СИСТЕМНИЙ АНАЛІЗ І ТЕОРІЯ ПРИЙНЯТТЯ РІШЕНЬ

СИСТЕМНЫЙ АНАЛИЗ И ТЕОРИЯ ПРИНЯТИЯ РЕШЕНИЙ

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COMBINATORIAL OPTIMIZATION UNDER UNCERTAINTY AND FORMAL MODELS OF EXPERT ESTIMATION

Previously, the author formalized the concepts of uncertainty, compromise solution, compromise criteria and conditions for a quite general class of combinatorial optimization problems. The functional of the class' problems contains linear convolution of weights and arbitrary numerical characteristics of a feasible solution. It was shown that the efficiency of the presented algorithms for the uncertainty resolution is largely determined by the efficiency of solving the combinatorial optimization problem in a deterministic formulation. A part of the formulated compromise criteria and conditions uses expert weights. Previously, the author and his disciples also formulated combinatorial optimization models, optimality criteria, criteria for decisions' consistency. The models allow to evaluate and justify the degree of stability and reliability of the estimated values of empirical coefficients using a formally ill-conditioned empirical pairwise comparison matrix of arbitrary dimension. The matrix may contain zero elements. The theoretical research and statistical experiments allowed to choose the most efficient of these optimization models. In this article, on the base of earlier results by the author and his disciples, we formalize and substantiate the efficiency of the proposed sequential procedure for expert estimation of weights that determine compromise criteria and conditions. The procedure is an integral part of the algorithm introduced by the author to solve combinatorial optimization problems under uncertainty of the mentioned class. We give unified algorithm for efficient uncertainty resolution that includes original and efficient formal procedure for expert coefficients' estimation using empirical matrices of pairwise comparisons.

Keywords: combinatorial optimization, uncertainty, compromise criteria, compromise conditions, empirical matrix of pairwise comparisons, consistent decision.

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КОМБІНАТОРНА ОПТИМІЗАЦІЯ В УМОВАХ НЕВИЗНАЧЕНОСТІ ТА ФОРМАЛЬНІ МОДЕЛІ ЕКСПЕРТНОГО ОЦІНЮВАННЯ

Раніше автором для досить загального класу задач комбінаторної оптимізації, функціонал яких містить лінійну згортку ваг і довільних числових характеристик допустимого розв'язку, були формалізовані поняття невизначеності, компромісного розв'язку, компромісних критеріїв та умов. Було показано, що ефективність наведених алгоритмів розв'язання невизначеності в значній мірі визначається ефективністю розв'язання задачі комбінаторної оптимізації в детермінованій постановці. Частина сформульованих компромісних критеріїв і умов використовує експертні вагові коефіцієнти. Раніше також автором і його учнями були сформульовані моделі комбінаторної оптимізації, критерії оптимальності, критерії узгодженості рішень, що дозволяють за формально погано узгодженою емпіричною матрицею парних порівнянь довільної розмірності, яка, можливо, містить нульові елементи, знаходити та обґрунтовувати ступінь стійкості та достовірності знайдених значень емпіричних коефіцієнтів. Проведені теоретичні дослідження та статистичні експерименти дозволили виділити з цих моделей оптимізації найбільш ефективні. У даній статті на основі отриманих раніше автором і його учнями результатів формалізовано і обґрунтовано ефективність запропонованої послідовної процедури знаходження експертних вагових коефіцієнтів, що визначають компромісні критерії та умови, як складової частини алгоритму розв'язання для введеного автором класу задач комбінаторної оптимізації в умовах невизначеності. Наводиться єдиний алгоритм ефективного розв'язання невизначеності, який включає в себе оригінальну ефективну формальну процедуру знаходження експертних коефіцієнтів за емпіричними матрицями парних порівнянь.

Ключові слова: комбінаторна оптимізація, невизначеність, компромісні критерії, компромісні умови, емпірична матриця парних порівнянь, узгоджене рішення.

А. А. ПАВЛОВ

КОМБИНАТОРНАЯ ОПТИМИЗАЦИЯ В УСЛОВИЯХ НЕОПРЕДЕЛЕННОСТИ И ФОРМАЛЬНЫЕ МОДЕЛИ ЭКСПЕРТНОГО ОЦЕНИВАНИЯ

Ранее автором для достаточно общего класса задач комбинаторной оптимизации, функционал которых содержит линейную свертку весов и произвольных числовых характеристик допустимого решения, были формализованы понятия неопределенности, компромиссного решения, компромиссных критериев и условий. Было показано, что эффективность приведенных алгоритмов разрешения неопределенности в значительной степени определяется эффективностью решения задачи комбинаторной оптимизации в детерминированной постановке. Часть сформулированных компромиссных критериев и условий использует экспертные весовые коэффициенты. Ранее также автором и его учениками были сформулированы модели комбинаторной оптимизации, критерии оптимальности, критерии согласованности решений, позволяющие по формально плохо согласованной эмпирической матрице парных сравнений произвольной размерности, возможно, содержащей нулевые элементы, находить и обосновывать степень устойчивости, достоверности найденных значений эмпирических коэффициентов. Проведенные теоретические исследования и статистические эксперименты позволили выделить из этих моделей оптимизации наиболее эффективные. В данной статье на основе полученных ранее автором и его учениками результатов формализована и обоснована эффективность предложенной последовательной процедуры нахождения экспертных весовых коэффициентов, определяющих компромиссные критерии и условия, как составной части алгоритма решения для введенного автором класса задач комбинаторной оптимизации в условиях неопределенности. Приводится единый алгоритм эффективного разрешения неопределенности, который включает в себя оригинальную эффективную формальную процедуру нахождения экспертных коэффициентов по эмпирическим матрицам парных сравнений.

Ключевые слова: комбинаторная оптимизация, неопределенность, компромиссные критерии, компромиссные условия, эмпирическая матрица парных сравнений, согласованное решение.

Introduction. We studied in [1] a class of combinatorial optimization problems of the following form:

$$\min_{\sigma\epsilon\Omega}(\max) \sum_{i=1}^{s} \omega_i k_i(\sigma)\,. \qquad (1)$$

Here, ω_i are numbers, Ω the set of feasible solutions; σ an arbitrary feasible solution, $k_i(\sigma)$ is i-th arbitrary numerical characteristic of σ. We recommended in [1] to use the proposed approach to solve problems of the form (1) under uncertainty only for the following case: for the problems of the form (1) there exists an exact (or approximation) algorithm which is qualitatively more efficient in terms of speed and/or accuracy than an arbitrary solving method for the problem (1) in which the structure of the feasible solutions domain differs from Ω of (1), e.g., in additional constraints on inequalities. For a problem of the form (1), solution of which uses expert coefficients, we formulated in [1] the combinatorial optimization problem statement under uncertainty as follows.

There are L sets of weights $\{\omega_i^l, i=\overline{1,s}\}$, $l=\overline{1,L}$. Here, $\overline{a,b}$ denotes the interval of integer numbers from a to b. Each of the sets may be a set of coefficients $\omega_1,\dots,\omega_s$ of the problem (1) at the stage of implementation of its solution. Probabilities $P_l>0$, $l=\overline{1,L}$, $\sum_l P_l=1$, may be specified for each possible set of weights (such probabilities do not exist if the uncertainty is not described by a probabilistic model). We need to find a feasible solution satisfying one of the conditions given below.

Here we investigate the problem (1) for the case of $\min_{\sigma\epsilon\Omega} \sum_{i=1}^{s} \omega_i k_i(\sigma)$. We consider the case of $\max_{\sigma\epsilon\Omega} \sum_{i=1}^{s} \omega_i k_i(\sigma)$ in [1]. Let us introduce the notation:

$$f_{\rm opt}^l = \min_{\sigma\epsilon\Omega} \sum_{i=1}^{s} \omega_i^l k_i(\sigma)\,,$$

$$\{\sigma_l\} = \arg\min_{\sigma\epsilon\Omega} \sum_{i=1}^{s} \omega_i^l k_i(\sigma)\,,$$

$$L_l = \sum_{\substack{m=1\\ m\neq l}}^{L} \left(\sum_{i=1}^{s} \omega_i^m k_i(\sigma_l) - f_{\rm opt}^m\right), l=\overline{1,L}.$$

Remark 1. If $\{\sigma_l\}$ consists of more than one solution, then we leave the one on which we achieve $\min_{\{\sigma_l\}} L_l$ and denote this solution as σ_l. We consider in [1] obtaining σ_l for the case when Ω is finite.

Suppose that

$$\Delta_l = \sum_{i=1}^{s} \omega_i^l k_i(\sigma) - f_{opt}^l \geq 0,$$

$$f_{\rm opt}^l = \min_{\sigma\epsilon\Omega} \sum_{i=1}^{s} \omega_i^l k_i(\sigma)\,. \qquad (2)$$

We need to find:

1) a feasible solution $\sigma(\Delta_1,\dots,\Delta_L)$ that has

$$\Delta_i \leq l_i, i=\overline{1,L} \qquad (3)$$

where l_i are given numbers;

2) a feasible solution that satisfies the condition:

$$\min_{\sigma\epsilon\Omega} \sum_{l=1}^{L} a_l \left(\sum_{i=1}^{s} \omega_i^l k_i(\sigma) - f_{\rm opt}^l\right) \qquad (4)$$

where $\forall a_l > 0$ are the coefficients specified by an expert.

Remark 2. For the general case, we formulated in [1] five compromise criteria and conditions for $L=2$ and four compromise criteria and conditions for $L>2$.

Remark 3. We formulated similar criteria, conditions, and algorithms for the case of the problem (1) when we reach the optimal functional value at $\max_{\sigma\epsilon\Omega}$.

The compromise solutions proposed in [1] are based on the following statement and its corollaries.

Statement 1 [1]. The following is true for arbitrary $a_l>0$, $l=\overline{1,L}$:

$$\arg\min_{\sigma\epsilon\Omega} \sum_{l=1}^{L} a_l \left[\sum_{i=1}^{s} \omega_i^l k_i(\sigma) - f_{\rm opt}^l\right]$$

$$= \arg\min_{\sigma\epsilon\Omega} \sum_{i=1}^{s} \left(\sum_{l=1}^{L} a_l \omega_i^l\right) k_i(\sigma). \qquad (5)$$

Corollary 1. Solving the problem $\min_{\sigma\epsilon\Omega} \sum_{l=1}^{L} a_l \times$ $\times \left(\sum_{i=1}^{s} \omega_i^l k_i(\sigma) - f_{opt}^l\right)$ reduces to solving one problem of the form (1):

$$\min_{\sigma\epsilon\Omega} \sum_{i=1}^{s} \left(\sum_{l=1}^{L} a_l \omega_i^l\right) k_i(\sigma). \qquad (6)$$

Corollary 2. Suppose that $\sigma(a_1,\dots,a_L,\Delta_1,\dots,\Delta_L) \neq$ $\neq \sigma_1 \vee \dots \vee \sigma_L$; $\exists \sigma(a_1,a_2,\dots,a_{i-1},a'_i, a_{i+1},\dots,a_L, \Delta'_1,\dots,\Delta'_L) \neq \sigma_1 \vee \dots \vee \sigma_L$, $a'_i \neq a_i$. Then the following holds:

$$(a'_i - a_i)(\Delta'_i - \Delta_i) < 0,$$

$$(\Delta'_i - \Delta_i)[(a_1\Delta'_1 + \dots + a_{i-1}\Delta'_{i-1} +$$

$$+ a_{i+1}\Delta'_{i+1} + \dots + a_L\Delta'_L) - (a_1\Delta_1 + \dots +$$

$$+ a_{i-1}\Delta_{i-1} + a_{i+1}\Delta_{i+1} + \dots + a_L\Delta_L)] < 0. \qquad (7)$$

where $\sigma(a_1,\dots,a_L,\Delta_1,\dots,\Delta_L)$ is an optimal solution of the problem (6) with given expert coefficients $a_i>0, i=\overline{1,L}$. $\Delta_i(\Delta'_i), i=\overline{1,L}$ are specified in (2) for sets $a_1,\dots,a_L$; $a_1,\dots,a_{i-1},a'_i,a_{i+1},\dots,a_L$, respectively.

It is shown in [1] that if $\sigma(1,\dots,1,\Delta_1,\dots,\Delta_L) \neq$ $\neq \min_{j=\overline{1,L}}\{\sigma_j\}$, then, by virtue of Corollary 2 (by logic of the inequalities (7)), we can organize a sequential procedure for solving a problem of the form (6), increasing $\forall a_i$ at each iteration if $\Delta_i > l_i$ or decreasing $\forall a_j$ if $\Delta_j < l_j$ (here, $\sigma(\Delta_1,\dots,\Delta_L)$ is the current solution). As a result, we either find a solution $\sigma(\Delta_1,\dots,\Delta_L)$ that satisfies the condition $\Delta_i \leq l_i, i=\overline{1,L}$ or obtain a set of solutions $\{\sigma\}^1$, each of which violates the compromise condition (1). In [1], we proposed to choose the solution from the set $\{\sigma_1, \dots, \sigma_L\} \cup\cup \{\sigma\}^1$ as the compromise solution if it reaches

$$\min \sum_t C_{jt}(\Delta_{jt} - l_{jt}),\ \forall t \Delta_{jt} > l_{jt}, \quad (8)$$

where $C_j > 0, j = \overline{1,L}$ are expert coefficients.

Thus, the efficiency of a compromise solution that meets the compromise condition 1) or the compromise criterion 2) essentially depends on the relevance of expert coefficients $a_j > 0, C_j > 0, j = \overline{1,L}$, used in optimization models, to the essence of the practical problem, the formal model of which is the combinatorial optimization problem under uncertainty.

Formal models for expert coefficients' estimation by an empirical matrix of pairwise comparisons. The author and his disciples formulated in [2–4] combinatorial optimization models, optimality criteria, criteria for decisions' consistency. The models allow to evaluate and justify the degree of stability and reliability of the estimated values of empirical coefficients using a formally ill-conditioned empirical pairwise comparison matrix of arbitrary dimension. The matrix may contain zero elements. Theoretical research and statistical experiments [2–4] allow to use the following formal procedures to estimate the empirical coefficients $a_j > 0, C_j > 0, j = \overline{1,L}$.

1. The initial data is an empirical matrix $\mathbf{\Gamma}$ (for estimation of weights $a_j > 0, j = \overline{1,L}$, or $C_j > 0, j = \overline{1,L}$). In general case, we denote weights by $\omega_j > 0, j = \overline{1,L}$. γ_{ij} is (ij)-th coefficient of the matrix $\mathbf{\Gamma}$. It determines, according to the expert, how many times the value of ω_i is greater than the value of ω_j.

To construct and substantiate optimization criteria and models for estimation of weights $\omega_i, i = \overline{1,L}$, by an empirical matrix of pairwise comparisons $\mathbf{\Gamma}$, we use the set A of coefficients $\gamma_{ij}, i,j = \overline{1,L}$, of the matrix of pairwise comparisons $\mathbf{\Gamma}$ which satisfy the conditions:

1) $\{\gamma_{ij}, i = \overline{1,L}\} \sqsubset A$;

2) $\forall i \neq j\ \gamma_{ij} \in A$ if $\gamma_{ij} \geq 1$, $\gamma_{ij} \notin A$.

We propose to use the measure

$$\frac{1}{\gamma_{ij}}\left|\frac{\omega_i}{\omega_j} - \gamma_{ij}\right| \quad (9)$$

as the consistency measure for estimated $\omega_i, i = \overline{1,L}$.

2. We propose to use the following models as optimization models for weights' estimation.

Model 1 requires sequential solving of the following linear programming (LP) problems:

$$\min_{\substack{\Delta^1_{ij}, \Delta^2_{ij} \\ \forall (ij) \in |A|}} \left(\sum_{(ij) \in |A|} \sum \Delta^1_{ij} - C_{lm} \sum_{(ij) \in |A|} \sum \Delta^2_{ij} \right), \quad (10)$$

$$\ln \gamma_{ij} + \Delta^2_{ij} \leq W_i - W_j \leq \ln \gamma_{ij} + \Delta^1_{ij},$$

$$\forall (ij) \in |A|,$$

$$0 \leq \Delta^1_{ij} \leq \ln(1 + l \cdot \Delta_1(l)), \forall (ij) \in |A|, \quad (11)$$

$$0 \geq \Delta^2_{ij} \geq \ln(1 - m \cdot \Delta_2(m)), \forall (ij) \in |A|,$$

$$W_i \geq 0, i = \overline{1,L}, \quad (12)$$

where W_i, Δ^1_{ij}, Δ^2_{ij} are the LP problem coefficients;

l,m natural numbers successively taking values (1; 1), (1; 2), (2; 1), (2; 2), etc.;

$\Delta_1(x), \Delta_2(x)$ given numeric scalar functions taking non-negative values, of a natural argument;

C_{lm} coefficient determined from the formula:

$$\ln(1 + l \cdot \Delta_1(l)) = C_{lm} \ln \frac{1}{1 - m \cdot \Delta_2(m)}; \quad (13)$$

$|A|$ the set of pairs (ij), $i \neq j$, for each of which $\gamma_{ij} \geq 1$ ($\gamma_{ij} \in |A|$).

When using problems (10)–(12) for weights' estimation, we set the functions $\Delta_1(x), \Delta_2(x)$ in such a way that the values of $l \cdot \Delta_1(l)$ and $m \cdot \Delta_2(m)$ at each iteration (for each attempt of solving the LP problem (10)–(12)) increase in a small ratio to their previous value. At the first iteration they take the minimum possible values. Due to this method of functions $\Delta_1(x), \Delta_2(x)$ specification, we can achieve the most consistent weights, with respect to measure (9), for each specific problem. This is possible due to the peculiarities of the current matrix of pairwise comparisons.

Iterations stop at the first successful solution of the LP problem (10)–(12) (the first LP problem with a non-empty set of constraints). After that, we find the weights of the objects $\omega^*_i, i = \overline{1,L}$, from the relationship:

$$\omega^*_i = e^{W^*_i}, i = \overline{1,L}. \quad (14)$$

Model 2 is the LP problem of the form:

$$\min \sum_{(ij) \in |A|} \sum r_{ij} \cdot y_{ij},$$

$$-y_{ij} \leq \omega_i - \gamma_{ij}\omega_j \leq y_{ij}, y_{ij} \geq 0, \quad (i,j) \in |A|$$

$$\omega_i \geq a \geq 1, i = \overline{1,L} \quad (15)$$

where a is a specified number;

$\omega_i, i = \overline{1,L}$, $\gamma_{ij}, (ij) \in |A|$ the LP problem variables;

r_{ij} weighted coefficient.

We propose to solve the LP problem (15) for coefficients r_{ij} that take the following values:

1) $$r_{ij} = \frac{1}{\gamma_{ij} - 1}, \forall (ij) \in |A|_{\gamma_{ij} > 1}; \quad (16)$$

2) $$r_{ij} = \frac{1}{\sqrt{\gamma_{ij} - 1}}, \forall (ij) \in |A|_{\gamma_{ij} > 1}; \quad (17)$$

3) $$-y_{ij} \leq \omega_i - \gamma_{ij}\omega_j \leq y_{ij}, y_{ij} \geq 0. \quad (i,j) \in |A| \quad (18)$$

Remark 4. For cases $\gamma_{ij} - 1 \in [0,\varepsilon]$ where $\varepsilon > 0$ is a given number, we replace $\gamma_{ij} - 1$ by ε in expressions (16)–(18).

Remark 5. We can also use other analytical expressions for coefficients r_{ij} that satisfy the condition: $r^1_{ij} < r^2_{ij}$

follows from $\forall\gamma_{ij}^1 > \gamma_{ij}^2 > 1+\varepsilon$ (where $\varepsilon > 0$ is a given number).

Remark 6. An analysis of statistical experiments in [4] concluded that the following analytical expression for r_{ij} is statistically efficient:

$$r_{ij} = \frac{1}{\sqrt[3]{(\gamma_{ij}-1)^2}}, \gamma_{ij} > 1 \ (\varepsilon = 0.1).$$

Nevertheless, in each individual case, we recommend to solve a set of LP problems (15) in which r_{ij} are determined from expressions (16)–(18).

3.1. *Choosing the best solution.* As a result of implementing item 2, we obtain a set of weights $\{\omega_i^p, i = \overline{1,L}\}$, $p = \overline{1,k}$, where k is the number of different found sets of weights $\omega_i, i = \overline{1,L}$. The solution is a set of weights $\omega_i^m, i = \overline{1,L}$, on which we reach the minimum:

$$\min_{p=\overline{1,k}} \sum_{(ij)\in|A|} \sum \frac{1}{\gamma_{ij}} \left| \frac{\omega_i^p}{\omega_j^p} - \gamma_{ij} \right|. \quad (19)$$

3.2. We need to perform the following operations [2, 4] to determine whether the set of weights $\omega_i^m, i = \overline{1,L}$, is a stable solution obtained by the empirical matrix of pairwise comparisons $\mathbf{\Gamma}$.

We build M combinatorial optimization problems, in each of which we replace γ_{ij}, $\forall(ij) \in |A|$, with arbitrary γ_{ij}^l, $\forall(ij) \in |A|$, $l = \overline{1,M}$, that satisfy the conditions

$$\gamma_{ij}^l - \gamma_{ij} = \pm \left| \frac{\widehat{\omega}_i}{\widehat{\omega}_j} - \gamma_{ij} \right|, \forall(ij) \in |A|,$$

where M is a sufficiently large natural number.

Remark 7. Combinatorial optimization problem to obtain $\omega_i^l, l = \overline{1,M}$, coincides with the one for which we obtained $\omega_i^m, i = \overline{1,L}$, up to a replacement of γ_{ij} by γ_{ij}^l.

Let $\widehat{\omega}_i^l$, $i = \overline{1,L}$, be the estimates of the alternatives' weights obtained by l-th optimization model (l-th set of optimization models), $l = \overline{1,M}$. The weights ω_i^m, ω_i^l, $l = \overline{1,M}$, $i = \overline{1,L}$, are equally normalized.

Weights' estimates $\widehat{\omega}_i^m$, $i = \overline{1,L}$, are statistically justified, stable, and well consistent, if numbers $\rho(\widehat{\boldsymbol{\omega}}^m, \widehat{\boldsymbol{\omega}}^l)$, $l = \overline{1,M}$, are acceptable. Here, $\widehat{\boldsymbol{\omega}}^m = (\omega_1^m, \ldots, \omega_L^m)^{\mathrm{T}}$, $\widehat{\boldsymbol{\omega}}^l = (\omega_1^l, \ldots, \omega_L^l)^{\mathrm{T}}$, $l = \overline{1,M}$, and $\rho(\widehat{\boldsymbol{\omega}}^m, \widehat{\boldsymbol{\omega}}^l)$ can be interpreted as the degree of difference between vectors $\widehat{\boldsymbol{\omega}}^m$ and $\widehat{\boldsymbol{\omega}}^l$ [3].

We proposed in [3] to use this measure:

$$\rho(\widehat{\boldsymbol{\omega}}^m, \widehat{\boldsymbol{\omega}}^l) =$$

$$= \sqrt{\sum_{i=1}^L \left(\frac{\omega_i^m}{\sqrt{\sum_{j=1}^L (\omega_j^m)^2}} - \frac{\omega_i^l}{\sqrt{\sum_{j=1}^L (\omega_j^l)^2}} \right)^2}, l = \overline{1,M}. \quad (20)$$

We studied a well-conditioned empirical matrix of pairwise comparisons in [4], according to the computational procedure proposed by Saaty [5–12]. We used Model 1 to estimate the weights. It turned out, as a result of simulation, that $\rho(\widehat{\boldsymbol{\omega}}, \widehat{\boldsymbol{\omega}}^l) \in (0, 0.12)$ in 100 % of cases, where $l = \overline{1,M}$ and $\widehat{\boldsymbol{\omega}} = (\omega_1, \ldots, \omega_L)^{\mathrm{T}}$ are the weights estimated by the simulated well-conditioned matrices of pairwise comparisons.

Thus, if $\rho(\widehat{\boldsymbol{\omega}}^m, \widehat{\boldsymbol{\omega}}^l) \in (0, 0.12)$ $\forall l = \overline{1,M}$, then the estimated values ω_i^m of weights ω_i, $i = \overline{1,L}$, are stable, and the empirical matrix of pairwise comparisons $\mathbf{\Gamma}$ is equivalent to a well-conditioned empirical matrix of pairwise comparisons by the efficiency of the weights' estimation in the sense of [5–12].

Remark 8. The implementation of item 3.2 is laborious and can be omitted if the guaranteed obtaining of stable and well consistent values of $\omega_i, i = \overline{1,L}$, is not mandatory.

Conclusions. In this article, we have proposed an efficient algorithm for uncertainty resolution for a quite general class of combinatorial optimization problems under uncertainty which use expert weights. The algorithm includes efficient algorithmic procedure for expert weights' estimation based on previous results of the author and his disciples.

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