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# MATHEMATICAL MODEL OF A SYNCHRONOUS MACHINE BASED ON THE METHOD OF AVERAGE VOLTAGES ON AN INTEGRATION STEP

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**Abstract:** The mathematical model of a synchronous machine developed by means of the authors' method of average voltages on an integration step is described. This model is formed in phase coordinates and takes into consideration non-linearity of a magnetization characteristic. The features of the mathematical model developed are high calculation performance and numerical stability that enables the developed model to be used for real-time modeling of complicated electrical power systems. The described mathematical model has been used as a part of the real-time mathematical model of the electric power generation system of South-Ukrainian Nuclear Power Plant. The computer simulation results are presented.

**Key words:** mathematical model, synchronous machine, method of average voltages on an integration step, South-Ukrainian Nuclear Power Plant.

## 1. Introduction

The area of application of mathematical modeling methods is constantly expending. Today, mathematical models are being used not only to analyse complicated electrical engineering systems but also to control and diagnose them. Hybrid models have good prospects for being applied since they combine mathematical (digital) models and physical objects. Such models can be used for solving the whole complex of tasks related to the analysis of operating regimes, diagnostics and tuning of control systems, training of maintenance staff. In this case, the mathematical models must work in real-time mode and, also, be characterized by high calculation performance and numerical stability. The development of such mathematical models for the elements of electrical engineering systems, in particular for synchronous machines widely used in power systems, is of interest at this time.

## 2. Statement of problem

There are known mathematical models of synchronous machines in phase coordinates that take into consideration non-linearity, asymmetry of windings, influence of damper system [1, 2]. Such models, together with the models of semiconductor converters and other typical component elements, can be used for the creation of computer models of complicated electrical power systems. However, they require a considerable amount of calculations on the integration step, accuracy and numerical stability being largely determined by the chosen method of integration. This complicates their application in control systems and hybrid models, which must function in a real-time mode.

Simpler models in orthogonal coordinates provide high performance of calculations but they have a limited area of application because of lower completeness of description, and impossibility of asymmetry realization.

Consequently, it is vital to develop mathematical models of synchronous machines that would combine high description completeness inherent in the models in phase coordinates, and would require a diminished amount of calculations.

In order to increase the performance of computations and to provide real-time calculations, such models must allow long-term calculations at a greater integration step. This article is concerned with the creation of such models.

## 3. Synchronous machine equations

Let us consider a nonsalient pole synchronous machine (SM) without taking into account the influence of rotor damper circuits.

Let us present each phase of SM's stator as an electric branch as shown in Fig. 1.

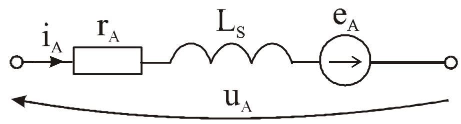


*Fig. 1. Electric branch of a SM's stator.*

The value of equivalent inductance can be found by the transformation from the known equivalent circuits of SM in dq-axes as

$$L_S = L_d - \frac{L_{ad}^2}{L_{ff}},$$

where $L_{ff} = L_{ad} + L_{of}$ .

In order to take into consideration the non-linearity of a magnetization characteristic, the inductance $L_{ad}$ is defined as a function of magnetization current.

The e.m.f. in SM's stator phases electric branches are determined as

$$e_A = -\frac{L_{ad}}{L_{ff}}\left( u_{fA} - r'_f\, i_{fA} + \frac{z_p w_R}{\sqrt{3}}\left(y_C - y_B\right) \right),$$
$$e_B = -\frac{L_{ad}}{L_{ff}}\left( u_{fB} - r'_f\, i_{fB} + \frac{z_p w_R}{\sqrt{3}}\left(y_B - y_C\right) \right), \quad (1)$$
$$e_C = -\frac{L_{ad}}{L_{ff}}\left( u_{fC} - r'_f\, i_{fC} + \frac{z_p w_R}{\sqrt{3}}\left(y_B - y_A\right) \right),$$

where $r'_f$ is the resistance of a field winding referred to the stator winding; $u_{fA}$, $u_{fB}$, $u_{fC}$ are the projections of excitation voltage on the stator's winding axis, which are determined as:

$$u_{fA} = u'_f \cos(g),$$
$$u_{fB} = u'_f \cos(g - r), \quad (2)$$
$$u_{fC} = u'_f \cos(g + r).$$

$r = 2p/3$, $u'_f$ stands for the excitation voltage, referred to the stator winding; $i_{fA}, i_{fB}, i_{fC}$ are the projections of an excitation current on the stator winding axis; $w_R$ represents the angular speed; $z_p$ is the number of pole pairs.

The flux linkage of the stator's phases in (1) will be equal to:

$$y_A = L_{ad} i_A + L_{ff} i_{fA},$$
$$y_B = L_{ad} i_B + L_{ff} i_{fB}, \quad (3)$$
$$y_C = L_{ad} i_C + L_{ff} i_{fC}.$$

The currents of stator and rotor windings are determined through the following differential equations

$$\frac{di_A}{dt} = \frac{u_A + e_A - r_A i_A}{L_S}, \quad \frac{di_B}{dt} = \frac{u_B + e_B - r_B i_B}{L_S}$$
$$\frac{di_C}{dt} = \frac{u_C + e_C - r_C i_C}{L_S}, \quad (4)$$

$$\frac{di_{fA}}{dt} = \frac{e_A}{L_{ad}} - \frac{L_{ad}}{L_{ff}}\frac{di_A}{dt}, \quad \frac{di_{fB}}{dt} = \frac{e_B}{L_{ad}} - \frac{L_{ad}}{L_{ff}}\frac{di_B}{dt}$$
$$\frac{di_{fC}}{dt} = \frac{e_C}{L_{ad}} - \frac{L_{ad}}{L_{ff}}\frac{di_C}{dt}. \quad (5)$$

The electromagnetic torque of SM will be equal to:

$$M_{SM} = \frac{z_p}{\sqrt{3}}\left( y_A\left(i_{fC} - i_{fB}\right) + y_B\left(i_{fA} - i_{fC}\right) + y_C\left(i_{fB} - i_{fA}\right) \right). \quad (6)$$

The rotation speed of SM is determined by using the equation below:

$$\frac{dw_R}{dt} = \frac{M_{SM} - M_e}{J}, \quad (7)$$

where $M_e$ is the external torque.

**4. The algebraization of SM's equations based on the method of average voltages on an integration step**

For the algebraization of SM's equations, let us use the method of average voltages on an integration step that is notable for its stability and high accuracy, even at a large integration step value [3].

According to this method, the equation for electric branch that contains the source of e.m.f., inductance, capacity and resistance is written as

$$\frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} \left(u + e - u_R - u_C - u_L\right) dt = 0, \quad (8)$$

where $u$, $e$, $u_R$, $u_C$, $u_L$ represent the instantaneous values of the applied voltage, e.m.f., voltages on resistance, ideal capacitor and inductance, $t_0$ stands for the time value at the beginning of an integration step, $\Delta t$ denotes the integration step value.

Having integrated (8), the following equation has been obtained:

$$U + E - U_R - U_C - U_L = 0, \quad (9)$$

where

$$U = \frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} u \cdot dt, \quad E = \frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} e \cdot dt,$$
$$U_R = \frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} u_R \cdot dt, \quad U_C = \frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} u_C \cdot dt,$$
$$U_L = \frac{1}{\Delta t}\left(y_1 - y_0\right)$$

are the average values of the applied voltages and the voltages on the branch elements, respectively; $y_0, y_1$ stand for the flux linkages at the beginning and at the end of an integration step.

The instantaneous values of the voltages on the resistance and capacitor is presented as

$$u_R = u_{R0} + \Delta u_R, \quad u_C = u_{C0} + \Delta u_C, \quad (10)$$

where $u_{R0}, u_{C0}$ are the values of voltages at the beginning of an integration step:

$$u_{R0} = R \cdot i_0 ;\ \frac{du_{C0}}{dt} = \frac{i_0}{C} .$$

$$\Delta u_R = \sum_{k=1}^{\infty} \frac{d^{(k)} u_{R0}}{dt^{(k)}} \cdot \frac{(t-t_0)^k}{k!} ;$$

$$\Delta u_C = \sum_{k=1}^{\infty} \frac{d^{(k)} u_{C0}}{dt^{(k)}} \cdot \frac{(t-t_0)^k}{k!} \quad (11)$$

represent the increments of these voltages beginning from $t > t_0$ ;

$\frac{d^{(k)} u_{R0}}{dt^{(k)}}$, $\frac{d^{(k)} u_{C0}}{dt^{(k)}}$ are the k-order derivatives of the voltages at the time moment $t = t_0$ .

When analysing, it is necessary for currents, voltages and flux linkages to be calculated at the end of an integration step. For this purpose, the character of a current curve should be defined within the integration step. It can be a straight line, a parabola, or an m-order polynomial in general. In this case, the increment of the current on the integration step will be equal to:

$$\Delta i = i_1 - i_0 = \sum_{k=1}^{m} \frac{\Delta t^k}{k!} \cdot \frac{d^{(k)} i_0}{dt^{(k)}} . \quad (12)$$

With (12) being taken into consideration, the dependences between the derivatives of voltages and currents are as follows:

$$\frac{d^{(k)} u_{R0}}{dt^{(k)}} = R \cdot \frac{d^{(k)} i_0}{dt^{(k)}} ,$$

$$\frac{d^{(k)} u_{C0}}{dt^{(k)}} = \frac{1}{C} \cdot \frac{d^{(k-1)} i_0}{dt^{(k-1)}} . \quad (13)$$

On the basis of (10) – (13) in [1], an equation for the branch has been obtained, The equation contains such unknowns as the branch current at the end of the integration step $i_1$ and the average value of the applied voltage $U$ on the step:

$$U + E - u_{R0} - u_{C0} + \left( \frac{R}{m+1} + \frac{\Delta t}{C} \cdot \frac{2-(m+1)(m+2)}{2(m+1)(m+2)} + \frac{L_0}{\Delta t} \right) i_0 -$$
$$- \sum_{k=1}^{m-1} \left( \frac{R \Delta t^k}{(k+1)!} \cdot \frac{m-k}{m+1} + \frac{\Delta t^{k+1}}{C(k+2)!} \cdot \frac{(m+1)(m+2)-(k+1)(k+2)}{(m+1)(m+2)} \right) \frac{d^{(k)} i_0}{dt^{(k)}} -$$
$$- \left( \frac{R}{m+1} + \frac{\Delta t}{C(m+1)(m+2)} + \frac{L_1}{\Delta t} \right) i_1 = 0 , \quad (14)$$

where $i_0$ is the branch current at the beginning of the integration step; $L_0$, $L_1$ are the branch inductances at the beginning and at the end of the integration step, $m$ represents the order of a polynomial by which the current curve is described on the integration step (order of method); $\Delta t$ stands for the integration step value.

As an example, in accordance with the 2nd-order method of average voltages on an integration step, an electric branch that contains resistance R, inductance L and e.m.f. is described by the following equation

$$U + E - R i_0 + \left( \frac{R}{3} + \frac{L_0}{\Delta t} \right) i_0 - \frac{R \Delta t}{6} \frac{d i_0}{dt} - \left( \frac{R}{3} + \frac{L_1}{\Delta t} \right) i_1 = 0 . (15)$$

Having applied equation (15) to the SM's model, we shall obtain the following equations for the SM's stator (indexes 0 and 1 correspond to the variable value at the beginning and at the end of the integration step respectively):

$$i_{A1} = \left( \frac{r_A}{3} + \frac{L_{S1}}{\Delta t} \right)^{-1} \times$$
$$\times \left( U_A + E_A - r_A i_{A0} + \left( \frac{r_A}{3} + \frac{L_{S0}}{\Delta t} \right) i_{A0} - \frac{r_A \Delta t}{6} \frac{d i_{A0}}{dt} \right),$$
$$i_{B1} = \left( \frac{r_B}{3} + \frac{L_{S1}}{\Delta t} \right)^{-1} \times$$
$$\times \left( U_B + E_B - r_B i_{B0} + \left( \frac{r_B}{3} + \frac{L_{S0}}{\Delta t} \right) i_{B0} - \frac{r_B \Delta t}{6} \frac{d i_{B0}}{dt} \right), \quad (16)$$
$$i_{C1} = \left( \frac{r_C}{3} + \frac{L_{S1}}{\Delta t} \right)^{-1} \times$$
$$\times \left( U_C + E_C - r_C i_{C0} + \left( \frac{r_C}{3} + \frac{L_{S0}}{\Delta t} \right) i_{C0} - \frac{r_C \Delta t}{6} \frac{d i_{C0}}{dt} \right).$$

The average values of e.m.f. on an integration step are defined as

$$E_A = e_{A0} + \frac{d e_A}{dt} \frac{\Delta t}{2} ,\ E_B = e_{B0} + \frac{d e_B}{dt} \frac{\Delta t}{2}$$
$$E_C = e_{C0} + \frac{d e_C}{dt} \frac{\Delta t}{2} . \quad (17)$$

The derivatives of e.m.f. in the stator phases will be determined from (1):

$$\frac{d e_A}{dt} = - \frac{L_{ad}}{L_{ff}} \left( \begin{aligned} & \frac{d u_{fA}}{dt} - r'_f \frac{d i_{fA}}{dt} + \frac{z_p w_R}{\sqrt{3}} \left( \frac{d y_C}{dt} - \frac{d y_B}{dt} \right) + \\ & + \frac{z_p}{\sqrt{3}} \frac{d w_R}{dt} (y_C - y_B) \end{aligned} \right),$$

$$\frac{d e_B}{dt} = - \frac{L_{ad}}{L_{ff}} \left( \begin{aligned} & \frac{d u_{fB}}{dt} - r'_f \frac{d i_{fB}}{dt} + \frac{z_p w_R}{\sqrt{3}} \left( \frac{d y_A}{dt} - \frac{d y_C}{dt} \right) + \\ & + \frac{z_p}{\sqrt{3}} \frac{d w_R}{dt} (y_A - y_C) \end{aligned} \right) \quad (18)$$

$$\frac{de_C}{dt} = -\frac{L_{ad}}{L_{ff}}\left(\begin{array}{l}\frac{du_{fC}}{dt} - r'_f\frac{di_{fC}}{dt} + \frac{z_p w_R}{\sqrt{3}}\left(\frac{dy_B}{dt} - \frac{dy_A}{dt}\right) + \\ +\frac{z_p}{\sqrt{3}}\frac{dw_R}{dt}\left(y_B - y_A\right)\end{array}\right),$$

where the derivatives of excitation current projections on the stator winding axis are determined from (5); the derivatives of stator flux linkage will be equal to:

$$\frac{dy_A}{dt} = L_{ad}\frac{di_A}{dt} + L_{ff}\frac{di_{fA}}{dt}, \quad \frac{dy_B}{dt} = L_{ad}\frac{di_B}{dt} + L_{ff}\frac{di_{fB}}{dt},, \quad \frac{dy_C}{dt} = L_{ad}\frac{di_C}{dt} + L_{ff}\frac{di_{fC}}{dt}; \qquad (19)$$

the derivatives of excitation voltage projections on the stator winding axis, according to (2):

$$\frac{du_{fA}}{dt} = \frac{du'_f}{dt}\cos(g) - z_p w_R u'_f \sin(g),$$
$$\frac{du_{fB}}{dt} = \frac{du'_f}{dt}\cos(g - r) - z_p w_R u'_f \sin(g - r), \qquad (20)$$
$$\frac{du_{fC}}{dt} = \frac{du'_f}{dt}\cos(g + r) - z_p w_R u'_f \sin(g + r).$$

The projections of an excitation current on the stator winding axis at the end of an integration step are determined from the formulas below:

$$i_{fA1} = i_{fA0} + \frac{E_A}{L_{ad}}\Delta t - \frac{L_{ad}}{L_{ff}}\left(i_{A1} - i_{A0}\right),$$
$$i_{fB1} = i_{fB0} + \frac{E_B}{L_{ad}}\Delta t - \frac{L_{ad}}{L_{ff}}\left(i_{B1} - i_{B0}\right), \qquad (21)$$
$$i_{fC1} = i_{fC0} + \frac{E_C}{L_{ad}}\Delta t - \frac{L_{ad}}{L_{ff}}\left(i_{C1} - i_{C0}\right).$$

**5. The algorithm of equation solving**

The algorithm for solving mathematical model equations on an integration step is as follows.

1. The projections of an excitation voltage on the stator winding axis $u_{fA}, u_{fB}, u_{fC}$ are determined from (2); the flux linkage of stator phases $y_A, y_B, y_C$ are determined from (3) based on the values of stator and field currents at the beginning of an integration step; the e.m.f. in the stator phases $e_A, e_B, e_C$ are determined from (1).

2. The derivatives of stator currents are determined from (4).

3. The electromagnetic torque of SM is determined from (6)

4. The derivative of SM's rotation speed is determined from (7).

5. The derivatives of excitation current projections on the stator winding axis are found from (5).

6. The derivatives of stator flux linkage are obtained from (19), and the derivatives of excitation voltage projections on the stator winding axis are determined from (20).

7. The derivatives of stator e.m.f. are determined from (18).

8. The average values of the stator e.m.f. on an integration step $E_A, E_B, E_C$ are determined from (17).

9. The values of stator currents $i_{A1}, i_{B1}, i_{C1}$ at the end of an integration step are determined from (16).

10. The projections of an excitation current on the stator winding axis $i_{fA1}, i_{fB1}, i_{fC1}$ are determined from (21) (the amplitude of these currents is equal to the excitation current).

The input computational information includes voltages applied to the stator phases, values of the variables at the beginning of an integration step, electromagnetic parameters, an external torque on the shaft of SM.

**5. The model application results**

The described mathematical model of a synchronous machine has been used for the mathematical modeling of parallel-working turbogenerators SG2 and SG3 as part of the mathematical model of the electric power generation system of South-Ukrainian Nuclear Power Plant (Fig. 2), which contains three 1000 MW turbogenerators SG1, SG2 and SG3 operating in parallel. The nominal data of the turbogenerators are: $I_n =$ 26.7 kA, $U_n =$ 24 kV, $\cos j_n = 0.9$.

The excitation system of the turbogenerators contains a brushless exciter (the excitation systems of the turbogenerators SG2 and SG3 are not shown in Fig. 2).

The mentioned mathematical model of the electric power generation system of South-Ukrainian Nuclear Power Plant works in a real-time mode as part of the digital diagnostic complex designed for the testing and diagnostics of real excitation systems by connecting the physical excitation system to the real-time computer model of a power unit; tuning of excitation controllers and protection systems; analysis of turbogenerator's operating regimes and detection of reasons for abnormal situations; training of maintenance staff of a power plant (works as a trainer) [4].

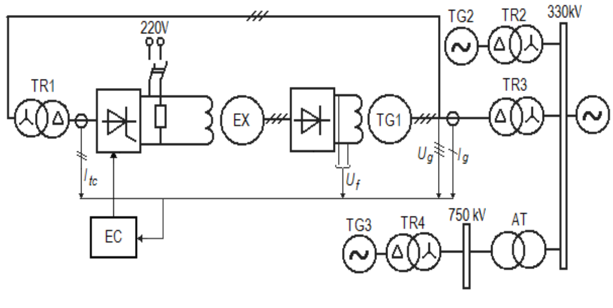


*Fig. 2. The functional scheme of an electric power generation system: EX – brushless exciter; TR1 – input transformer of the main generator's excitation system; TR2, TR3, TR4 – generator transformers, AT – autotransformers, EC – excitation controller.*

The features of the implemented in the DDC model of an electric power generation system are as follows: taking into consideration the non-linearity of electric machines (modelled in phase coordinates) and semiconductor converters (every thyristor (or diode) is replaced by the branches with resistance and inductance); taking into consideration the asymmetry and interferences between all component parts; the possibility of model's interacting with real physical equipment through analogue and discrete signals.

A necessity of applying superfast mathematical models of generators operating in parallel is explained by the necessity of increasing the speed of computation and providing the continuos real time model operation for a long period of time (for about twenty-four hours).

Figures 3–6 depict the simulation results in the form of time dependencies of variables for the regimes of initial excitation of the synchronous generator TG2, its connection to a power network and loading by active power.

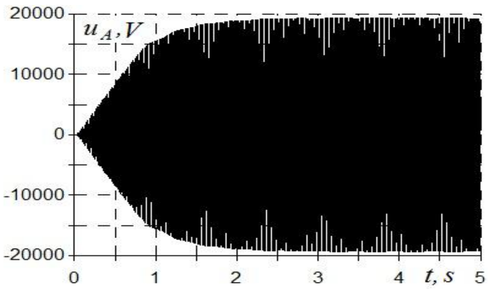


*Fig. 3. Stator phase voltage of TG2 in the regime of initial excitation.*

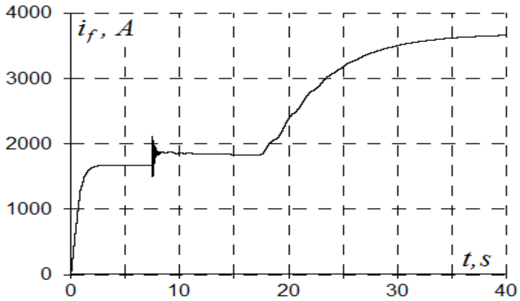


*Fig. 4. Excitation current of TG2 during the initial excitation, connecting to a power network and loading by active power.*

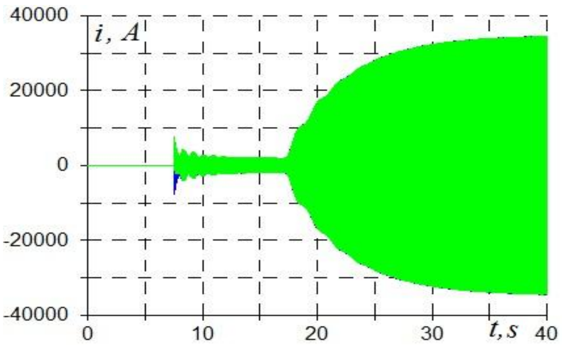


*Fig. 5. Stator currents of TG2 during the initial excitation, connecting to a power network and loading by active power.*

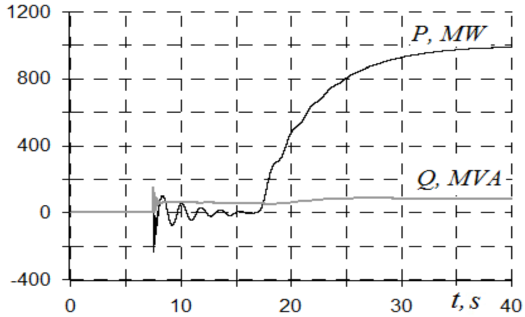


*Fig. 6. Active and reactive output power of TG2 during the initial excitation, connecting to a power network and loading by active power.*

Fig. 7 shows a computation oscillogram of reactive power on the outputs of TG1 and TG2 for the regime of output reactive power redistribution between the generators as the result of increasing TG2's voltage set point. The results demonstrate that with the reactive power on the output of TG2 increasing, the reactive power generated by TG1 is reduced. The value of reactive power is determined by the value of voltage in a power line and the voltage set point signals in the excitation controllers of the generators.

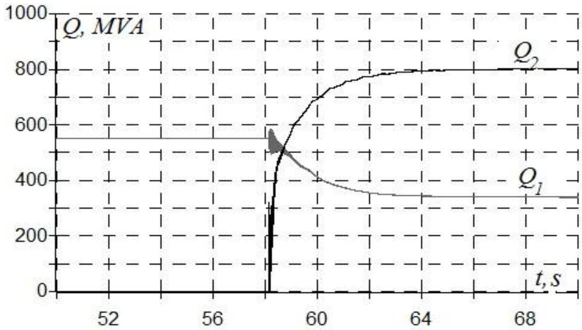


*Fig. 7. Computation oscillograms of reactive power redistribution between TG1 and TG2.*

## 6. Conclusion

The developed mathematical model of a synchronous machine is notable for a diminished amount of calculations on an integration step and, at the same time, keeps all the advantages of the mathematical models in phase coordinates.

The use of the method of average voltages on an integration step for the creation of this mathematical model provides high numerical stability, possibility to increase the step value resulting in high computational accuracy. This substantially enhances computation performance of the model and provides good prospects of using it in real-time systems.

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## МАТЕМАТИЧНА МОДЕЛЬ СИНХРОННОЇ МАШИНИ НА ОСНОВІ МЕТОДУ СЕРЕДНІХ НАПРУГ НА КРОЦІ ІНТЕГРУВАННЯ

Омелян Плахтина, Андрій Куцик

Описано математичну модель неявнополюсної синхронної машини, створену з використанням авторського методу середніх напруг на кроці чисельного інтегрування. Математичну модель створено у фазних координатах з урахуванням нелінійності характеристики намагнічування. Особливістю цієї моделі є її висока швидкодія та числова стійкість, що дає змогу використовувати її для математичного моделювання в реальному часі складних електротехнічних систем.

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