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METHOD OF REGULARIZING THE PROBLEM OF RECOVERY OF INPUT SIGNALS OF DYNAMIC OBJECTS

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Abstract. The task of signal recovery is one of the most important for automated diagnostics and control systems. This task is computationally complex, especially when there are a lot of heterogeneous errors in the signals and recovery is to be performed in real time. The article deals with the application and investigation of a modified algorithm for the method of quadrature formulas for the numerical solution of the Volterra integral equations of the I kind in solving the problem of signal recovery in real time. A method is proposed for selecting the parameters of regularizing links of computing means.

Key words: Signal reconstruction, interference, real time, Volterra equations of the I kind, regularization.

1. Introduction

The integral formulation of the problem of reconstructing the input signal of a stationary dynamic object is described by the Volterra equation of the I kind

$$\int_0^t K(t-s)\,y(s)\,ds = f(t), \qquad (1)$$

where the functions $y(t)$ and $f(t)$ represent respectively the input and output signals, and a kernel (function $K(t)$) is the impulse response function of the object.

The problem of solving equation (1) belongs to a class of ill-posed problems, since the presence of errors in its right side and the kernel usually causes numerical instability of the solution process, which makes it necessary to use regularization methods [1].

In many cases, in order to increase the stability of the solution process of the integral equation of type (1) A.N. Tikhonov's regularization method [2] is used, reducing the problem to solving the integro-differential equation, one of the conditions for which is $K(0) \neq 0$. However in real objects usually $K(0) = 0$, which limits the possibilities of the method.

In this article, to ensure the stability of the computing process, a possibility of using the Lavrentiev regularization method is considered with a specific modernization of the regularization parameter search process.

2. Application of Lavrentiev method

According to the Lavrentiev method, the following equation is solved instead of equation (1):

$$a\,y(t) + \int_0^t K(t-s)\,y(s)\,ds = f(t). \qquad (2)$$

The problem of determining the regularization parameter a is time- and effort-consuming [3]. In the work [3] multiple ways of determining a are shown, including the method of model experiments (examples) for the Fredholm integral equation of the I kind.

Let us consider the possibility of applying the method of model experiments to determining the regularization parameter a in the solution of the Volterra integral equation of the I kind. The procedure of determining the parameter a consists of the following steps.

1. For solving the integral equation (1) a model equation is created

$$\int_0^t Q(t-s)\,y_Q(s)\,ds = f_Q(t), \qquad (3)$$

in which $Q(t)$ coincides with the predetermined function $K(t)$ and solution $y_Q(t)$ is given (selected) in such a way, that the right side $f_Q(t)$ is as close as possible to $f(t)$, i.e.

$$f(t) \approx f_Q(t) = \int_0^t Q(t-s)\,y_Q(s)\,ds. \qquad (4)$$

2. In practice, measurement mistakes are inevitable and instead of the equation with exactly known right side $\bar{f}(t)$ we obtain an approximate right side, i.e.

$$f(t) = \bar{f}(t) + \Delta\bar{f}(t),$$

where $\Delta\bar{f}(t)$ is an error. Having the law (e.g., normal) of $\Delta\bar{f}(t)$ distribution, we can write down

$$f(t) = \bar{f}(t) + x\,\bar{f}(t),$$

where x is a normally distributed random number. Therefore, the right part of the model equation (3) we perturb with an error at which values $\left\|\Delta f_Q\right\| / \left\| f_Q\right\|$ and $\left\|\Delta f_Q\right\| / \left\| f_Q\right\|$ are approximately equal, i.e. instead of equation (3) we have

$$\int_0^t Q(t-s)\,\overline{y}_Q(s)\,ds = f_Q^*(t), \tag{5}$$

where $f_Q^*(t) = f_Q(t) + x\, f_Q(t)$.

Applying the Lavrentev regularization method allows us to represent equation (3) as

$$a\, y_{Qa}(t) + \int_0^t Q(t-s)\, y_{Qa}(s)\,ds = f_Q^*(t). \tag{6}$$

3. By repeated numerical solving equation (6), e.g. by using quadrature formulas, $a_{\text{opt}Q}$ is determined for some values of a, at which

$$\sum_{i=0}^{m} \left| y_{Qa}(t_i) - y_Q(t_i) \right|^2 = \min, \quad i = \overline{1,m}, \tag{7}$$

where m is the number of sampling points.

4. The obtained value $a_{\text{opt}Q}$ is used to solve integral equation (1).

Numerical simulations show that using the method of modeling experiments for the development of the algorithm for solving a numerical equation for signal restoration problem (1) allows determining the effective values of the parameter a which regularizes the problem.

When solving the considered problem numerically, it is important to understand possible errors of the result.

Note that for a linear integral equations the error of solutions can be expressed using the fundamental error formulas. Indeed, the machine solution can be represented as depending on a number of quantities q_1, q_2, ..., q_n, characterizing the model parameters, input actions, etc., whose deviations cause errors of the result. If there are deviations, real solution can be decomposed into the limited Taylor series and can be represented as

$$\begin{aligned} &Y(t, q_1 + \Delta q_1, \mathbf{K}, q_n + \Delta q_n) \approx Y(t, q_1, \mathbf{K}, q_n) + \\ &+ u_1(t) q_1 + u_2(t) q_2 + \mathbf{K} + u_n(t) q_n, \end{aligned} \tag{8}$$

where $u_1(t)$, $u_2(t)$, ..., $u_n(t)$ are influence (or sensitivity) coefficients.

Subtracting the exact solution of $Y(t, q_1, q_2, ..., q_n)$ from (8), we obtain

$$\Delta Y(t) + u_1(t)\Delta q_1 + u_2(t)\Delta q_2 + \mathbf{K} + u_n(t)\Delta q_n.$$

3. Determination of sensitivity coefficients

To determine the sensitivity coefficients, the corresponding equations can be obtained. We assume that the parameters q_1, q_2, ..., q_n are determined by the internal properties of the model, i.e. they are part of the kernel of the solved machine equation, which in this case has the form

$$a\,Y(t) + \int_0^t K_M(t-s, q_1, \mathbf{K}, q_n)\, Y(s)\,ds = f(t). \tag{9}$$

($Y(t)$ is the sought approximate solution)

Differentiating both sides of (9) in respect to the parameter q_i ($i = 1, ..., n$), we obtain

$$a\,\frac{\partial Y(t)}{\partial q_i} + \int_0^t \frac{\partial}{\partial q_i} K_M(t-s, q_1, \mathbf{K}, q_n)\, Y(s)\,ds = 0$$

or

$$\begin{aligned} &a\,\frac{\partial Y(t)}{\partial q_i} + \int_0^t \left[\frac{\partial K_M(t-s, q_1, \mathbf{K}, q_n)}{\partial q_i} Y(s) + \right. \\ &\left. + K_M(t-s, q_1, \mathbf{K}, q_n) \frac{\partial Y(s)}{\partial q_i} \right] ds = 0. \end{aligned}$$

Introducing the notation

$$\frac{\partial Y(s)}{\partial q_i} = u_i(t),$$

$$\frac{\partial K_M(t-s, q_1, \mathbf{K}, q_n)}{\partial q_i} = K'_{Mq_i}(t-s, q_1, \mathbf{K}, q_n),$$

we obtain the sought equations

$$\begin{aligned} &a\,u_i(t) + \int_0^t K_M(t-s, q_1, \mathbf{K}, q_n)\, u_i(s)\,ds = \\ &= -\int_0^t K'_{Mq_i}(t-s, q_1, \mathbf{K}, q_n)\, Y(s)\,ds. \end{aligned} \tag{10}$$

The function $Y(s)$ on the right side of equation (10) can be expressed in terms of approximate solutions. As it can be seen from (10), to determine the sensitivity coefficients we can decide to use the basic solved equation, since the kernel of equation (10) coincides with its kernel.

4. Estimation of error

As it is done in the case of differential equations, the equation for the error can be obtained for linear integral equations. We assume that while solving (2), equation actually being solved looks like this

$$\tilde{y}(t) + \int_0^t \tilde{G}(t-s)\, y(s)\,ds = \tilde{f}(t), \tag{11}$$

where $\tilde{G}(t-s) = \tilde{K}(t-s)/a$, $\tilde{f}(t) = f(t)/a$.

We assume that $\tilde{G}(t-s)$ takes into account initial modeling errors (methodical and instrumental ones) and represents the sum

$$\tilde{G}(t-s) = G(t-s) + \Delta G(t-s).$$

The right side $\tilde{j}(t)$ of equation (11) contains the external disturbance error and equals

$$\tilde{j}(t) = j(t) + \Delta j(t);$$

$\tilde{y}(t)$ is an approximate solution determined by the relation

$$\tilde{y}(t) = y(t) + \Delta y(t).$$

where $\Delta y(t)$ is the total error of the solution. Then, by subtracting expression (2) from (11), we obtain

$$\Delta y(t) + \int_0^t \{[G(t-s) + \Delta G(t-s)][y(s) + \Delta y(s)] - $$
$$- G(t-s)\, y(s)\} ds = \Delta j(t).$$

Expanding brackets under the integral and considering errors $\Delta G(t-s)$ and $\Delta y(t)$ so small that their product can be neglected, we obtain the sought equation

$$\Delta y(t) + \int_0^t G(t-s)\Delta y(s)\, ds = \Delta j(t) - \int_0^t \Delta G(t-s)\, y(s)\, ds$$

or

$$a\Delta y(t) + \int_0^t K(t-s)\Delta y(s)\, ds = \Delta f(t) - \int_0^t \Delta K(t-s)\, y(s)\, ds.$$

It is difficult to use this equation for calculating the error $\Delta y(t)$ because of the uncertainty generally occurring in primary errors, as well as due to the fact that instead of true solution $y(s)$ on the right side we must use the approximate one. However, it is applicable for a qualitative study of errors, since it particularly shows that various components of the total error can be defined separately (leaving in the right side only $\Delta f(t)$, we can determine the inherited error of the result, and leaving only the integral – the numerical algorithm error). In addition, the equation for the error allows us to make its assessment. Let us give an example of such an assessment.

If (t, s) belongs to the region D, $0 \le t \le d, 0 \le s \le t$ and you can set constraints

$$\max_{(t,s)\in D} |K(t-s)| \le K, \quad \max_{(t,s)\in D} |\tilde{K}(t-s)| \le \tilde{K},$$
$$\max_{(t,s)\in D} |\Delta K(t-s)| \le d, \quad \max_{t\in[0,d]} |\tilde{y}(t)| \le f,$$

$$\max_{t\in[0,d]} |\Delta f(t)| \le h,$$

then, using the results of [4], we obtain the estimate

$$\Delta y(t) \le \left[f d \frac{e^{\frac{1}{a}(K-\tilde{K})t} - 1}{K - \tilde{K}} + m \right] e^{\frac{1}{a}\tilde{K}t}.$$

5. Conclusion

Thus, the use of the Lavrentiev regularization method in solving the Volterra integral equations of the I kind provides required stability of the signal recovery process, and the method of model experiments allows determining the values of the regularization parameter. Expressions obtained on basis of the accuracy analysis of solved equations are the basis of deterministic and probabilistic error estimate of the sought solution.

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СПОСІБ РЕГУЛЯРИЗАЦІЇ ЗАДАЧІ ВІДНОВЛЕННЯ ВХІДНОГО СИГНАЛУ ДИНАМИЧЕЧНОГО ОБ'ЄКТА

Андрій Верлань, Ю. Стертен, Юрій Фуртат

Задача відновлення сигналів є однією з першочергових для автоматизованих систем діагностики і управління. Це обчислювально складна задача, особливо за наявності в сигналах великої кількості гетерогенних завад і необхідності проводити відновлення в реальному часі. У статті розглянуто питання застосування та дослідження модифікованого алгоритму методу квадратурних формул чисельного рішення інтегральних рівнянь Вольтерра першого роду у разі розв'язання задачі відновлення сигналів у реальному часі. Запропоновано спосіб вибору параметрів регуляризуючих зв'язків моделювальних ланцюгів обчислювальних засобів.

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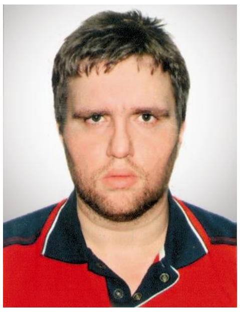

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