha}\partial_{\bar{z}};\ \beta_1^{(2)} = -i\left(e^{2i\alpha}\partial_z + e^{-2i\alpha}\partial_{\bar{z}}\right);$$
$$\beta_2^{(1)} = 2\partial_1\left(e^{2i\alpha}\partial_z^2 - e^{-2i\alpha}\partial_{\bar{z}}^2\right);\ \beta_2^{(2)} = 2\partial_2\left(e^{2i\alpha}\partial_z^2 - e^{-2i\alpha}\partial_{\bar{z}}^2\right),$$

where $\partial_z = \partial/\partial z,\ \partial_{\bar{z}} = \partial/\partial\bar{z},\ z = x_1 + ix_2,\ \bar{z} = x_1 - ix_2,\ \Delta$ – Laplace operator.

Subintegral functions $f_j,\ g_j,\ j=1,\ 2$ at small values of argument are not regular. To find their characteristics let us write the expressions for these functions in details, using the complex variables, $\zeta = x_1^0 + ix_2^0$:

$$f_1 = \frac{1}{2\pi\mu}\left(\frac{k_2}{2r}K_1(k_2 r)\operatorname{Re}\left(\bar{Z}\left(\frac{\bar{Z}}{Z}-1\right)e^{2i\alpha}\right)-\left(\frac{c_1}{c_2}\right)^2\frac{k_1}{2r}K_1(k_1 r)\operatorname{Re}\left(2Z+\bar{Z}\left(1+\frac{\bar{Z}}{Z}\right)e^{2i\alpha}\right)+\right.$$
$$\left.+2\left(K_2(k_2 r)-\left(\frac{c_1}{c_2}\right)^2K_2(k_1 r)\right)\operatorname{Re}\left(\frac{\bar{Z}}{Z^2}e^{2i\alpha}\right)\right);$$

$$f_2 = \frac{1}{2\pi\mu}\left(\frac{k_2}{2r}K_1(k_2 r)\operatorname{Im}\left(\bar{Z}\left(\frac{\bar{Z}}{Z}+1\right)e^{2i\alpha}\right)-\left(\frac{c_1}{c_2}\right)^2\frac{k_1}{2r}K_1(k_1 r)\operatorname{Im}\left(2Z-\bar{Z}\left(\frac{\bar{Z}}{Z}-1\right)e^{2i\alpha}\right)-\right. \qquad (7)$$
$$\left.-2\left(K_2(k_2 r)-\left(\frac{c_1}{c_2}\right)^2K_2(k_1 r)\right)\operatorname{Im}\left(\frac{\bar{Z}}{Z^2}e^{2i\alpha}\right)\right);$$

$$g_1 = \frac{1}{2\pi\mu}\left(\frac{k_2}{2r}K_1(k_2 r)\operatorname{Im}\left(\bar{Z}\left(\frac{\bar{Z}}{Z}-1\right)e^{2i\alpha}\right)-\left(\frac{c_1}{c_2}\right)^2\frac{k_1}{2r}K_1(k_1 r)\operatorname{Im}\left(\bar{Z}\left(\frac{\bar{Z}}{Z}+1\right)e^{2i\alpha}\right)-\right.$$

$$-2\left(K_2(k_2r)-\left(\frac{c_1}{c_2}\right)^2 K_2(k_1r)\right)\mathrm{Im}\left(\frac{\bar{Z}}{Z^2}e^{2i\alpha}\right)\Bigg);$$

$$g_2=\frac{1}{2\pi\mu}\left(-\frac{k_2}{2r}K_1(k_2r)\,\mathrm{Re}\left(\bar{Z}\left(\frac{\bar{Z}}{Z}+1\right)e^{2i\alpha}\right)+\left(\frac{c_1}{c_2}\right)^2\frac{k_1}{2r}K_1(k_1r)\,\mathrm{Re}\left(\bar{Z}\left(\frac{\bar{Z}}{Z}-1\right)e^{2i\alpha}\right)-\right.$$

$$-2\left(K_2(k_2r)-\left(\frac{c_1}{c_2}\right)^2 K_2(k_1r)\right)\mathrm{Re}\left(\frac{\bar{Z}}{Z^2}e^{2i\alpha}\right)\Bigg);$$

where $Z=z-\zeta$.

To find irregular components in presentations (7) let us use asymptotic expressions for the 3-d level Bessel function [7]:

$$K_1(r)=1/r+\left(-\ln 2+\ln r+\gamma-1/2\right)r/2+O(r^3),$$
$$K_2(r)=2/r^2-1/2+\left(\ln 2-\ln r-\gamma+3/4\right)r^2/8+O(r^4),$$

where γ – Euler's constant.

Using the formulas of Plemelj-Sokhotski [4] at boundary transition in dependencies (7), we will obtain the system of integral equations for finding unknown at the function boundary *p1, p2*:

$$\frac{1}{2}\mathrm{Re}\left(ip\frac{ds}{d\zeta}\right)+\int_L f_j\left(\mathbf{x},\,\mathbf{x}^0\right)p_j ds=\tilde{\varphi};\;\frac{1}{2}\mathrm{Im}\left(ip\frac{ds}{d\zeta}\right)+\int_L g_j\left(\mathbf{x},\,\mathbf{x}^0\right)p_j ds=0; \tag{8}$$

where $p=p_1+ip_2$ – unknown function.

To solve the obtained system of integral equations (8) let us use the numerical algorithm, based on the method of mechanical quadrature and collocations. Specified quadrature formulas will be used to the integrals of Cauchy type. We will obtain the system of linear algebraic equations as follows:

$$\begin{aligned}&\frac{1}{2}\mathrm{Re}\left(ip_s\frac{|g_s'|}{g_s'}\right)+h\sum_{n=1}^{N}f_1\left(\mathbf{x}_s,\mathbf{x}_n^0\right)p_{1n}|g_n'|+h\sum_{n=1}^{N}f_2\left(\mathbf{x}_s,\mathbf{x}_n^0\right)p_{2n}|g_n'|=\tilde{\phi}_\kappa;\\&\frac{1}{2}\mathrm{Im}\left(ip_s\frac{|g_s'|}{g_s'}\right)+h\sum_{n=1}^{N}g_1\left(\mathbf{x}_s,\mathbf{x}_n^0\right)p_{1n}|g_n'|+h\sum_{n=1}^{N}g_2\left(\mathbf{x}_s,\mathbf{x}_n^0\right)p_{2n}|g_n'|=0;\end{aligned} \tag{9}$$

where $g_n=g(\varphi_n)$, $g_s=g(\varphi_s)$, $\phi_n=hn$, $\varphi_s=\varphi_n+h/2$, $h=2\pi/N$, $g(\varphi)$ – parametric boundary of the region L; N – number of nodal points. Parameter φ is chosen taking into account Sidi's nonlinear transformation so that while describing the boundary the region was to the left. Such approach happened to be efficient for investigations of the constant oscillations of section plates [2].

Having found unknown values from the system (9) the calculation of circular loads at the tunnel cavity boundary was performed numerically basing on the dependencies [4]:

$$\tilde{\sigma}_\theta=(\sigma_{11}+\sigma_{22})/4-\left(e^{-2i\alpha}\left(\sigma_{11}-\sigma_{22}+2i\sigma_{12}\right)+e^{2i\alpha}\left(\sigma_{11}-\sigma_{22}-2i\sigma_{12}\right)\right)/4,$$

which with the potential image being substituted (4), taking into account expressions for the images (5) in the operation appearance is written as follows:

$$\tilde{\sigma}_\theta = \int_L \left(q_j \left(\mathbf{x}, \mathbf{x}^0 \right) p_j \right) ds, \tag{10}$$

here

$$q_j = \left(\gamma_1^{(j)} K_0 \left(k_2 r \right) + \gamma_2^{(j)} \left(K_0 \left(k_1 r \right) - K_0 \left(k_2 r \right) \right) / k_2^2 \right) / 2\pi\mu;$$
$$\gamma_1^{(1)} = (1 - e^{2i\alpha}) \partial_z + (1 - e^{-2i\alpha}) \partial_{\bar{z}}\, ; \; \gamma_1^{(2)} = i \left((1 + e^{2i\alpha}) \partial_z - (1 + e^{-2i\alpha}) \partial_{\bar{z}} \right);$$
$$\gamma_2^{(1)} = \partial_1 \left(\Delta - 2 \left(e^{2i\alpha} \partial_z^2 + e^{-2i\alpha} \partial_{\bar{z}}^2 \right) \right); \; \gamma_2^{(2)} = \partial_2 \left(\Delta - 2 \left(e^{2i\alpha} \partial_z^2 + e^{-2i\alpha} \partial_{\bar{z}}^2 \right) \right).$$

Having extracted irregular components in (10) and taking advantage of the formulas of Plemelj-Sokhotski at boundary transition, we will obtain dependencies for the calculation of circular stresses at the cavity boundary:

$$\tilde{\sigma}_\theta = \frac{\nu}{2(1-\nu)} \mathrm{Im} \left(p \frac{ds}{dt} \right) + \int_L h_j \left(\mathbf{x}, \mathbf{x}^0 \right) p_j ds\, . \tag{11}$$

Determination of originals obtained basing on formulas (11) of stresses was performed using the reversible Fourier transform:

$$\sigma_\theta(\mathbf{x}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\sigma}_\theta(\mathbf{x}, \omega) e^{i\omega t} d\omega\, ,$$

which for the numerical calculation of the circular stresses is realised due to the discrete Fourier transform, which was found to be efficient for investigation of the impulse wave diffraction under antiplane deformation [3]:

$$\sigma_\theta \left(T_k \right) = \frac{2}{T} \mathrm{Re} \left(\sum_{n=0}^{K-1} \tilde{\sigma}_\theta \left(\omega_n \right) \exp \left(2\pi i \frac{n \cdot k}{K} \right) - \sum_{n=0}^{K-1} \tilde{\sigma}_\theta \left(\omega_n \right) \right), \tag{12}$$

where K - number of the discrete sampling elements, $\omega_n = 2\pi n / T$ - sampling frequencies, $T_k = \frac{kT}{K}$, $k = 0..K-1$ - fixed time terms. To optimize the numerical calculations dynamic circular stresses (12) can be easily calculated taking advantage of the fast discrete Fourier transform on the basis of the Cooley-Tukey algorithm [9] at $K = 2^m$, m - integer positive real number.

In [10] the accuracy control of the developed approach while comparing of the obtained numerical results with the available data for the case of plane stress state described in the papers by Guz O.M., Kubenko V.D., etc., has been performed. The obtained results confirm high accuracy of the proposed method of the analytical-numerical calculation.

Results of the investigations. On the basis of the developed approach the modeling of the dynamic stress state of non-finite elastic medium, weakened by the system of tunnel cavities with circular and elliptic cross-section (Fig. 1) under impact load as (2), which is applied to the cavity boundary of the circular cross-section, has been developed. Numerical calculations have been performed for the case, when the circular cavity radius is *0.2a*, and semiaxles of elliptic cavity cross-section are *0.2a* and *a.*

Numerical calculations are performed for N=250 nodal points at the tunnel cavity cross-sections boundary and $K=2^{13}$ number of the discrete sampling components in the Fourier transform. Calculation of the dynamic stresses is performed for the sandy shale with the density $\rho=3\cdot10^3$ kg/m^3, Young's modulus $E=3.1\cdot10^4$ *Mpa* and Poisson's ratio $\nu=0.33$.

Dependence on the relative values of the dynamic stresses on the dimensionless time parameter tc_1/a at the cavity boundary is shown in Fig.2 for the case $\gamma=30°$. Here the values with the sign 1 are the stress values at the cavity boundary of the circular cross-section, the values with the sign 2 - at the cavity boundary of the elliptic cross-section. The value $\sigma(M)$ is the stress value in point *M*, which is in the distance *a* from the center of the elliptic cavity. While calculating it was considered that $\sigma_0 = 1MPa$.

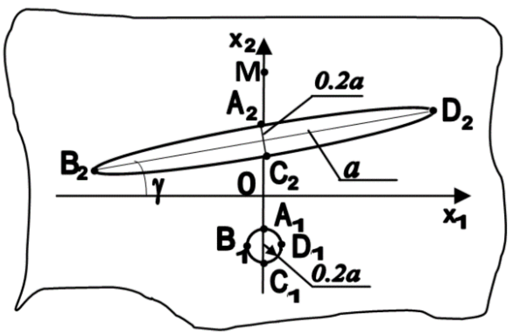

Figure 1. Cross-sectional model of elastic medium

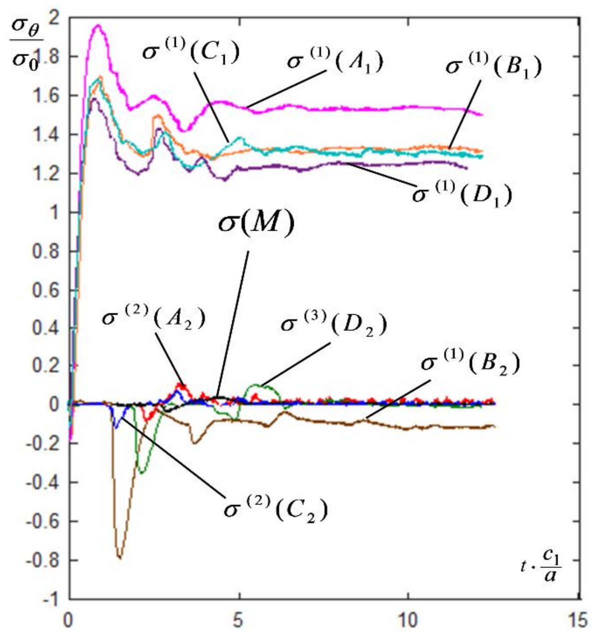

Figure 2. Distribution of dynamic stresses at the boundary of tunnel cavities

In Fig. 2 it is seen, that the wave, which is initiated by the impact load, reaches the cavity boundary of the elliptic cross-section at the moment *t=1.77 sec*. The cavity boundary having been reached, which is the wave radiator itself, the reflected part of the wave propagates from its boundary. This fact is confirmed by the Fresnel-Huygens principle.

The wave split by the non-uniformity propagates further along the cavity boundary. At the time *t=1.85 sec* it reaches B_2 at the cavity boundary of elliptic cross-section, and at *t=2.27 c* - point D_2.

The waves reflected from the tunnel cavities boundaries affect sufficiently the further body stress state, which is of the oscillation nature in the time range $t\in(3.94;8.05)$.

The change of the dynamic stresses in point M in time confirms the fact, that the impact load intensity, applied to the cylinder cavity boundary, is totally decreased in the layers located in the distance *3a* from its center.

Matching of the dissipated wave fields propagation with the available principles of the wave mechanics confirms additionally the efficiency of application of the developed approach to the investigation of the dynamic stress state of bodies weakened by the systems of cavities of non-canon cross-section.

Conclusions. The integral and discrete Fourier transform happened to be efficient for the investigation of the non-stationary dynamic tasks. High accuracy of the proposed approach is provided by application of the discrete transformation only at the stage of searching for the dynamic stress originals and matching of the obtained results with the main principles of the wave mechanics.

The developed approach on the basis of the boundary integral equation method and the theory of the complex variable function makes it possible to investigate the time dependencies of the dynamic stress change in the bodies with tunnel cavities or system of cavities practically of any cross-section.

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УДК 539.3

ДОСЛІДЖЕННЯ ВПЛИВУ УДАРНИХ ХВИЛЬ НА ДИНАМІЧНИЙ НАПРУЖЕНИЙ СТАН СЕРЕДОВИЩА, ПОСЛАБЛЕНОГО СИСТЕМОЮ ТУНЕЛЬНИХ ПОРОЖНИН

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Резюме. *Розвинено методику дослідження розподілу динамічних напружень у пружних тілах з системами тунельних порожнин на основі інтегрального та дискретного перетворення Фур'є за часом. В області Фур'є-зображень використання методу граничних інтегральних рівнянь та апарату теорії функції комплексної змінної дало можливість розробити ефективний алгоритм визначення динамічного напруженого стану для тіл з порожнинами практично довільного перерізу. На основі запропонованої методики проведено числові розрахунки динамічної концентрації напружень на границі тунельних порожнин кругового та еліптичного перерізу й побудовано часові розподіли динамічних напружень за дії ударного навантаження. Вивчено вплив відбитих хвиль на динамічний напружений стан у тунельних порожнинах.*

Ключові слова: *нестаціонарна задача, тунельна порожнина.*

Отримано 18.08.2017