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A DEEPER INSIGHT IN SOME EFFECTS IN PROJECT RISK MANAGEMENT

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Abstract: This document shows a possible way how to deal with insecurities in the time schedule of a project plan. It shows that Program Evaluation and Review Technique (PERT), the most popular approach to handle this, bears some severe disadvantages. Furthermore it offers an alternative to overcome them by using Monte Carlo simulation. Finally it can be claimed that a complete change of paradigm is necessary: If you have any insecurities as inputs, everything becomes insecure. This might on the first sight convey the impression that the whole situation converts more complex, but we should rather accept this as the opportunity to apply all the well-known instruments from statistics.

Keywords: risk management in projects, critical path method, PERT, Monte Carlo simulation.

1. INTRODUCTION

In every project there is the need to implement some kind of risk management (c.f. [1, 2]), which normally contains the following cyclic phases:

(1) risk management planning
(2) risk identification
(3) qualitative risk analysis,
(4) quantitative risk analysis,
(5) risk response planning, and
(6) risk monitoring and control.

Especially in the steps (3) and (4) some analytical/statistical methods are needed because you have to deal with uncertainties/insecurities and therefore with densities and distributions.

Risks in projects can occur in different dimensions, such as time, costs, quality etc. A risky event that may happen is normally characterized by two aspects: The probability of occurrence and the impact that is a consequence of this event. Both will have some probability distributions that need to be estimated in advance.

In this contribution we will only consider uncertainties related to time. A commonly used approach to deal with this is PERT (c.f. [3, 4]), which has been developed 50 years ago. But there are some weaknesses, disadvantages, errors, and inaccuracies in using this method. We will discuss them and show how to overcome them by using Monte Carlo simulation (c.f. [5, 6]). It will be performed by analyzing an example of a concrete but fictitious project plan.

2. THE PERT APPROACH

Let us look at the following example of a network plan and consider uncertainties in time. We assume that these uncertainties are already characterized in the steps (2) and (3) by estimating optimistic (OD), most likely (MD), and pessimistic (PD) durations (3-point-estimates) see Table 1.

Table 1. Project Plan.

Activity	Predecessors	OD	MD	PD
A	-	2	3	4
B	-	3	6	9
C	-	2	5	10
D	-	4	6	9
E	A, B, C	3	7	10
F	C, D	2	7	9
G	E	2	3	4
H	E, F	3	6	8
I	F	3	5	9
J	F	2	7	10
K	G, H, I	2	6	8
L	I, J	3	5	8

First of all we will solve the problem by using the well-known standard PERT method. PERT was developed by the United States Navy together with the OR department of Booz, Allen and Hamilton in the 1950s. Purpose of this development was to support the deployment of the Polaris-Submarine weapon system (c.f. [7]).

PERT uses beta-distributions with the density f_b given by (1)

$$f_b(x|a,b,p,q) = \frac{(x-a)^{p-1}(b-x)^{q-1}}{B(a,b,p,q)}, \quad (1)$$

with B being the beta-function

$$B(a,b,p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}(b-a)^{p+q-1}, \quad (2)$$

and Γ being the well-known gamma-function:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt, \quad (3)$$

In order to fit a beta distribution in the way that min = OD, max = PD, and mode = MD, estimates for the expected duration (ED) and the standard deviation (STD) of the beta distribution are needed. They can be obtained by

$$ED = \frac{OD + 4 \cdot MD + PD}{6}, \quad STD = \frac{PD - OD}{6} \quad (4)$$

which leads to the transformations

$$\begin{aligned} a &= OD \\ b &= PD \\ p &= \frac{ED - OD}{PD - OD} \cdot \left[\frac{(ED - OD)(PD - ED)}{STD^2} - 1 \right], \\ q &= \frac{PD - ED}{ED - OD} \cdot p \end{aligned} \quad (5)$$

Fig. 1 shows three examples of beta distributions with different combinations of values for OD, MD, and PD (1/3/20, 1/8/15, and 2/18/20). These distributions are quite intuitive and similar to triangular distributions, but smoother than these. Although its mathematical description is more complicated than that of triangular distributions, it has some very useful and simple properties (e.g. (4)).

Most of the mathematical background is primarily necessary to generate the individual distributions in the Monte Carlo simulation and for the understanding of the whole approach, but not for the application of PERT. The original PERT approach only uses the formulas (4) and then creates the critical path based on the EDs of the individual activities. Table 2 shows the given means (EDs) and variances. By this PERT has the advantage of a sophisticated mathematical background, but a very simple application. Fig. 2 shows the critical path that follows.

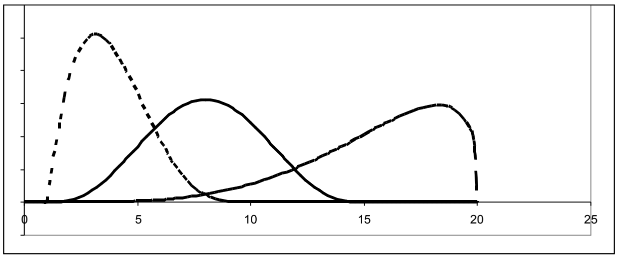

Fig. 1 – Different beta distributions.

Table 2. Project Plan with expected durations and variances assuming Beta distributions.

Activity	Predecessors	ED	VAR
A	-	3.000	0.111
B	-	6.000	1.000
C	-	5.333	1.778
D	-	6.167	0.694
E	A, B, C	6.833	1.361
F	C, D	6.500	1.361
G	E	3.000	0.111
H	E, F	5.833	0.694
I	F	5.333	1.000
J	F	6.667	1.778
K	G, H, I	5.667	1.000
L	I, J	5.167	0.694

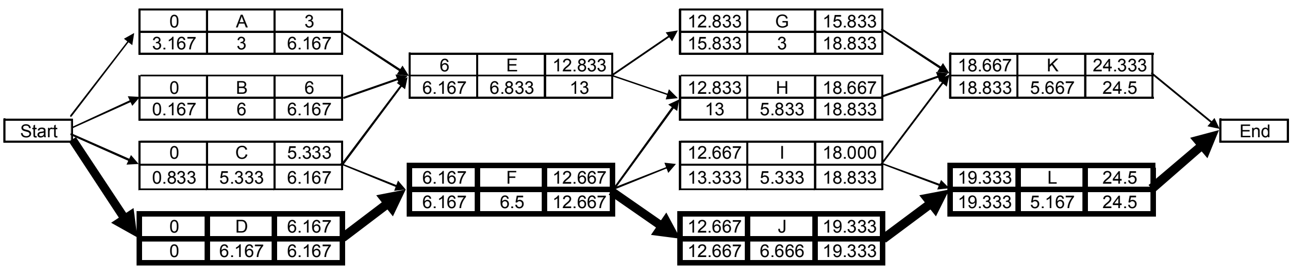

Fig. 2 – Critical path (bold arrows) of the project.

In the first row of each task we see the values of the early start time (EST), the label, and the early finish time (EFT) of each activity, whereas the second row shows the late start time (LST), the duration, and the late finish time (LFT). So we get the indicated critical path (bold arrows) with a total length of 24.5 days.

Then the cumulated distribution (convolution) along the critical path (D-F-J-L) is observed. Because all the distributions are assumed to be independent, due to the Central Limit Theorem, the result tends to a normal distribution with mean and variance equal to the sum of the individual values on the critical path. In our example we get a mean of 24.5 and a standard deviation of 2.128. The resulting density is shown on the left side in Fig. 3.

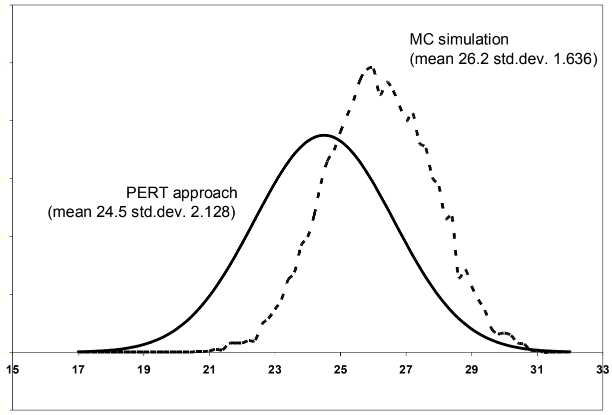

Fig. 3 – Results of MC simulation with beta distributions versus PERT results.

Although this approach has been widely used in the last 50 years, Harvey Maylorsays(c.f. [8]): "Moreover, many of the traditional methods of project planning such as PERT […] have never been the subject of any evaluation – not least because, until recently, there was no alternative." But today, there is a powerful alternative: Monte Carlo simulation (c.f. [9, 10]).

3. MONTE CARLO SIMULATION

As it is commonly known, the Monte Carlo simulation is a method that relies on repeated random sampling from given distributions (c.f. [6]).Because of their reliance on repeated computations and random or pseudo-random numbers, Monte Carlo methods are most suited to calculations by computers. The main idea in applying Monte Carlo methods lies in the fact that one has to model the problem just for one instance and can then create as much instances as you like by creating a loop. In our case we create 10,000 instances and the result is shown in the graph on the right hand side of Fig. 3. All the assumed beta distributions for the durations of the individual activities are exactly the same in both approaches (PERT and Monte Carlo simulation) and therefore the tworesultsarefully comparable.

The continuous line is the already mentioned normal distribution that resulted out of the original PERT approach with a mean of 24.5 and a standard deviation of 2.128. The dotted line shows the distribution of the results of the Monte Carlo simulation with a mean of 26.2 and a standard deviation of 1.636. It is obvious that the Monte Carlo simulation leads to an average that is almost 2 units higher than those of the original PERT approach, but with a smaller dispersion. The background for this will be shown in the following simple illustrative example.

4.THE EFFECT OF THE SHIFTING THE DISTRIBUTION

To illustrate the above effect shown in Fig. 3 we create a very simple example: Let us assume that we only have two parallel and independent tasks with estimated durations that are normally distributed with a mean of 20 and a standard deviation of 4 for both tasks. The distribution function of that normal distribution shall be denoted by Φ, the corresponding density by φ. Then the PERT approach would lead to two parallel critical paths with both having a normal distribution with the given parameters and therefore to a distribution for the whole project that is again Φ. Since in fact the duration of the whole project is nothing else than the maximum of the two independent tasks, the real distribution function is Φ^2, with the corresponding density function $[\Phi^2]' = 2 \cdot \Phi \cdot \varphi$. In Fig. 4 the two distributions are compared and the similarity to Fig. 3 is quite obvious.

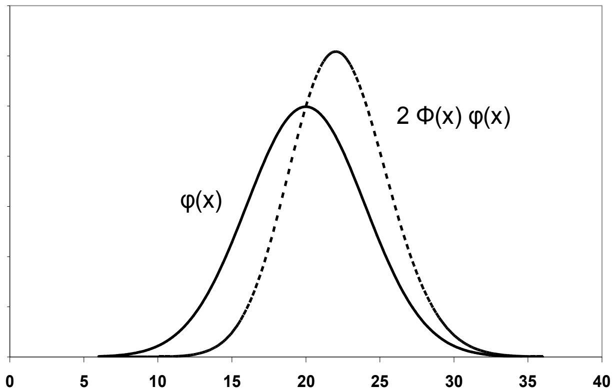

Fig. 4 – PERT and theoretical result.

Since in the analytic determination of the critical path there is always the necessity to calculate maximums, this will lead to the use of order statistics. Therefore if only distributions for the durations are known, this will not lead to only one unique critical path, but to different parallel critical paths that occur with some probability. And as the maximum characterizes the real final end of the project, this will be in general later than that of the PERT approach. This shifts the whole distribution to the right. On the other hand the upper extremes of the final end (the right tail of the distribution) will be almost the same in the PERT and the theoretical approach. Therefore the variance is reduced. In the simple example, the mean of the theoretical distribution rises to about 22 and the standard deviation reduces to about 3.2. The increase of the mean and the reduction of the variance obviously depend on the individual structure of the given project plan. But it is quite evident that the PERT approach systematically underestimates the real risk.

5.THE CRITICAL FIELD

As seen in the simple example before, when moving from the PERT result to the "real" distribution created by Monte Carlo simulation, we get the same effect with a higher mean and a lower standard deviation. This relies in the fact that thePERT approach is a little inconsequential: Although it is a stochastic approach, it uses the deterministic assumption for the construction of the critical path (cf. Fig. 5). But as soon as you move from the stochastic approach to the deterministic one, there is only little chance to get a valid result.

The main reason for this can be seen in the simple example: There is no longer one unique critical path – we have two parallel critical paths. And we have the same in our main example: We will get critical paths that vary from case to case. Fig. 6 shows in how many of the 10,000 cases the individual tasks belong to the critical path.

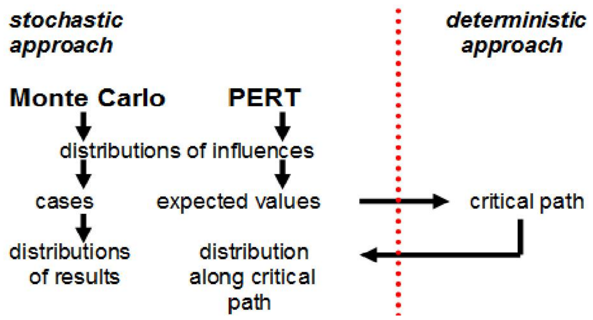

Fig. 5 – Stochastic versus deterministic approach.

In Fig. 6 the shading indicates the probability of a task to be critical: The darker the shading, the higher is the probability that the activity belongs to the critical path.

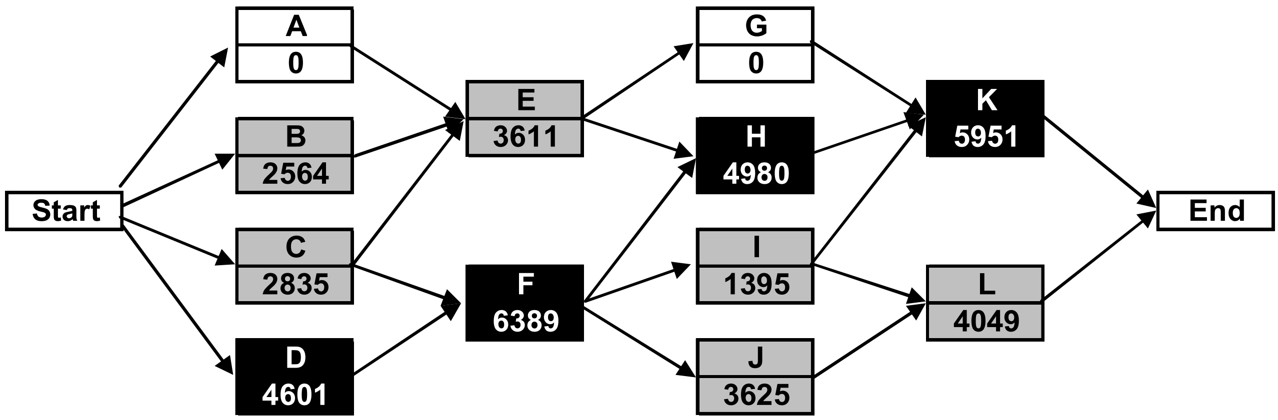

Fig.6 – How often is an activity critical in the main example?

It can be seen that we have tasks that are never critical (like A and G), some are sometimes critical (like B, C, E, I, J, and L) and some are quite often critical (like D, F, H, and K). But a comparison with Fig. 2 shows that the latter are not identical with the critical path of the deterministic approach that was used by PERT. Therefore in the stochastic approach it is no longer reasonable to use the term "critical path" – we always have *critical fields*.

6. BETTER UNDERSTANDING OF BUFFERS

Since we only have distributions for the durations and not just one unique critical path, we also do not have deterministic buffers. Apart from the special case that an activity will be critical with a probability of 100% (which does not occur in the given example), we will only get distributions for the buffers.

But of course, for buffer allocation in the practical operation of a project, we should get deeper insight into these distributions. Using Monte Carlo simulation these underlying distributions can be easily generated.

A few examples for these distributions are presented in Fig. 7 – 9. G is an example for an activity that is never critical and therefore the probability of a buffer with size 0 is zero (c.f.

Fig. 7). The most probable buffer size for activity G is 4.

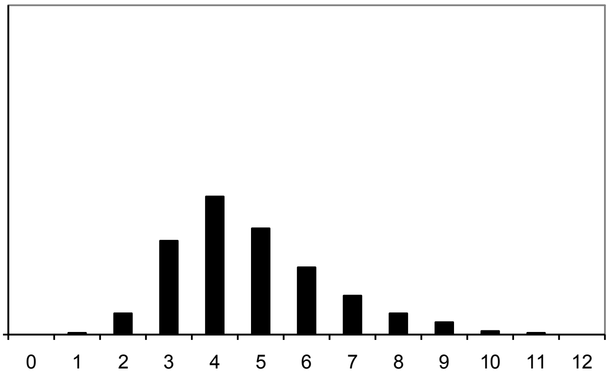

Fig. 7 – Distribution of buffers of activity G.

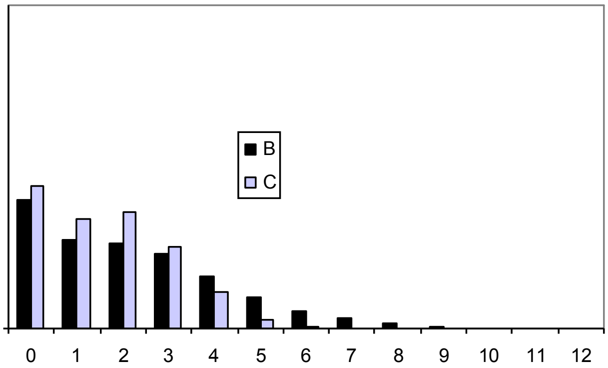

Fig. 8 – Distributions of buffers of activities B and C.

Fig. 8 shows the distributions of activities B and C, two activities that are medium critical with probabilities of 25-30%. The buffer distributions of these two activities are quite different: For activity B the probability slowly decreases and buffers of 7, 8, or even 9 can occur with a positive probability, whereas for activity C the probability for buffers of 1 to 3 is quite high but then decreases quite fast. Therefore the distributions of the buffers really offer additional information than only the probability of being critical.

Fig. 9 shows the buffer distribution of activity K, an activity that is critical by 60%. But we can see that there is a chance to get buffers up to 4. Additionally one can analyze these buffer distributions in many ways. For example one can look at the different moments of these distributions (c.f. Table 3), to detect that all the distributions are (right) skewed. This is something that frequently occurs in distributions that are derived in the context of risk management (c.f. [11]).

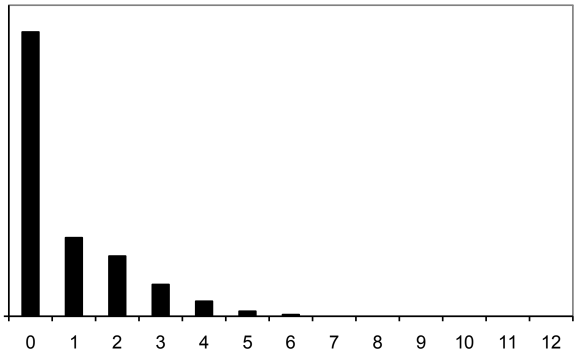

Fig. 9 – Distribution of buffers of activity K.

Table 3. Moments of buffers.

	buffers		
activity	mean	standard deviation	skewness
A	5.35	1.79	0.77
B	0.25	2.01	0.83
C	1.57	1.35	0.50
D	1.11	1.36	1.24
E	1.80	1.88	0.99
F	0.71	1.18	1.75
G	4.80	1.76	0.81
H	1.07	1.36	1.25
I	2.32	1.58	0.45
J	1.69	1.81	1.06
K	0.83	1.23	1.54
L	1.45	1.67	1.19

7. CONSEQUENCES FOR THE CONTROL PROCESS

In practice the critical path is most often the common guideline for the controlling process. But if you understand that in the case of uncertainties there exists no unique critical path, you need a total change in paradigm. It is easy to show that different realizations within the ongoing process lead to different changes in the critical path. Moreover because of the back and forth calculation of the critical path, it is also possible that a single uncertain event at the end of the project may change the whole critical path, even in the very beginning of the project. (Here some analogies can be found to the Wagner-Whitin approach in dynamic lot sizing.)

To show this in our example, let us assume that we observed exactly the realizations that led to the critical path shown in Fig. 2 apart from the fact that in activity K a duration of 8 was realized. This will lead to a critical path that is shown in Fig. 10. As it is clearly visible, this single change leads to a totally different critical path: Some activities even at the start of the project that were formerly critical are no longer critical and the other way around.

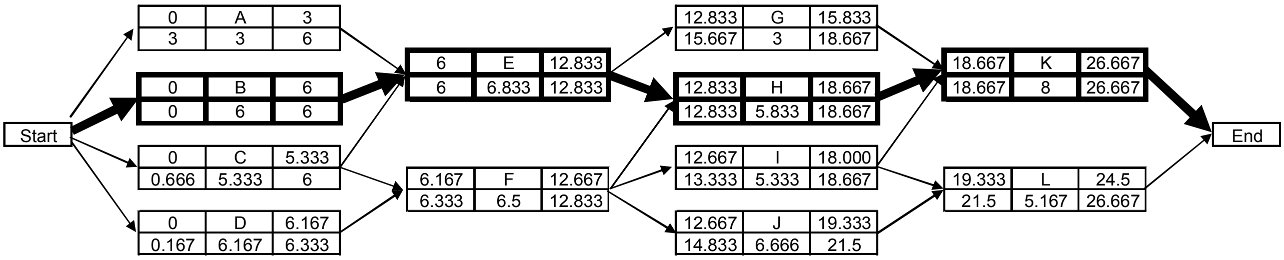

Fig. 10 – Net and critical path – now with a duration of 8 in activity K.

It is also interesting to look what happens in the case of different realizations in the starting phase. As we have already seen (Fig. 6), activity A will never be critical, even if we get here the worst/longest realization that is possible. On the other hand even a realization of a relatively short duration in activity B, C, or D may lead to the fact that this activity becomes critical. You always have to keep in mind that the fact of being critical is determined within the whole timeframe, not only by the realized duration of an activity itself or coincident or past activities: also possible durations in the future determine whether an activity is critical or not. The total retrograde calculation of the whole project from start to end is necessary, especially if you have to face uncertainties in the realizations. Therefore simulations with a Monte Carlo model can deliver interesting results that may prevent project managers from a misinterpretation of the current situation.

8. CONCLUSION AND REMARKS

If one accepts uncertainties in a project this has in most cases consequences for the whole planning, monitoring, and controlling process. But as projects are usually characterized by items like “uniqueness” or “complexity”, it makes little sense to ignore uncertainties. The allowance of stochastic influences leads to the situation that everything in your project can only be described by densities and distributions: start and end of the individual activities, length of the buffers, critical paths etc. Although this seems to be in some way more complicated, it offers the opportunity to apply the whole spectrum of statistical tools, especially multivariate methods.

If one understands the monitoring and controlling process within a project mainly as the comparison between the nominal/target/planned values and the actual/performance values – as it is often seen – this leads to a new situation: *There are no fixed target values, but rather distributions*. Therefore this is in some way a change of paradigm and can no longer be handled by further acting as having mainly a deterministic approach and just adding some “risk features”, as it is done for instance in PERT. Therefore this leads necessarily to totally different techniques to apply. Let us just mention here project management in specific applications like software engineering (c.f. [12]) or R&D (c.f. [13, 14]), where the use of uncertainties is inevitable.

But the approach of using Monte Carlo simulation models and exclusively densities and distributions, offers the possibility of a high flexibility. You can frame conditions, dependencies, and requirements for the parameters of the model by using the capabilities of the syntax of the underlying programming language. Also different scenarios can easily be analyzed or sensitivity analyses for the involved parameters can be performed.

This method can also be expanded to project portfolios. If you want to solve problems where usually techniques like the Critical Chain approach were applied, the use of Monte Carlo simulation models and exclusively assuming distributions for the parameters may offer new and deeper insights into the dynamics within the project portfolio.

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