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ANALYSIS OF ELECTROMAGNETIC PROCESSES IN A TURBOGENERATOR WITH EQUIVALENT STATOR TOOTH ZONES AT NO-LOAD

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Abstract: While developing the mathematical field models of electric devices, to simplify the statement of the problem, it is common to apply chain schemes and substitute equivalent media for the complex structures of a construction. Identifying the influence of such simplifications on the conformity of the reproduction of electromagnetic processes by the models is an important problem, particularly when the given methods concern zones that are the sources of an electromagnetic field in the objects. Besides, when developing the mathematical field models, the coordinate systems in which the process is considered are neglected. In this paper, 2-D mathematical field models for calculating a quasi-stationary electromagnetic field in the cross-section of a turbogenerator at no-load with an equivalent stator zone are suggested. These models provide the calculation of the electromagnetic field in all zones of the device carried out on the basis of a vector potential in both the single coordinate system of the moving rotor, and the coordinate systems of the rotor and stator simultaneously. The results obtained show that it is unacceptable to apply either substitution schemes or the equivalent replacement of the complex structures of electric devices in the case when they are the sources of the electromagnetic field in the object under investigation. Moreover, the analysis of the results confirms the necessity to consider the systems of coordinates of movable and immovable media when developing mathematical field models of electrodynamic devices.

Key words: electromagnetic field, vector potential, turbogenerator, movable media, system of coordinates.

1. Introduction

In [3], there is given the mathematical field model of the turbogenerator designed for calculating a no-load mode at the given voltage of the excitation winding in which the tooth structures of the stator and rotor are substituted by nonlinear equivalent anisotropic media possessing the recalculated electromagnetic characteristics. Such an approach is widely used in the theory of electromagnetic circuits while determining the parameters of the equivalent replacement of series and parallel connections of electric and magnetic resistances.

Work [4] proposes mathematical field models of the turbogenerator at no-load with an equivalent tooth zone, the slot zone of the rotor winding, and the real tooth structure of the stator in both the coordinate system of the moving rotor and the physical reference systems of the rotor and stator simultaneously.

The turbogenerator models considered in [3, 4] are based on the theoretical approaches to the calculation of electromagnetic processes in electrodynamic devices described in [1, 2].

Many authors of scientific works very often propose applying the schemes of substitution to the media which are the sources of electromagnetic field when developing the mathematical models of electric devices [5-8].

The results obtained in works [3, 4] show that even in the case of modeling of the electromagnetic field of the turbogenerator in the simplest no-load mode, if there are no currents of conductivity in the stator tooth zone, the electromagnetic processes occurring in the real media of the teeth and slots of the stator significantly differ from the processes in the equivalent zones. The values of the current in the excitation winding being equal, there is a 9 % difference in the integral voltages of the stator winding phase in the nonlinear models [3] and [4]. This is caused by the substitution a continuous anisotropic medium possessing the recalculated electromagnetic characteristics for the tooth zone with a stator winding.

At no-load, currents of conductivity in the rotor winding are the source of the electromagnetic field in the turbogenerator. That is why the development of mathematical field models which take into account a real tooth structure with an excitation winding evokes such a great interest. The models of this kind make it possible to estimate how the replacement of the real zone of the rotor winding connected to the external source of power with an equivalent medium influences the main characteristics of the turbogenerator at no-load.

Another important issue while developing mathematical field models of electrodynamic devices is considering the coordinate systems in which the main calculated values are formed. This issue is really vital for the analysis of electromagnetic processes in movable and immovable elements of the device on the basis of Maxwell's equations in a time domain.

This article deals with the development of two field models of the turbogenerator with the equivalent stator tooth zone both in one reference system of the moving rotor and in the coordinate systems of the rotor and the stator simultaneously. The proposed models are developed on the basis of Maxwell's equations with respect to potentials at a given voltage in the rotor winding. They are intended for the direct time integration of the formed system of equations.

2. Statement of the problem

An important problem arising from the development of mathematical field models of electric devices is the level of detailing the mathematical description of electromagnetic processes in the elements of their construction.

The problem indicated above acquires its significance while reproducing the electromagnetic processes in the zones that are the sources of the electromagnetic field in the mode under investigation. Thus, the task of developing the mathematical field models of electric devices taking, as full as possible, into consideration the real structure of the elements which contain electric windings connected to an external source of power remains important.

In addition, there exists the topical necessity of considering the coordinate systems of moving and stationary components when calculating the electromagnetic field in moving and stationary zones of electrodynamic devices. Ignoring the systems of reference, as the results obtained in works [3, 4] show, causes reproducing the processes which do not correspond to the phenomena in the real media of the devices.

In practice, the most effective models are the mathematical field models of electric devices developed at given input voltages rather than currents. The currents in the winding zones are only one of the consequent effects of a general electromagnetic phenomenon and therefore are to be determined directly by the laws of electromagnetic processes in real object zones in a specific mode under consideration.

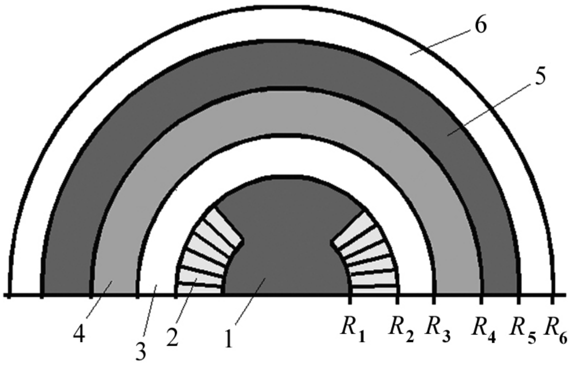

Fig. 1. Calculated zones of the turbogenerator cross-section.

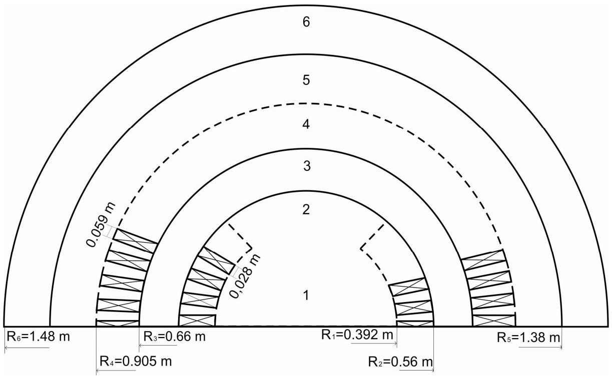

Fig. 2. Main geometrical zone dimensions of the turbogenerator cross-section.

The suggested mathematical models of the turbogenerator at no-load with the equivalent substitution of the stator tooth zone, developed at the specified voltage of the excitation winding in the rotor's coordinate system and in the physical reference systems, imply accepting the following main assumptions:

- the electromagnetic process in all zones of the turbogenerator cross-section can be considered plane-parallel;
- the calculation of the electromagnetic field is performed in a quasi-stationary approximation;
- the end phenomena in the device are not taken into considerationне;
- the magnetic material hysteresis is not considered.

As in earlier works [3, 4], the TGV-500 turbogenerator with nine complete slots and one incomplete slot of the rotor at each side of the winding being placed on the stator pole division has been chosen as a prototype. The calculated zones of the turbogenerator cross-section are shown in Fig. 1, where 1 stands for the massive rotor body zone; 2 denotes the zone of slots and teeth of the excitation winding; 3 is the air gap between the stator and rotor; 4 represents the equivalent stator tooth zone; 5 stands for the stator body zone; 6 is the air zone outside the turbogenerator.

The angular segment of the pole division of the device in which all rotor slots belonging to each part of the winding are located is approximately equal to 60º. Practically in the same segment there are 8 stator slots. To simplify the models, the number of rotor slots at both winding sides on the stator pole division is chosen to be equal to 8.

For more accurate consideration of the geometry and reciprocal position of the stator and rotor tooth zones, it is necessary to choose the greatest possible and at the same time minimal value (to provide the sufficient numerical integration stability of the main system of differential equations) of the step of the discretization grid, which will enable to perform the more detailed description of the real structure of the device. In terms of practice, this may lead to the emergence of a large number of nodes along the angular coordinate on the turbogenerator pole division that in turn may result in a significant increase in the system of calculated equations, and respectively in a decrease in its numerical stability. Otherwise, it is necessary to use two separate discretization grids for the stator and rotor with different steps along the angle and perform the recalculation of unknown electromagnetic quantities on their boundary.

The correspondence of the geometrical dimensions of the formed cross-sectional zones of the device model, illustrated in Fig. 1, to the geometrical dimensions of the construction elements of the real TGV-500 turbogenerator is represented in Fig. 2.

3. Mathematical model of the turbogenerator with an equivalent stator tooth zone in the coordinate system of the moving rotor

Taking into consideration the accepted assumptions and for the purpose of optimization of the mathematical description of the electromagnetic processes occurring in the model, as well as in order to simplify the task of determining boundary and edge conditions, the main system of calculated equations is formed with respect to electromagnetic field potentials by using the calibration $\nabla\varphi = 0$.

The given model implies that electromagnetic phenomena in the massive rotor body, its slots and teeth, as well as in the air gap between the stator and rotor are considered in the coordinate system of the moving rotor. Electromagnetic processes in both the equivalent stator tooth zone and its body, as well as in the air gap outside the turbogenerator are analyzed in the reference system of the stator reduced to the coordinate system of the moving rotor.

The initial equation of the mathematical model has the following form [1-4]

$$\frac{\partial \mathbf{A}}{\partial t} = -\Gamma^{-1}\nabla\times\left(\mathrm{N}\nabla\times\mathbf{A}\right), \tag{1}$$

where $\mathbf{A}$ is the vector potential of the electromagnetic field; Γ is the matrix of static electrical conductivities; N denotes the matrix of the static inverse magnetic penetrability of the medium; ∇ represents the Hamiltonian operator.

Expression (1) is also used to calculate the electromagnetic field in both the body and teeth of the rotor.

Electromagnetic processes in the slots with an electroconductive winding of the rotor are simulated on the basis of the given ratio

$$\frac{\partial \mathbf{A}}{\partial t} = -\Gamma^{-1}\left(\nu_0\nabla\times\nabla\times\mathbf{A}\pm\boldsymbol{\delta}\right), \tag{2}$$

where ν_0 is the inverse magnetic copper penetrability; δ stands for the extraneous current density vector

$$\delta = \frac{w_f i_f}{S}, \tag{3}$$

with w_f, i_f being the number of windings and current of the excitation winding; S represents the integral area of the rotor winding slots.

In the air gaps between the stator and rotor, the value of vector potential of electromagnetic field can be calculated by using the ratio below

$$0 = \nu_0\,\nabla\times\nabla\times\mathbf{A}, \tag{4}$$

where ν_0 is the inverse magnetic air penetrability.

The function $\mathbf{A}$ in equations (1), (2), (4) belongs to the coordinate system of the moving rotor.

The simulation of electromagnetic processes in the equivalent tooth zone and the body of the stator, as well as in the zone of outside air gaps is performed in the reference system reduced to that of the rotor.

As conductivity currents are absent in the tooth zone and body of the stator when the turbogenerator operated at no-load, the vector potential value in these zones is determined by the following expression

$$0 = \nabla\times\mathrm{N}\nabla\times\mathbf{A}'. \tag{5}$$

where $\mathbf{A}'$ is the vector potential of the electromagnetic field in the coordinate system reduced to the rotor one.

The electromagnetic field in the air gap zones outside the device is described by the following ratio

$$0 = \nu_0\,\nabla\times\nabla\times\mathbf{A}'. \tag{6}$$

As we have already considered in publications [3, 4], if there are no conduction currents in the stator winding, it is sufficient to calculate the turbogenerator electromagnetic field on the basis of the mathematical field model in the coordinate system of the moving rotor at no-load in all the zones of the device cross-section for one fixed reciprocal position of the moving and stationary systems of coordinates for getting a complete pattern of the electromagnetic process. To determine the value of electromagnetic quantities for the rest of the rotor and stator reciprocal positions, it is necessary to fulfill the certain angular coordinate displacement of the resulting spatial-time distributions of the electromagnetic field in the equivalent tooth zone and body of the stator, as well as in the zone of the outside air gap. This is the way in which, in the zones above, the calculated variables values will be transformed into the physical coordinate system of the static stator.

The developed mathematical model, like those given in [3, 4], employed a new advanced approach to the determination of boundary conditions of the field equations described in [2]. It implies matching the first and second spatial derivatives of the function **A** on the left and right from the separation line with real magnetic characteristics of the media and their periodicity on the pole division of the device.

The boundary conditions on the internal zone edges along the radial and tangential directions are used in the following form [3, 4]

$$A_i = \frac{\nu_{i-1}\, A_{i-1} + \nu_{i+1}\, A_{i+1}}{\nu_{i-1} + \nu_{i+1}};$$

$$A'_i = \frac{\nu_{i-1}\, A'_{i-1} + \nu_{i+1}\, A'_{i+1}}{\nu_{i-1} + \nu_{i+1}}, \tag{7}$$

where the index i represents the node number corresponding to the direction perpendicular to the media separation line.

Boundary conditions along the radii of the mathematical field model of the turbogenerator are formed on the basis of the spatial periodicity of the electromagnetic process on the pole division of the device for the function of vector potential and its first and second derivatives in respective systems of reference [2]

$$A_{k=1} = 2A_{k=2} + 2A_{k=n-1} - A_{k=3} - A_{k=n-2} - A_{k=n};$$

$$A_{k=n+1} = A_{k=2} + A_{k=4} + A_{k=n-1} - 2A_{k=3} - 2A_{k=n};$$

$$A'_{k=1} = 2A'_{k=2} + 2A'_{k=n-1} - A'_{k=3} - A'_{k=n-2} - A'_{k=n};$$

$$A'_{k=n+1} = A'_{k=2} + A'_{k=4} + A'_{k=n-1} - 2A'_{k=3} - 2A'_{k=n}, \tag{8}$$

where k is the index corresponding to the spatial grid nodes along the angular coordinate.

The value of the vector potential of the electromagnetic field on the external edge of the device is determined from the following ratio

$$A'_{i=m+1} = 2A'_{i=m} - A'_{i=m-1}, \tag{9}$$

where i is the index corresponding to the spatial grid nodes in the cylindrical system of coordinates along the radius.

Anisotropic electromagnetic properties of the tooth zone and laminated body of the stator are recalculated by means of the ratios below [3, 4]

$$\nu_\alpha = \frac{d_f + \nu_0 \cdot d_0/\nu}{d_f + d_0} \cdot \nu; \quad \nu_r = \frac{d_f + d_0}{d_f + \nu \cdot d_0/\nu_0} \cdot \nu;$$

$$\gamma = \frac{\gamma_{Cu} \cdot d_0}{d_f + d_0}, \tag{10}$$

where γ is the electrical conductivity of an equivalent medium directed towards the axis; γ_{Cu} is the copper conductivity; ν_r, ν_α represent the static inverse magnetic medium penetrability in radial and tangential directions; d_f, d_0 are either the width of the tooth and slot, or the ferromagnetic sheet and isolation of the laminated stator.

In the mathematical model designed for calculating the no-load mode of the turbogenerator, the non-linear characteristics of the magnetic rotor and stator materials are represented by the following cubic splines [3, 4]

$$\nu(B) = \sum_{m=1}^{3} a_i^{(k)} (B_k - B)^m, \; k = 1, 2, \ldots, n, \tag{11}$$

where n is the number of segmentations along the axis B which take into account real properties of the rotor and stator materials. In this case, the value of the module and magnetic induction vector components in all zones of the device cross-section is found from a following ratio

$$B_r = \frac{1}{r}\frac{\partial A}{\partial \alpha}; \quad B_\alpha = -\frac{\partial A}{\partial r}; \quad B = \sqrt{B_r^2 + B_\alpha^2} \tag{12}$$

where B_r, B_α, B are the radial and tangential components and the module of a magnetic induction vector in grid nodes of both the rotor coordinate system and the transferred stator system of coordinates.

Currents in the excitation winding in the transition process at no-load are calculated on the basis of the equation below [3, 4]

$$\frac{di_f}{dt} = \left(u_f - r_f i_f - w_f k_f l_r \sum_{i=1}^{n} \frac{\partial A_{Ri}}{\partial t} \right) \Bigg/ L_f, \tag{13}$$

with u_f, r_f being the voltage and active resistance in the rotor winding; L_f denoting the inductance of scattering frontal parts of the winding; l_r representing the axis rotor length; A_{Ri} standing for the value of the vector potential function in the rotor coordinate system in the grid nodes being located the winding zone; k_f being the coefficient involving the number of nodes along the angle α which get into the rotor winding zone.

Equations for determining the stator winding voltages have the following form

$$u_i = w_i k_i l \sum_{m=1}^{n} \frac{\partial A'_{Si}}{\partial t}, \; i = \text{A, B, C}, \tag{14}$$

where w_i is the number of the stator windings in each phase; l denotes the axis winding length; A'_{Si} stands for the value of the vector potential function in the nodes of a spatial discretization grid connected with the coordinate system of the moving rotor within the stator windings.

4. Mathematical model of the turbogenerator with an equivalent tooth zone of the stator in the rotor and stator systems of coordinate simultaneously

The given mathematical model of the turbogenerator developed in moving and stationary systems of coordinates simultaneously implies that electromagnetic processes in the rotor body, its slots and teeth, as well as in the air gap between the rotor and stator are described by variables in the coordinate system of the moving rotor. At the same time, electromagnetic phenomena in the equivalent stator tooth zone and its body, as well as in the air gap outside the turbogenerator are considered by means of the vector potential function which belongs to the reference system of the static stator. In the model, it is necessary to provide the physical reciprocal mechanical displacement of the rotor media relative to the stator.

The mathematical modeling of electromagnetic processes in the tooth zone and the massive body of the rotor is done with the help of expression (1). The value of vector potential of electromagnetic field in the slots with electroconductive rotor winding is found by using equation (2). Electromagnetic phenomena in the air gap between the stator and rotor are calculated by means of ratio (4). In the equations given above, the function of vector potential of the electromagnetic field **A** is connected with the rotor system of coordinates.

Electromagnetic processes in the equivalent stator tooth zone and the stator body are found from the equation below

$$0 = \nabla \times \mathrm{N} \nabla \times \mathbf{A}\,, \qquad (15)$$

in which the vector potential of the electromagnetic field belongs to the stator system of coordinates.

The calculation of the electromagnetic field in the air gap outside the turbogenerator is performed in the coordinate system of the stator using the following expression

$$0 = \nu_0 \nabla \times \nabla \times \mathbf{A}\,. \qquad (16)$$

The value of the vector potential of the electromagnetic field on the external boundaries of the calculated zones of the turbogenerator cross-section along the radii on the pole division of the device in physical systems of coordinates is determined from the expressions below

$$A_{k=1} = 2A_{k=2} + 2A_{k=n-1} - A_{k=3} - A_{k=n-2} - A_{k=n}\,;$$

$$A_{k=n+1} = A_{k=2} + A_{k=4} + A_{k=n-1} - 2A_{k=3} - 2A_{k=n}\,. \qquad (17)$$

The mathematical model being developed, the boundary conditions outside the turbogenerator are expressed in the following form

$$A_{i=m+1} = 2A_{i=m} - A_{i=m-1}\,. \qquad (18)$$

For all created internal boundaries between the device cross-section zones, the boundary conditions are found with the help of the equation below

$$A_i = \frac{\nu_{i-1}\, A_{i-1} + \nu_{i+1}\, A_{i+1}}{\nu_{i-1} + \nu_{i+1}}\,. \qquad (19)$$

The electromagnetic characteristics of the continuous anisotropic medium of the equivalent stator tooth zone are recalculated by means of ratio (10).

The value of the current in the rotor excitation winding is calculated using expression (13).

The phase voltages of the stator winding are obtained from the ratio below

$$u_i = w_i k_i l \sum_{m=1}^{n} \frac{\partial A_{Si}}{\partial t}\,,\ i = \mathrm{A, B, C}, \qquad (20)$$

where A_{Si} is the value of the vector potential function in the nodes of the spatial discretization grid in the physical coordinate system of the stator.

In the mathematical field model of the

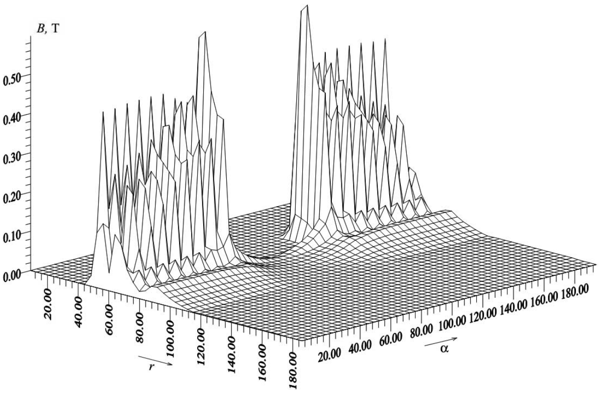

Fig. 3. Spatial distribution of the magnetic induction vector module in the rotor coordinate system on the turbogenerator pole division at t = 1 sec of the transition process at no-load.

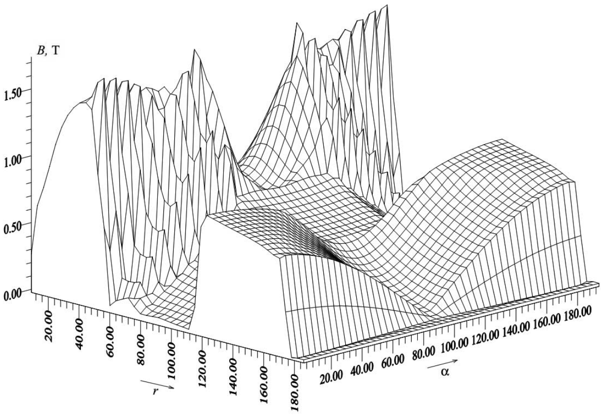

Fig. 4. Spatial distribution of the magnetic induction vector module in the rotor coordinate system on the turbogenerator pole division at t = 100 sec of the transition process at no-load.

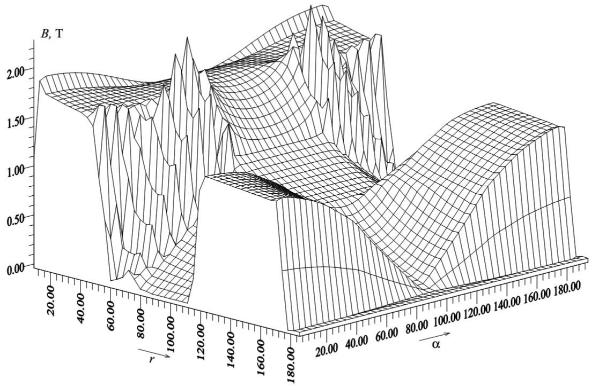

Fig. 5. Spatial distribution of the magnetic induction vector module in the rotor coordinate system on the turbogenerator pole division at t = 650 sec of the transition process at no-load.

turbogenerator in the phase coordinates, there occurs a mechanical displacement of the rotor media relative to those of the stator together with the appropriate discretization grids fulfilling the following condition

$$\Delta\alpha = \omega\Delta t\,, \qquad (21)$$

where $\Delta\alpha$ is the angular step of discretization grids; Δt is the step of time integration of the system of differential equations; ω is the angular rotary speed.

5. Calculation results

On the basis of the developed mathematical field models described above, the computer calculation (computation) of the no-load mode of the turbogenerator at the given voltage of the excitation winding has been done. The obtained spatial distributions of the module and the components of the magnetic induction vector, as well as the value of the excitation winding current and the stator windings voltage are shown in Fig. 3 – 10.

Figures 3, 4 and 5 show the spatial distributions of the magnetic induction vector module on the turbogenerator pole division at the points of time $t = 1$, $t = 100$ and $t = 650$ sec of the no-load transition process, respectively, obtained on the basis of the mathematical field model developed in the moving rotor coordinate system.

Figure 6 illustrates the spatial distribution of the module of a magnetic induction vector on the pole division of the turbogenerator at the point of time $t = 650$ sec of the no-load transition process obtained on the basis of the mathematical field model developed in the coordinate systems of the rotor and stator simultaneously.

Figure 7 represents the spatial distribution of the module of a magnetic induction vector on the pole division of the turbogenerator at the point of time $t =$ 664,13592 sec of the no-load transition process obtained on the basis of the mathematical field model developed in the coordinate systems of the rotor and stator simultaneously.

Figure 8 shows the spatial distribution of the module of a magnetic induction vector on the pole division of the turbogenerator with a conductive tooth zone of the stator at the point of time $t =$ 664,01102 sec of the no-load transition process obtained on the basis of the mathematical field model developed in the coordinate systems of the rotor and stator simultaneously.

Figure 9 depicts the time dependence of the excitation winding current i_f of the turbogenerator in the transition process at no-load.

Figure 10 demonstrates the time values of A-phase voltage in the stator winding after completing the transition process in linear (1) and nonlinear (2) models with equivalent rotor tooth zones, as well as in the model with the real tooth structure of the rotor (3).

On the given spatial distributions along the radius, the grid nodes correspond to the following zones of the turbogenerator cross-section: 0÷54 is the ferromagnetic rotor body zone; 54÷75 is the rotor tooth zone; 75÷87 is the air gap zone between the stator and rotor; 87÷117 is the equivalent stator tooth zone; 117÷174 is the stator

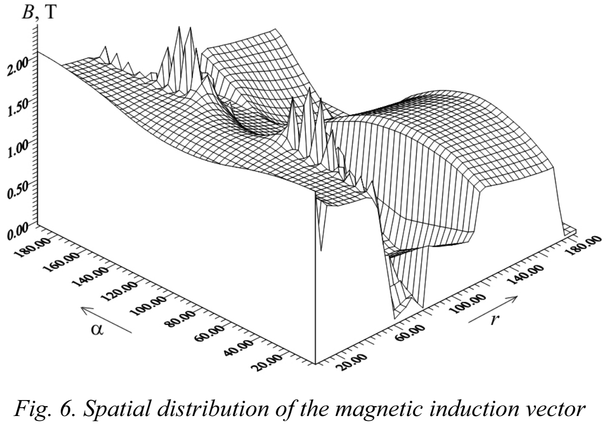

Fig. 6. Spatial distribution of the magnetic induction vector module in phase coordinate system of rotor and stator simultaneously on the turbogenerator pole division at t = 664,13258 sec of the transition process at no-load.

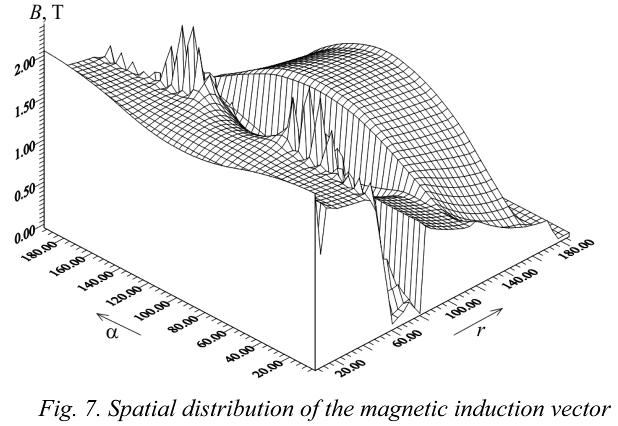

Fig. 7. Spatial distribution of the magnetic induction vector module in phase coordinate system of rotor and stator simultaneously on the turbogenerator pole division at t = 664,13592 sec of the transition process at no-load.

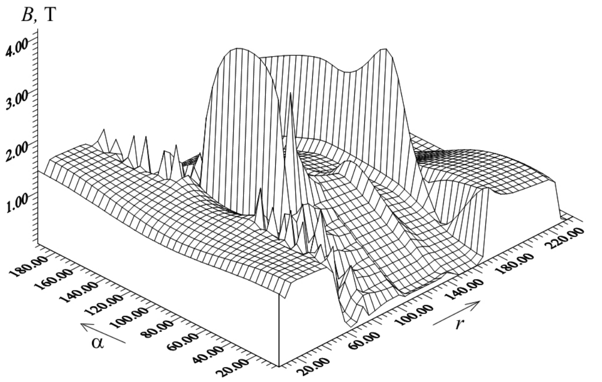

Fig. 8. Spatial distribution of the magnetic induction vector module on the turbogenerator pole division with a conductive stator tooth zone in phase coordinate systems at t = 664,01102 sec of the transition process at no-load.

body zone; 174÷186 is the air gap outside the turbogenerator. The number of the discretization grid nodes along the angular coordinate on the pole division of the turbogenerator is equal to 192.

Along the angular coordinate, the nodes of the coordinate grid of the rotor tooth structure correspond to the following zones: 4÷8; 12÷16; 20÷24; 28÷32; 36÷40; 44÷48; 52÷56; 60÷64 are the slots of the right side of the winding; 132÷136; 140÷144; 148÷152; 156÷160; 164÷168; 172÷176; 180÷184; 188÷192 are the slots of the left side of the winding; 2÷4; 8÷12; 16÷20; 24÷28; 32÷36; 40÷44; 48÷52; 56÷60; 136÷140; 144÷148; 152÷156; 160÷164; 168÷172; 176÷180; 184÷188; 192÷194 are the rotor teeth; 64÷132 is a big rotor tooth.

Having analyzed the results given in Fig. 5, Fig. 6, and Fig. 7, we can see that the spatial distribution of the module of the magnetic induction vector in the equivalent tooth zone and body of the stator in Fig. 6 and Fig. 7 becomes a certain value displaced along the angular coordinate compared with Fig. 5. This displacement corresponds to the real reciprocal location of the rotor and stator at a given fixed moment of time.

Since the results displayed in Fig. 5 are obtained by means of the mathematical field model of the turbogenerator in the transformed system of coordinates, they confirm the conclusions drawn in the earlier works. Therefore, to calculate the no-load mode of the turbogenerator, it is reasonable to use the mathematical field model in the transformed system of coordinate. In the given model, transition to a physical reference system is implemented by the method of a necessary angular coordinate displacement of the obtained electromagnetic quantities [1]. This is what the results shown in Figures 6 and 7 demonstrate. In this case the relative error of the difference of spatial values of electromagnetic quantities calculated using the two developed models is less than 1 %.

The presence of electroconductive media in the tooth zone of the stator of the turbogenerator mathematical model at no-load in the phase systems of coordinate developed on the basis of the equations of the electromagnetic field, leads to reproducing a short circuit mode in the stator windings instead of the no-load one. This proves the spatial distribution of the module of the magnetic induction vector presented in Fig. 8.

As the magnetic materials of the rotor and stator in the no-load regime of the turbogenerator in the presented models at the same values of the excitation current do not reach a deep saturation zone unlike the models given in works [3, 4], the maximal difference in values of the spatial distributions of the electromagnetic field of the device for linear and nonlinear models is not more than 10%, with the integral values of the stator winding voltages in the defined models differing by less than 0,5%.

Comparing the values of the spatial distributions of the magnetic induction vector module obtained from the mathematical models of the turbogenerator with the equivalent tooth zone and the real rotor tooth structure, as well as the integral values of the stator winding voltages (Fig. 10), at the same currents in the excitation winding for the models mentioned above, shows an about 35 % in error in the linear variant and 23 % in error in the nonlinear one. The given values of the errors were calculated on the basis of the known fixed quantities of the voltages for the real no-load TGV-500 turbogenerator in the mode under investigation.

Respectively, the application of the familiar method used in the theory of circuits for recalculating the electromagnetic characteristics of the equivalent anisotropic zones while developing simplified mathematical models of electric devices is the cause of significant errors. This confirms the conclusion that the

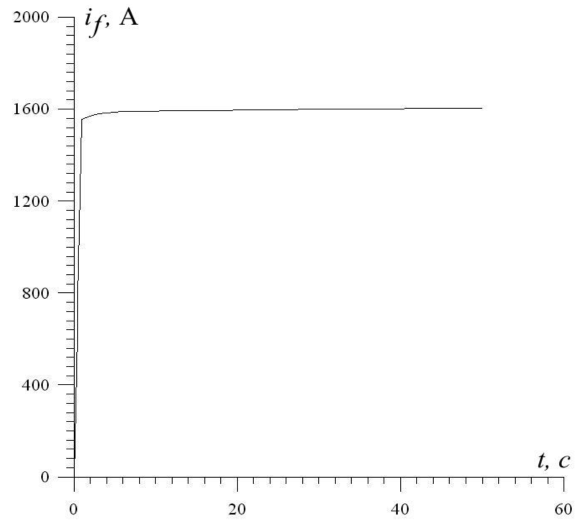

Fig. 9. Time values of an excitation current induced in the rotor winding of the turbogenerator.

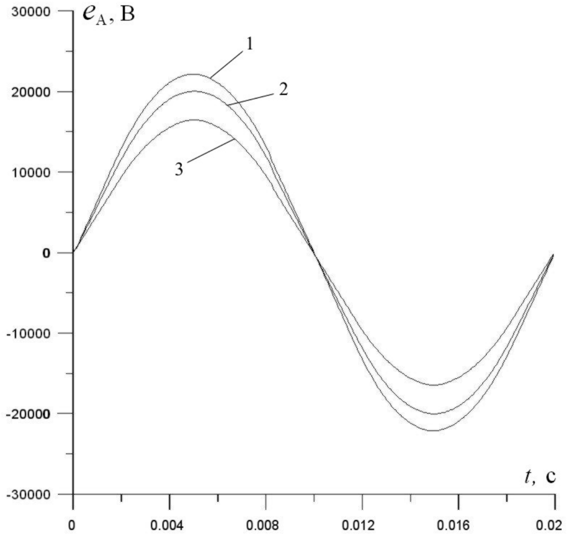

Fig. 10. Time values of A-phase voltage in the stator's winding after completing the transition process in linear (1) and nonlinear (2) models with an equivalent tooth zone of the rotor, and in nonlinear models with a real tooth structure of the rotor (3).

description of electromagnetic phenomena in sophisticated elements of the device construction by introducing into the model imaginary media with anisotropic equivalent characteristics is hardly adequate.

These results show that the developed mathematical models for the calculation of the no-load mode of the turbogenerator at the specified voltage on the excitation winding considering the real tooth structure of the rotor are the most accurate, as their error in linear and nonlinear variants is less than 0,1 %.

The analysis of electromagnetic processes in the cross-sectional zones of the turbogenerator in the no-load transition process had been performed by the time when the spatial distributions of the electromagnetic field vectors in the device were established, and the fixed values of the voltages in the stator windings of the turbogenerator were achieved.

5. Conclusions

This work suggests mathematical field models of the turbogenerator with the real rotor tooth structure and the equivalent stator tooth structure in one moving and physical systems of the coordinate of the rotor and stator simultaneously at the given voltage of the excitation winding to calculate the transitional process at no-load. Since the excitation winding in the turbogenerator at no-load is the source of the field and there is no current in the body and winding of the stator, the developed models are the most optimal for the analysis of electromagnetic processes in the turbogenerator in the mode under consideration.

The results of the analysis of electromagnetic processes in the turbogenerator using the developed models with the real rotor tooth zone and the equivalent stator tooth zone in the transformed and physical reference systems confirm the necessity of taking into account the reference systems of moving and static bodies when developing the mathematical field models of electric devices on the basis of the theory of electromagnetic field. Without this condition being satisfied, the development of the mathematical field models of any electrodynamic device becomes impossible.

The results of the computer simulation show that the most effective tool to be used for analyzing electromagnetic phenomena in the turbogenerator at no-load is the mathematical field model designed in the system of coordinates of the moving rotor. The mathematical model of the device in physical coordinate systems of the rotor and stator based only on the methods of the field theory is more difficult to be practically implemented and less efficient while a device at no-load being studied, as it implies the mechanical displacement of nonlinear anisotropic media. This is the case when the relative error of the difference of spatial-time values of the electromagnetic quantities found with the use of two developed models is within 1%.

The application of the equivalent replacement of the rotor tooth zone by a continuous anisotropic medium while developing the mathematical field models of the turbogenerator at no-load is the cause of a computational error being within 23% comparing with the results of the models that consider both the real rotor tooth zone and the equivalent stator tooth structure. This is the evidence that while developing mathematical field models of electric devices, the main task is to take into account the real design of constituent elements of the object construction that are the sources of the electromagnetic field in the mode under modeling.

The analysis performed shows that the mathematical models of the turbogenerator at no-load developed on the basis of the methods of the theory of the electromagnetic field at the given voltage of the excitation winding considering the real rotor tooth structure are the most precise as they provide less than 0,1% in error of the integral values of the stator windings voltages in the linear and nonlinear variants. It is the disadvantage of the models that the processes obtained in the equivalent stator tooth zone do not correspond to the processes occurring in the real slots and teeth of the stator. For a more complete and adequate reproduction of electromagnetic phenomena in the stator tooth zone at no-load, it is necessary to create the mathematical field model of the turbogenerator with the real tooth structure of the rotor and stator simultaneously.

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РОЗРАХУНОК ЕЛЕКТРОМАГНІТНИХ ПРОЦЕСІВ ТУРБОГЕНЕРАТОРА З ЕКВІВАЛЕНТНОЮ ЗУБЦЕВОЮ ЗОНОЮ СТАТОРА В РЕЖИМІ НЕРОБОЧОГО ХОДУ

Ярослав Ковівчак

На основі рівнянь Максвелла в потенціалах розроблено двовимірні польові математичні моделі розрахунку квазістаціонарного електромагнітного поля у поперечному перерізі турбогенератора в режимі неробочого ходу з еквівалентною зубцевою зоною статора. Запропоновані моделі передбачають розрахунок електромагнітного поля у всіх зонах пристрою як в одній системі координат рухомого ротора, так і в системах координат ротора та статора одночасно. Проведено порівняльний аналіз отриманих результатів.

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