УДК

JOZEF POLACEK, LUBOS VARGOVCIK

Unis a.s, Czech Republic

A MATHEMATICAL MODEL OF A SMALL CLASS TURBOPROP ENGINE

Results and process of a development of a mathematical model of a small turboprop engine are presented in this article. The presented model is based on engine characteristics and other necessary input data which came from the EU project ESPOSA of an engine. This engine is currently under the development and testing by present European manufacturer. The model is derived as generic, dynamic and nonlinear. There is chosen a conception, where final set of governing equations is based on a connection of submodels governing equations of elementary engine parts, i.e. compressor, turbine, nozzle. The lumped parameters method is applied to these submodels where their properties are described with characteristics, which are standard in the aircraft industry. Ist and 2nd derived mathematical models are comprised of a dynamic part which respects the rotating mass dynamics only. Result of this assumption is a system of equations, both differential and nondifferential. Dynamic part of 3th derived model respect also gas temperature and pressure dynamics. Its system of equations is fully differential.

Key words: mathematical model, small turboprop engine, ESPOSA.

1. Fundamentals of turboprop engine modeling philosophy

Problems in the development of mathematical models for turboprop engines are of high interest in today's control industry. There are several methods how to build a mathematical model of the arbitrary gas turbine engine in order to its control system synthesis, see [1], [2].

There were some initially failed trials of this activity at our company in history, aiming how to reach a usable result in relatively short time. Finally, an industrial standard of mathematical model design which is based on connection of submodels governing equations of basic engine parts, i.e. compressor, turbine, nozzle was accepted. The lumped parameters method is applied on these submodels where their properties are described by well-known kind of characteristics.

1.1 Model requirements

The main purpose of engine model creation is its application in the process of engine control software design and testing. Therefore, the basic set of requirements which must be met by the mathematical model of an engine is given:

 <u>determinism</u> – the results are dependent only on the engine inputs and have to be-same in each case where the inputs are same;

 <u>continuity in time</u> – its desirable discretization is one of next steps;

- two-parts architecture: <u>static model</u> (so called engine deck) and <u>dynamic model</u>;

- <u>interactivity</u> - responds to an impulse from an engine environment;

must run in <u>real time;</u>

1.2 Model assumptions

Physical nature of any industrial power plant should be correctly described by its distributed parameters. This can be realized via system of partial differential equations. This task is solved by various software based on finite volume methods (Ansys–Fluent, Comsol–Multyphysics or others) in the common industrial level. This approach is excessive and impractical in order to control algorithm design. Anyway, in order to finding of acceptable equilibrium between an accuracy and a complexity, there must be accepted following assumptions and restrictions:

- The engine = <u>distributed physical system</u>. Its parameters are continuously distributed in a space and time (Y=f(x,y,z,t)). Accepted assumption is a discretization of chosen control volume for particular components described by <u>lumped</u> <u>parameters</u>, see fig. 1.

- Simplification of physical properties. Thermal properties of air (dry air) and products of combustion of air with hydrocarbon fuels are described only as a function of temperature for the sake of simplicity.

 -1^{st} and 2^{nd} generation of his mathematical model considers only the dynamics of rotating masses. There are neglected temperature and pressure dynamics. These dynamics are considered in a 3^{th} model generation.

- Effect of a heat accumulation inside the metal components of the engine is not considered.

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ISSN 1727-0219 Вестник двигателестроения № 2/2015

 A very simple model of combustion chamber is applied. It is set as a constant of a pressure drop and combustor effectivity.

- Characteristics of the engine components are described in a form of a pressure drop and adiabatic effectivity as a functions of reduced variables. The well-known theory of similarity is applied.

- Only the adiabatic processes are considered.

1.3 Model composition

Process of the model component architecture is composed of the following steps. The first step is a decomposition of the whole power plant into the main groups and components in next step, see fig. 1. Model of the engine is assembled from components which are ordered along the engine gas path, see fig. 2



Fig. 1. Scheme of typical engine components modeled as discrete control volumes



Fig. 2. Scheme of the engine gas path components

These components are joined together in the sense of serial/parallel block connections which are subsequently merged into a static and dynamic model, see fig. 3.

2. Derivation of the engine model

Borders between components in the gas path are numbered according to the aerospace standard SAE ARP755C, see fig. 4.



Fig.3. Scheme of the model with static and dynamic parts



Fig. 4. Station numbering of the engine

2.1 Static model

Governing equations for temperatures, pressures and mass flows were assembled and adjusted in these numbered stations. The common laws of energy and mass conservation and various semiempirical estimations of dimensionless values of efficiencies, pressures and speed ratios of mass flow parameters were applied.

2.2 Dynamic model

A dynamic model were assembled with the help of the static model. The governing equations were rewritten from equations of the static model, see two examples below.

1. Dynamic equilibrium of gas generator rotating mass, [rpm/s]:

$$\frac{\mathrm{dn}_{gg}}{\mathrm{d\tau}} = \frac{\Delta P_{gg}}{n_{gg} J_{gg} \left(\frac{\pi}{30}\right)^2}.$$

2. Dynamic equilibrium of free turbine rotating mass, [rpm/s]:

$$\frac{\mathrm{dn}_{\mathrm{prop}}}{\mathrm{d\tau}} = \frac{\Delta P_{\mathrm{lpt}}}{n_{\mathrm{prop}} (J_{\mathrm{lpt}} \mathrm{ksi}_{\mathrm{prop}} + J_{\mathrm{prop}}) \left(\frac{\pi}{30}\right)^2}$$

2.3 Component characteristics

Chosen block values are incorporated in the engine model. This characteristics deals with maps of pressure ratios and efficiency which are dependent on a speed ratio and mass flows parameters. These characteristics are used in calibrated forms, in the 2nd and 3rd model, see fig. 5.

3. Results of calculation of the engine model

Calculation of important values (pressures, temperatures, ...) are realized only in the numbered stations, see fig. 4.

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reduced air mass flow (kg/s)

Fig. 5. Example of a calibrated compressor characteristic

3.1 Calculation of the steady state series

Calculations of above-mentioned values in the steady state are processed via the static model, so called engine deck. Examples of pressure and temperature gas path profiles are depicted in fig. 6. These values were calculated by the 1st engine

model. The 2nd engine model is more precise. It enables to calculate similar set of steady state values - examples of static characteristics as a function of steady state fuel flow and Mach number, see fig. 7. Tab.1 shows calculated steady-state series of chosen values in the points of equilibrium.



Tab. 1

Example of a rig test at referential altitude 4000 m (ISA). The steady-state series of chosen values are calculated as a function of steady-state fuel flow

ordermf.2mf.2mf.2mf.2mf.2mf.2mf.3 <thm< th=""><th>CASE</th><th>***</th><th>H.e</th><th>4000</th><th>(m)</th><th>***</th><th>delt.Te</th><th>0</th><th>(K)</th><th>***</th><th>M0.e</th><th>0</th><th>(1)</th><th>***</th><th>***</th><th>***</th><th>***</th></thm<>	CASE	***	H.e	4000	(m)	***	delt.Te	0	(K)	***	M0.e	0	(1)	***	***	***	***
(1)(order	mf.F	mf.2	beta.C	n.GG	n.LPT	n.PP	т.з	T.4	T.41	T.42	т.44	T.5	P.3	p.42	p.5	fi.PP
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16 56,552 1,1627308 0,8783181 36505,75 32819,75 2150 454,9684 1005,422 998,8348 800,03574 795,7026 717,0202 3071,6223 301189,73 11128,42 69882,237 0,1282615 19 57,875 1,127546 0,875297 32819,75 2150 455,789 100,1249 988,3191 717,2846 311202,4 11206,4 1290,49 9698,137 11126,4 1202,49 9695,131 10,810,49 989,137 10,910,48 988,3191 71,7867 31120,4 1120,49 96969,137 11,812,42 9699,347 0,1882547 210 58,75 1,94111 0,8789297 3674,73 1210 459,3867 101,4727 988,9103 801,4727 31519,82 11294,82 6970,972 11381,546 6970,374 0,991609 22 59,152 1,0120,172 0,87927 112,9873 32819,75 32819,75 32819,75 32819,75 32819,75 32819,75 32819,75 32819,75 32819,75 32819,75 3281	19	56,125	1,1561732	0,8781816	36401,782	32819,75	2150	454,0461	1003,5535	978,03807	801,97227	794,64636	716,92616	305135,51	110751,39	69676,892	0,1796699
17 57 1,1692559 0,8784564 36607,223 32819,75 2150 455,778 1007,3228 981,7204 80,1019 796,76107 717,13007 309187,73 111627,98 69887,055 0,188244 19 57,875 1,1821895 0,8782711 36799,072 32819,75 2150 455,6728 1001,022 985,3931 717,3807 311206,41 112504,21 649651,11 0,110458 20 58,3125 1,188778 0,8782824 38848,43 32819,75 2150 455,8322 1014,970 899,1023 807,44728 30177,97178 31177,911 113376,2 69701,060 0,164966 22 59,1875 1,2011010 0,879317 37139,753 32819,75 2150 460,2275 1016,639 800,5121 800,4797,11455,71 1381,4462 32100,98 114253,86 69704,97 0,201787 24 60,0251 1,21579 9,879663 3721,975 32819,75 2150 461,8699 1024,797 1153,466 69706,87 0,211877	10	56,5625	1,1627308	0,8783181	36505,755	32819,75	2150	454,9684	1005,4423	979,88345	803,03574	795,7026	717,02028	307165,23	111189,42	69682,237	0,1825615
18 57,475 1,175/46 0,875090 36705,247 32819,75 2150 457,7632 1009,1949 983,5488 805,1711 978,230 717,25446 311202,4 11202,49 9120,40 918,512 20 58,312 1,1885778 0,878524 36886,643 32819,75 2150 457,6522 101,207 988,9131 717,5903 31717,791 11378,726 31206,4 1120,49 96969,347 0,1982947 21 58,75 1,2014913 0,878931 37139,75 32819,75 2150 460,257 101,653 909,2693 809,4523 801,777 31319,752 1347,822 6970,497 0,219976 23 59,625 1,021420 0,879972 3728,973 32819,75 32819,75 32819,75 32819,75 32819,75 32819,75 32819,75 32819,75 2150 461,6539 102,979 914,4013 803,2982 7148,17152 32497,71 1151,6418 962,7578 118,7752 3174,717 1138,746 9970,556 2,0204,732 2,00378	13	57	1,1692559	0,8784564	36607,223	32819,75	2150	455,8798	1007,3228	981,72044	804,10159	796,76107	717,13007	309187,73	111627,98	69687,055	0,185424
19 57,87 1,1212185 0,872711 36799,072 32819,75 2150 457,6522 1011,052 985,3717 806,24982 978,8391 71,73978 313206,4 11250,496 6965,111 0,1010458 20 58,125 1,188578 0,8782973 8074,799 32819,75 2150 459,8326 1014,7904 989,91023 800,44522 801,772 71,75703 31717,911 11337,62 69701,069 0,164966 22 59,1875 1,2074129 0,873917 37139,753 32819,75 2150 461,0551 111,619 803,2829 718,2149 311455,71 13145,74 69704,97 0,2017875 24 60,055 1,215663 37219,748 32819,75 2150 461,6599 1022,468 904,2175 118,44062 32403,72 11461,41 69706,85 0,2013875 25 60,057 1,215680 0,88043 3745,289 32819,75 2150 462,4724 102,468 904,314 103,492 11404,41 69706,670 0,2114947 26 60,3751 1,52517 0,8804313 37452,489 3281	18	57,4375	1,175746	0,8785909	36705,247	32819,75	2150	456,77878	1009,1949	983,54881	805,17114	797,82307	717,25446	311202,3	112066,4	69691,347	0,1882547
20 58,151 1,1885778 0,878524 36888,64 32819,75 2150 458,352 101,292 987,19206 807,4133 799,9724 71,75670 315198,22 11294,159 96968,35 0,1397924 21 58,75 1,2011901 0,8791759 32819,75 2150 459,3867 1014,670 980,0123 800,727 71,75703 31717,751 11335,74 69703,274 0,1991609 23 59,625 1,2074129 0,879924 32819,75 2150 462,67245 1022,479 94,46018 814,18315 806,57373 178,1732 32497,17 1151,3648 806,57373 1149,4773 32819,75 2150 462,67245 1024,118 986,17374 815,0414,039 805,5737 179,	19	57,875	1,1821895	0,8787211	36799,072	32819,75	2150	457,66326	1011,062	985,37177	806,24982	798,89391	717,39876	313206,4	112504,29	69695,111	0,1910458
21 58,7 1,1949113 0,8789997 8697,479 32819,75 2150 459,3876 1014,790 989,1002 808,44522 801,072 717,7590 31717,91 11337,82 6970,069 0,1694966 22 59,1875 1,2074129 0,8793917 37139,753 32819,75 2150 460,2251 1018,5157 992,4315 810,48971 803,2982 718,21492 321100,98 114253,76 69704,97 0,2017875 24 60,0255 1,215630 0,879582 37298,048 32819,75 2150 461,65591 1022,9478 803,4392 718,44923 324073,17 115130,66 69706,855 0,204339 25 60,551 1,2196802 0,805433 37452,289 32819,75 2150 462,4724 1022,967 982,8797 812,8987 91,5717 115130,66 69706,870 0,2143947 26 60,3751 1,2234679 0,88043 37452,289 32819,75 2150 465,7092 102,887 815,5717 194,847 39706,71 1164,944,96 69700,070 0,2143947 26 62,6275 1,244409	20	58,3125	1,1885778	0,8788524	36888,634	32819,75	2150	458,53229	1012,927	987,19206	807,34133	799,97724	717,56701	315198,22	112941,59	69698,35	0,1937924
22 59,187 1,211901 0,8791789 3,7058,18 3,2819,75 2150 460,2551 1016,553 990,2403 800,31247 71,79748 319145,57 113815,74 9970,274 0,1991609 23 59,625 1,2074129 0,8799573 32819,75 2150 461,0551 1018,575 926,413 803,2989 718,1420 211,009 11425,26 67070,02 0,017875 25 60,51 1,2136773 0,879558 32819,75 2150 461,6659 1020,3797 984,4018 811,8151 806,5738 718,7753 32819,75 0,605 0,209359 11,610,39 805,5738 710,9561 326885,88 11557,112 6970,505 0,209459 27 61,375 1,216879 0,880143 3752,812 32819,75 2150 465,7062 1029,479 919,4737 32897,11 1151,404 690,4974 11,944737 32878,71 11614,96 69706,00 0,214946 29 62,25 1,2443043 0,881473 3767,71 32819,75	2:	. 58,75	1,1949113	0,8789997	36974,799	32819,75	2150	459,38676	1014,7904	989,01023	808,44522	801,0726	717,75903	317177,91	113378,62	69701,069	0,1964966
23 59,65 1,20/4129 0,8/93/17 3/139,74 3/819,75 2150 461,6531 1018,515 99,24431 810,689/1 803,389/2 718,7149 23/1100,98 114,254,76 69/06,97 0,20/18/5 24 60,0625 1,2196602 0,8799782 37298,048 32819,75 2150 462,6724 1022,246 996,27957 812,9878 805,5737 718,7732 32497,17 11513,066 69706,855 0,209338 26 60,9375 1,237687 0,880543 3745,288 32819,75 2150 462,4724 1022,967 999,3173 815,3494 807,3194 719,44737 32788,711 11601,279 67076,760 0,2114960 28 61,8125 1,237687 0,881783 3760,3175 32819,75 2150 465,5003 1003,662 1007,447 809,1137 71,98,04 30677,71 16464,96 69700,670 0,211497 30 62,6275 1,244503 0,81733 3760,3175 32819,75 2150 465,5003 1003,662 1007,447 81,54247 33264,591 32245,991 116804,963 670,470,211464	2	59,1875	1,2011901	0,8791759	37058,318	32819,75	2150	460,2275	1016,653	990,82693	809,56123	802,17971	717,97478	319145,57	113815,74	69703,274	0,1991609
24 60,0625 1,2135/73 0,879553 37219,953 32819,75 2150 640,8279 1020,3797 98,44018 811,81315 804,4310 718,4806 324394,72 114691,47 69706,162 0,204378 25 60,515 12,27175 0,880365 37375,583 32819,75 2150 642,67245 1022,468 986,27975 81,28789 805,5713 718,7735 23497,71 1151,0166 69706,505 0,2094359 27 61,375 1,216879 0,880434 3745,258 32819,75 2150 463,42412 1025,997 999,91377 719,4373 328787,71 116161,966 69706,506 0,214346 28 61,8125 1,2375863 0,8812973 37528,182 32819,75 2150 465,7602 1029,878 807,91147 179,4373 32849,76 0,2143494 30 62,625 1,2441567 0,88243 3767,271 32819,75 2150 465,7602 1029,479 810,93249 811,95441 304,0613 11737,88 69706,004 0,2143947 31 63,125 1,254346 0,88243 3767,271	23	59,625	1,2074129	0,8793917	3/139,/53	32819,75	2150	461,05511	1018,5157	992,64315	810,68971	803,29892	/18,21492	321100,98	114253,26	69704,97	0,201/8/5
26 60,957 1,2198602 0,8799742 37/298,048 32819,75 2150 462,67248 1002,4868 980,5795 812,59788 2805,5738 718,7733 719,4737 732878,71 11601,79 9706,760 0,2113906 28 61,8125 1,2243603 0,812783 37603,175 32819,75 2150 465,5003 1003,616 817,78488 810,3255 702,04547 33245,99 11689,66 69704,670 0,2113917 30 62,6275 1,244150 0,882173 3750,511 32819,75 2150 466,52033 103,6162 1007,3457 81,54241 710,6184 33440,681 11777,45 69700,91 0,221579 3356 64,4751 1,6605251 1122174 646,3	24	60,0625	1,2135773	0,8796563	37219,545	32819,75	2150	461,86999	1020,3797	994,46018	811,83151	804,43106	718,48062	323043,72	114691,47	69706,162	0,204378
6 60.937 1.2571/5 0.88365 37475,558 3.2819,75 2150 463,4626 1024,1188 998,10340 814,1039 806,7395 179,09561 3.20885,88 11557,112 9970,055 0.29945.99 27 61,375 1.216679 0.880413 3752,812 32819,75 2150 466,24142 1025,9967 999,3127 815,34948 807,9147 719,4473 328789,71 11645,966 69706,004 0.2143947 29 62,25 1,2434039 0.881243 3767,271 32819,75 2150 465,7602 1029,812 810,3255 70,04547 32454,99 116806,866 6970,607 0.2119416 31 63,125 1,244366 882243 3767,271 32819,75 2150 466,25031 103,628 1003,472 811,95441 31406,631 11737,786 69703,607 0.2119416 31 63,125 1,264356 0.882243 3767,271 32819,75 2150 467,26511 103,4527 101,902,275 811,81481 71,16760 33665	25	60,5	1,2196802	0,8799782	37298,048	32819,75	2150	462,67245	1022,2468	996,27957	812,98789	805,57738	718,77352	324973,17	115130,66	69706,855	0,2069333
22 61,375 1,2316879 0,8301473 3/15/2,429 32819,75 2150 406,74142 1023,950 999,3173 815,34948 807,91147 173,4473 328787,1 116012,79 9706,763 0,2113406 28 61,8125 1,237863 0,881783 37603,175 32819,75 2150 465,009 1027,882 1003,616 817,74848 810,3255 70,24547 332545,91 11689,66 69704,767 0,2163977 30 62,6285 1,244156 0,882517 37750,514 32819,75 2150 465,5003 1031,662 1007,3457 819,33249 811,56481 770,01841 334400,63 11373,788 69700,970 0,221579 32 63,5251 1,2643750 0,882219 37894,783 32819,75 2150 468,70451 1033,612 1007,3457 810,3326 311,66421 72,16702 33665,51 11821,52,66 69869,340 0,2232972 33 64,1751 1,274578 0,833397 3766,613 32819,75 2150 469,9721	20	60,9375	1,225/1/5	0,880365	37375,558	32819,75	2150	463,46269	1024,1188	998,10304	814,16039	806,73936	719,09561	326888,58	115571,12	69707,055	0,2094539
28 61,812 1,23,7883 0,812/93 3726,122 2380,75 2150 465,7092 1027,838 1007,71 118195,07 101,923	2.	61,375	1,2316879	0,8808143	37452,289	32819,75	2150	464,24142	1025,9967	999,93173	815,34948	807,91749	719,44737	328789,71	116012,79	69706,769	0,2119406
29 60,2451 1,243403 0,6817a3 37403,173 3819,73 2130 465,2033 1003,612 1005,616 611,7648 610,2535 720,2441 352243,39 118836,66 9074,76 0,121571 30 62,6275 1,244305 0,882617 37750,514 32819,75 2150 466,52033 103,662 1007,345 819,03249 811,5648 770,6714 3819,75 2150 467,25631 103,6612 1007,345 819,03249 811,5648 770,6714 3819,75 2150 468,0043 103,5615 109,2727 821,5848 817,6855 1121576 638640,63 113777,45 69700,91 0,221579 333 64 1,255975 0,8832219 37894,783 32819,75 2150 468,79457 1014,975 81,63727 722,1433 3846,72,1 103,616 81,67272 72,7423 31468,72,1 103,616 81,67278 72,7423 31468,72,1 103,616 81,79758 723,947 34468,73 119511,96 696578,42 0,2302466 62,551	20	61,8125	1,2375863	0,8812973	37528,182	32819,75	2150	465,0098	1027,8836	1001,7686	816,55719	809,11377	719,8304	330675,71	116454,96	69706,004	0,2143947
30 62,685 1,2491360 0,88243 3/97/7,21 34817,35 2120 4667,2650 101,052,47 611,054,47 611,0	23	62,25	1,2454095	0,881785	37603,175	32819,75	2150	465,76902	1029,782	1005,616	010 03240	810,32955	720,24547	332545,99	117227.00	69704,767	0,2108177
31 63,5625 1,2604352 0,8823074 3,830,73 2130 407,8033 103,5032 1007,933 820,936 314,10700 330665,51 11777,74 9700,74 0,213272 32 63,5625 1,2604352 0,8823074 32819,75 2150 468,00435 103,5032 1007,937 821,5924 812,167022 338065,51 118121,56 69695,254 0,2233229 33 64,4755 1,274478 0,8833295 3036,813 32819,75 2150 468,7946 1014,9273 824,1904 816,6702 722,1424 416,782,41 1109,2262 68,97218 103,9102 824,1904 816,6702 723,8121 3457,82,11 199,40,231 329,77 11850,26 69695,24 0,2233229 0,38046,82 1181748,24 1199,316 69687,842 0,230469 36 65,3125 1,2267456 0,88313 3817,322 32819,75 2150 470,3048 1043,508 1018,752 823,6965 820,602 74,4708 34020,27 1203,418 69678,697 0,233581 <td></td> <td>62,0873</td> <td>1,2491307</td> <td>0,882243</td> <td>37077,271</td> <td>32819,73</td> <td>2150</td> <td>467,326501</td> <td>1031,0928</td> <td>1003,473</td> <td>819,03249</td> <td>912 9195</td> <td>720,09184</td> <td>334400,03</td> <td>117337,88</td> <td>60700.01</td> <td>0,2192110</td>		62,0873	1,2491307	0,882243	37077,271	32819,73	2150	467,326501	1031,0928	1003,473	819,03249	912 9195	720,09184	334400,03	117337,88	60700.01	0,2192110
52 65,352 1,2004322 0,862,534 3,762,783 3,813/3 1,133,131 103,217 61,35247 61,36287 1,21,010 9305,340 0,222,222 33 64 1,250756 0,832373 32819,75 2150 466,07364 103,217 61,35376 101,109,227 62,13647 11,1010 238067,77 118213,06 6905,266 6023242 62,1266 666,252 722,7424 34167,24 11900,217 118213,06 6901,766 0,228535 35 664,875 1,2714578 0,8833397 37066,01 32819,75 2150 469,7211 1011,101 22,410904 816,6702 722,7424 34167,24 119008,216 6961,766 0,2285355 35 664,875 1,2724782 0,88313 3810,72 2150 470,20261 1041,815 1014,922 825,1048 817,958 1043,317 1016,838 820,6082 72,474078 34700,727 1036642,71 1911,916 6967,342 0,2331283 36 66,525 1,2225148 0,882487	3.	63,123	1,2348300	0,8820317	37730,314	32819,73	2150	467,20301	1035,6162	1007,3437	820,29890	012,0103	721,10700	330240,18	117777,43	60608 304	0,221379
34 64,4375 1,271478 0,883397 3766,763 32819,75 2150 666,97213 10374,97 10174,157 0,284,1904 816,6702 722,7424 34467,24 110002,61 6967,243 0,305,124 0,205,124	3.	63,3623	1,2604332	0,8823834	37822,983	32819,73	2150	468,00433	1027 4975	1011 1194	821,38244	915 2726	722,87022	220977 77	118213,08	69698,304	0,2239229
35 66,875 1,274578 0.883397 0.700,074 0.843397 0.105,037 0.61,058 0.833297 0.106,017 0.61,058 0.817,058	3.	64 4275	1 2714579	0,8832219	27066 021	22819,75	2150	468,73304	1037,4575	1011,1194	822,88003	815,3730	722,1903	241679.24	110092.61	69691,254	0,2202400
36 65,125 1,282,272 0,883213 38107,234 32819,75 2150 470,93048 1043,817 1014,9473 082,862 723,88121 345248,79 11938,29 666,785 1,282,272 0,883213 38107,234 32819,75 2150 471,6548 1043,817 1016,836 626,8371.4 819,288 723,88121 345248,79 11938,292 66678,651 223,9302 66678,651 222,92148 0,882713 382107,323 2350 723,88121 345248,79 11938,326 6678,651 223,9302 2063,416 6978,667 0,2333983 0,2333983 2018,3817 1016,3816 826,8371.44 819,288 723,88121 345248,79 11938,326 60678,657 12308,428 0,2333983 0,2339912 12308,428 0,233928 0,2384,14 60578,681 0,2339059 2150 473,7979 1051,207 12308,428 12338,26 12338,23 12338,23 12338,26 12338,23 12338,26 12338,26 12338,26 12338,26 12338,26 12338,26 12338,26 12338,26 12338,26<	31	64,4375	1 2768886	0,88333397	38036 813	32819,75	2150	409,47213	1041 415	1013,02	825 51048	817 97588	723 3047	343468 27	119511.96	69687 842	0,2285555
37 6,57 1,229148 0,830153 3817,37 2150 471,5548 1047,522 282,81656 822,06074 724,4778 347020,2 1233,72 1233,72 0,235,933 38 66,157 1,229148 0,8827613 3817,75 2150 472,37483 1047,3520 828,1656 820,60682 724,4778 347020,2 12036,72 0,235393 39 66,625 1,229148 0,8827453 3819,75 2150 472,37483 1047,3209 1020,6664 829,5073 821,9912 72,56743 3506,88 1212,23 6967,3422 0,235393 40 67,0525 1,333928 0,882497 3831,575 3219,75 2150 473,0995 1042,4932 832,5181 75,5674 3505,684 1212,32 6966,179 0,2421412 41 67,575 1,336973 0,881697 38219,75 2150 474,4990 1052,287 126,4156 352,9817 726,3156 35217,77 12,236,696 120,211,12 126,626 325,9813 726,3156 <td< td=""><td></td><td>65 3125</td><td>1 2922732</td><td>0.993213</td><td>38107 234</td><td>32819,75</td><td>2150</td><td>470,20201</td><td>1043 3917</td><td>1016 9396</td><td>926 93714</td><td>919 2995</td><td>723,3047</td><td>345249 79</td><td>119938.97</td><td>69693 495</td><td>0,2331293</td></td<>		65 3125	1 2922732	0.993213	38107 234	32819,75	2150	470,20201	1043 3917	1016 9396	926 93714	919 2995	723,3047	345249 79	119938.97	69693 495	0,2331293
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Fig. 7. Values of engine gas generator speed (nGG), compressor air mass flow (mf2), -propeller speed (nPP) and propeller blade angle (fiPP) as the function of steady-state fuel flow and Mach number

3.2 Calculation of the transient state series

Transient states of important values were calculated for many cases of input values combinations. These calculations were processed using MathCAD software as well as MATLAB/Simulink, see fig. 8. The MathCAD calculations are more convenient for the algorithm debugging. The MATLAB/Simulink model is advantageous for large and real-time data processing.



Fig. 8. Example of time series of compressor air mass flow (mf2) and gas generator speed (nGG) as a reaction to fuel mass

There was made an attempt to model development in a GSP software (Gas turbine Simulation Program) in order to get some kind of spectacular verification. An approximate example of this calculation is placed in fig. 9. This attempt for verification failed due to unclarity of input and other parameters setting.



Fig. 9. Jet engine example of a GSP results sheet. There is calculated time series of compressor pressure ratio (PRC), temperature after combustor (T4) and compressor mass flow (W2)

4. Possibilities for the next development

Development of the 3rd revision of the engine model successfully started in the first quarter of 2015. This approach was chosen in response to the problems with an implementation of previous two models into the dSPACE hardware-in-the-loop simulation platform. The 3rd generation of model is based on fully differential set of governing equations which is the main difference in comparison with the previous model generations. Previous models were constituted of mixed set of both differential and nondifferential governing equations. This mixed system contained algebraic loops as a source of problems. The major expectations for the 3rd model generation are connected with a significant shortening of calculation time in order to fulfilling of real-time requirements. Unfortunately, the lower accuracy in calculated results is expected as a

penalty of additional simplifications. This drawback has been accepted in order to fast processing of the 3rd engine model algorithm.

Development of the 3rd engine model algorithm is being finalized nowadays in MathCAD software. In order to a future progress, there is a need to start the transition and testing of the model in MATLAB/Simulink.

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Поступила в редакцию 16.06.2015

Jozef Polacek, Lubos Vargovcik. Математическая модель турбовинтового малоразмерного двигателя

В данной статье представлены результаты и описывается процесс разработки математической модели турбовинтового малоразмерного двигателя. Представленная модель основана на характеристиках двигателя и других необходимых входных данных, полученных по проекту Европейского Союза ESPOSA. Ланный двигатель в настоящее время находится на стадии разработки и испытаний европейским изготовителем. Полученная модель является типовой, динамической и нелинейной. Используется концепция, согласно которой окончательный набор определяющих уравнений основан на связи определяющих уравнений субмоделей элементарных узлов двигателя, например, компрессора, турбины, соплового аппарата. Для этих субмоделей используется метод сосредоточенных параметров, где их свойства описываются характеристиками, которые являются стандартными в авиации. Первая и вторая производные математические модели состоят из динамической части, которая распространяется только на динамику вращающихся масс. Результатом этого ограничения является система как дифференциальных, так и недифференциальных уравнений. Динамическая часть третьей производной модели также распространяется на динамику температуры и давления. Ее система уравнений является полностью дифференциальной.

Ключевые слова: математическая модель, малые турбовинтовые двигатели, ESPOSA

Jozef Polacek, Lubos Vargovcik. Математична модель турбогвинтового малорозмірного двигуна

У даній статті представлено результати і описується процес розробки математичної моделі турбогвинтового малорозмірного двигуна. Представлена модель заснована на характеристиках двигуна і інших необхідних вхідних даних, отриманих за проектом Європейського Союзу ESPOSA. Даний двигун у цей час перебуває в стадії розробки і випробувань європейським виготовлювачем. Отримана модель є типовою, динамічною і нелінійною. Використовується концепція, відповідно до якої остаточний набір визначальних рівнянь засновано на зв'язку визначальних рівнянь субмоделей елементарних вузлів двигуна, наприклад, компресора, турбіни, соплового апарата. Для цих субмоделей використовується метод зосереджених параметрів, де їх властивості описуються характеристиками, які є стандартними в авіації. Перша і друга похідної математичної моделі складаються з динамічної частини, що поширюється тільки на динаміку обертових мас. Результатом цього обмеження є система як диференціальних, так і недиференціальних рівнянь. Динамічна частина третьої похідної моделі також поширюється на динаміку температури і тиску. Її система рівнянь є повністю диференціальною.

Ключові слова: математична модель, малорозмірний турбогвинтовий двигун, ESPOSA.