# **ГАЗОДИНАМІКА**

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### Numerical simulation of droplets deformation and breakup in shearing flows

A mathematical model is presented that describes the deformation of a single drop suspended in another immiscible liquid under shear flow. The deformed droplet is assumed to be in the form of prolate ellipsoid of revolution. The drop deformation is regarded as motion of the centers mass of the half-drops, symmetrical with respect to the drop center. The effects of viscid and capillary forces on the drop deformation accounted for in modeling with the aid of the mechanical Voight's model. A simple criterion for destruction of droplets in shear flows has been obtained. The results of numerical calculations for droplet deformation in shear flows are presented in comparison with experimental data of other authors. It is shown that the model allows the prediction of behavior of deformed drops in shear flows over a wide range of flow regimes and physical parameters of the both liquid phases.

**Introduction.** The problem of drop deformation and breakup in shear and extensional flow is of academic and practical interest and was attracted close attention over the intervening decades. Drop breakup is important for a wide range of engineering, and biomedical applications including production and processing of emulsions, aerosols, and drug delivery systems. In addition to many of the practical concerns, the study of drop deformation remains a classical example of a free-boundary problem in fluid mechanics. From the theoretical point of view, the problem of deformation of drops is extremely complicated. The equations of motion must be solved for flow inside and outside the drop with boundary conditions on their surfaces, the form of which is a priori unknown, but is defined as part of the solution.

Since Taylor's pioneer work in the 1930s [1], there have many valuable results on the deformation and breakup of single Newtonian drops in well-defined flow fields, Reviews [2-4] give useful summaries on both deformation and breakup studies.

Novel useful results presented in resent works on this topic [5-9] are devoted to the modeling and optimization of the emulsification processes in relation to increasing the production efficiency of monodisperse emulsions.

The physical problem is determined by three dimensionless numbers: the drop Reynolds number  $\text{Re} = \rho_c GR^2/\mu_c$ , the capillary number  $Ca = \mu_c GR/\sigma$ , and the viscosity ratio  $\lambda = \mu_d/\mu_c$ . Here, *R* is the undeformed drop radius, *G* is the shear rate,  $\rho_c$  is density of the continuous phase,  $\mu_d$  and  $\mu_c$  are the dynamic viscosities of dispersed and continuous phases, and  $\sigma$  is the interfacial tension between the liquids.

The criteria for the droplet breakup in shear flows are usually associated with a critical capillary number  $Ca_{cr}$  [1-10]. At subcritical capillary numbers ( $Ca < Ca_{cr}$ ) the drop is stabilized in the final shape of prolate ellipsoid. At supercritical capillary numbers ( $Ca > Ca_{cr}$ ) the drop irreversibly stretches and rapidly breaks, forming daughter droplets and smallest fragments. It is known that  $Ca_{cr}$  is a complex function of  $\lambda = \mu_d / \mu_c$ . Both experimental and theoretical studies have focused mainly on the determination of criteria for drop breakup under creeping shear flow conditions ( $Ca_{cr} = f(\lambda)$ ) and size-distribution of drop fragments resulting from breakup at the supercritical flow regime [1, 3-7, 10]. One of the main conclusions from the studies is that the drop breakup becomes difficult when  $\lambda <<1$  and impossible in laminar shear flow when  $\lambda > 4$ 

Numerical simulations of the problem are generally based on a boundary integral method, by which the creeping flow equations inside and outside the drop are transformed into a form that only involves quantities at the drop surface [3, 4, 6, 8]. Droplet breakup was also studied with numerical simulations using a free energy lattice Boltzmann method [9]. In spite of the major accomplishments of numerous studies on the drop deformation and break up in shear flows, however, many important qualitative questions so far remain to be answered. In particular, what are the mechanisms for breakup and how do they depend on the system parameters including the degree of deformation of the drop?

**Formulation of the problem.** One objective of the given study is to examine in detail the time-dependent deformation of a single liquid droplet in shear flows with numerical simulations of the process. The study focuses on a deeper analysis of the critical conditions for the irreversible deformation of droplets, leading to their subsequent destruction. Below, we consider the principles of constructing a mathematical model, which describes the evolution of an initially spherical droplet in shear flow of another liquid over a wide range of flow regimes and physical parameters of the both liquid phases

This model is based on the main points of the previously developed mathematical model [11], which adequately describes the deformation of droplets in nonstationary accelerated liquid and gaseous flows. The basic position of that model is the assumption that at all stages of deformation the drop takes the form of an ellipsoid of revolution. In the model [11], deformation of a droplet was considered as the displacement of the centers of mass of the half-drops, separated by a plane, passing through the center of the drop and orthogonal to the flow velocity direction x. When the centers of the half-drops move along the coordinate x in the direction of the center of the drop, the spherical drop deforms into an oblate ellipsoid, otherwise, into a prolate ellipsoid. These main principles are used in the development of the model of a drop behavior in shear flows,

Let us consider a stationary shear flow of fluid with density  $\rho_c$  and viscosity  $\mu_c$ . The velocity vector of the stream  $\vec{v}$  is directed along the coordinate x, where as the velocity change occurs in the direction y, and the velocity gradient  $G = dv_x/dy = \text{const}$ . Components of the flow velocity vector:  $v_x = G \cdot y$  and  $v_y = 0$ . A drop, immersed in the liquid, moves with the flow in the direction x with a velocity equal to flow velocity  $v_x(y_0)$ , where  $y_0$  is the y-coordinate of the drop center. Combining the origin of the Cartesian coordinates with drop center ( $x_0 = 0$ ,  $y_0 = 0$ ), one can consider the motion of the flow only relative to the drop, regarding it as quiescent. For a shear flow the flow velocity field relative to the drop  $w_x(y - y_0) = v_x(y) - v_x(y_0)$  is symmetrical with respect to the drop center, and is independent of the position of the drop in the stream.

The stream, flowing around the droplet, causes the total dynamic action on the drop of both inertial and viscous friction forces. Obviously, the distribution of pressure along the drop surface must be symmetrical with respect to the origin  $(x_0, y_0)$ . The forces  $F_{\zeta}$  that act on each half-drop are equal in magnitude but opposite in direction.

The change in the drop shape, caused by the forces  $F_{\zeta}$  action, is counteracted by the capillary force  $F_{\sigma}$ , which tends to return the drop to its initial spherical shape, and also by the dissipative forces of the viscosity of the drop itself  $F_{\mu}$ , which are proportional to the rate of deformation. An analogue of this physical model, which takes into account the role of all these factors, is the mechanical Voigt's model, describing the behavior of a visco-elastic body. This model, schematically shown in Fig.1, represents recurrent capillary forces  $F_{\sigma}$  by action of an elastic spring, and dissipative viscous ones  $F_{\mu}$  by a damping element, when both the elements work in parallel. The half-drop mass  $m = 2/3\pi R^3 \rho_d$  is assumed to be concentrated in the geometrical center of the half-drop. The droplet deformation is regarded as the motion of the half-drops in the x direction. Figure 2 shows the main parameters of our model.

The deformed drop is considered as an ellipsoid of revolution with major semiaxis *a* and minor semi-axes  $b = \sqrt{R^3/a}$  (Fig.2). As in the base model [11], the degree of drop deformation is determined by the parameter a/R, which is often used in analyzing droplet deformation in shear flows [2,6,7,9]. In some cases it is more convenient to use the conventional definition D = (a - b)/(a + b) which was the proposed by Taylor [1].

When the droplet is stretched in the direction x the y - coordinate of the center of the half-drop is  $y_s = y_{s0} = \text{const}$ . The distance of the center of mass of each half-drop from the geometrical center of the drop itself is determined by the equation

$$r_s(\tau) = \sqrt{x_s^2(\tau) + y_{s0}^2} = 3a/8.$$
 (1)

The parameter  $r_s(\tau)$  defines the current position of the center of each half-drop. It is not difficult to calculate that  $r_s$  is connected with the length of the semi-axis *a* by the relation  $r_s = 3a/8$ . For undeformed drop (a = R), the position of the center of mass of hemi-sphere is determined by relation  $r_{s0} = 3R/8$ .



Fig.1

Fig.1. Schematic drawing of the droplet deformation in shear flows in the framework of the Voight's mechanical model for a viscous- elastic body.



As follows from Eq. (1), the direction  $\vec{r}$  never coincide with the flow direction x. The orientation of droplets is usually defined by the angle  $\alpha$  between the direction of the vector  $\vec{r}$  and the positive direction of the axis y [1-3, 5, 10]. The angle  $\alpha$ , shown in Fig.2, is important parameter of the model, because the effectiveness of the particular shear flow in deforming a drop is strongly dependent on drop orientation in the flow.

Both experiment and modeling show [1-4, 6, 7, 9] that infinitesimal deformation of an spherical drop in a simple shear flow occurs in the direction of 45° relative to the flow direction  $v_x$ . Hence, the initial coordinates of the center of mass of halfdrops  $x_{s0} = r_{s0} \cdot \sin 45^\circ = 3\sqrt{2R}/16$  and  $y_{s0} = r_{s0} \cdot \cos 45^\circ = 3\sqrt{2R}/16$ . As drop elongated, its principal axis a rotates towards the flow direction x. In accordance with Fig.2, the orientation angle  $\alpha$   $\alpha$  is related to the degree of drop deformation a/Rand the position of  $r_s$  by the expressions:

$$\alpha = \operatorname{arctg}(x_s/y_{s0}) = \operatorname{arccos}(\sqrt{2}(a/R)^{-1}) = (3\sqrt{2}R/8r_s).$$

As noted above, the process of drop deformation is determined by the combined effect of three forces. This are the hydrodynamic force  $F_{\zeta}$ , the capillary force  $F_{\sigma}$  and the viscous force  $F_{\mu}$ . Below we consider the influence of each of these forces on the process of deformation of a drop.

**Hydrodynamic force action.** The force  $F_{\zeta}$  that stretches the drop in the direction x is proportional to the drag of the half-drop to the stream, flowing around the droplet, and is determined by the relation  $F_{\zeta} = \overline{\zeta} \cdot \overline{p}_x S_{yz}$ . Here,  $\overline{\zeta}$  is the drag coefficient averaged over the drop surface,  $\overline{p}_x$  is the hydrodynamic pressure averaged over the surface  $S_{yz}$  and  $S_{yz}$  is the area of the drop projection onto the plane, passing through the center of the drop, and orthogonal to the direction x. This projection is an ellipse with a minor axis b and a major axis  $y_w$ , which is described by equation  $z = b \cdot \sqrt{1 - y^2/y_w^2}$ . Parameter  $y_w$  is the distance from the axis x to the plane XZ that contacts the surface of this ellipsoid (Fig.2). At each point of the drop surface the local pressure  $p_x(y) = \rho_c w_x^2(y)/2 = \rho_c (G \cdot y)^2/2$ . Then the force  $F_{\zeta}$  is defined as follows

$$F_{\zeta} = \int_{S_{xy}} \overline{\zeta} p_x \left( S_{zy} \right) \cdot dS_{zy} = \frac{\overline{\zeta} \rho_c G^2}{2} \int_{0}^{y_w} 2y^2 z(y) \cdot dy = \frac{\overline{\zeta} \cdot \rho_c \pi b \cdot \left( y_w \right)^3 G^2}{8}, \qquad (2)$$

where  $y_w = a \cdot ((R/a)^3 \sin^2 \alpha + \cos^2 \alpha)$ . The coefficient  $\overline{\zeta}$  is valued by the equation

$$\zeta = \frac{3}{2} \cdot \left( \frac{16}{\text{Re}} + \frac{2,2}{\text{Re}^{0.5}} + 0,6 \right) \cdot \left( \frac{1,5\mu_d + \mu_c}{\mu_d + \mu_c} \right).$$
(3)

Reynolds number for shear flows is defined as  $\text{Re} = \rho_c G R^2 / \mu_c$ .

**Viscous force action.** The effect of viscosity forces on the deformation of droplets streamlined by liquid flow has been analyzed in detail in [11], using the tensor equation for energy dissipation in unit volume of a viscous fluid. An equation had been obtained, which describes the rate of viscous energy dissipation  $dE_{\mu}/d\tau$  as a function of viscosity  $\mu_d$ , droplet radius *R*, and velocity gradient  $\nabla v$ . With reference to the problem at issue, the viscosity force is calculated by the equation

$$F_{\mu} = \frac{dE_{\mu}}{dr_s} = \frac{dE_{\mu}}{d\tau} \frac{dr_s}{d\tau} = \frac{4\pi R^3 \mu_{eff}}{r_s^2} \cdot \frac{dr_s}{d\tau}.$$
 (4)

Here  $\mu_{eff} = \mu_d + 0.6\mu_c$  is the effective viscosity, which takes into account the contribution of the attached mass of the continuous phase, adjacent to the drop surface, into the viscous force actions [11].

**Capillary force action.** The capillary force  $F_{\sigma}$  is considered as the ratio of the surface energy increment  $dE_s = \sigma \cdot dS$  caused by drop deformation, to the displacement of the center of mass of the half- drop  $dr_s$ . ( $F_{\sigma} = dE_s/dr_s = \sigma \cdot dS/dr_s$ ). The analysis, carried out in [11], shows that when the spherical droplet is deformed into the shape of an oblate or elongated ellipsoid, the capillary force  $F_{\sigma}$  is determined

from equations:



**Fig.3**. The characteristic change in the capillary force  $F_{\sigma}$  during the droplet transformation into oblate (left branch) and into prolate (right branch) ellipsoid.



**Fig.4**. The photos of sequence of droplet deformation and break-up in shear flow at subcritical (a) and supercritical capillary numbers (b), according to the data of [7].

$$F_{\sigma} = \sigma \frac{dS}{dr_s} = \frac{16\pi a\sigma}{3} \left[ \left(\frac{R}{a}\right)^3 - \frac{1 - 0.25 \cdot (a/R)^3}{e^3} \cdot \ln\left(\frac{1 + e}{1 - e}\right) + \frac{1.5}{e^2} \right]$$
(5a)

for oblate ellipsoid, and

$$F_{\sigma} = \sigma \frac{dS}{dr_s} = \frac{16\pi R^3 \sigma}{3a^2} \left[ \frac{0.5 \cdot (R/a)^{3/2} \left(1 - 4 \cdot (R/a)^3\right) \cdot \arcsin e}{e^3} + \frac{1.5}{e^2} - 1 \right].$$
 (5b)

for prolate ellipsoid, where  $e = \sqrt{1 - b^2/a^2} = \sqrt{1 - R^3/a^3}$  is eccentricity of an ellipse with half-axes *a* and *b*.

**Criteria for droplet destruction**. In the investigations of droplet behavior in shear flows the most difficult and least developed question is the justification of criterion for the transition from the subcritical deformation mode, when the drop stabilizes in the finite form of prolate ellipsoid, to the supercritical mode, when an irreversible elongation of the droplet occurs, resulting in its destruction. As has been specified above, this transition is customary associated with the critical capillary number  $Ca_{cr}$ , which is a very complex and analytically not described function of viscosity ratio  $\lambda$  and Reynolds number.

Within the framework of this model, a simple criterion for the destruction of droplets in shear flows has been obtained.

The dependence of the capillary force on the degree of drop deformation a/R, which has been calculated from the equations (5a) and (5b), is shown in Fig.3.

The data presented reveal an important, previously unknown feature of the capillary force influence on the drop deformation, when a/R > 1. From Fig.3 it can be observed that, irrespective of the physical properties of both liquids, dependence  $F_{\sigma} = f(a/R)$  for elongated drops has a maximum at the strictly determined value  $(a/R)_{cr} = 2,2$ , which can be considered as a physical constant. Exceeding this critical value must necessarily lead to irreversible deformation and the subsequent destruction of the excessively elongated drop even after stopping the dynamic action of the flow [3].

The obtained result convincingly explains the mechanism of the so-called "burst", which denotes the flow conditions, corresponding to the onset of rapid continuous elongation of a droplet. This effect was first observed in the Taylor experiments [1] and was subsequently recorded by other researchers [2-4, 7, 10]. It should be noted that the critical value of the above Taylor deformation parameter D corresponds to  $D_{cr} = 0.53$ .

As an illustration, Fig.4 shows photographs of dimethicone droplets during their successive expansion in the shear creeping flow of castor oil in the subcritical  $(Ca = 0.98Ca_{cr})$  (a) and the supercritical  $(Ca = 1.01Ca_{cr})$  flow regime (b), according to the data of [7]. It can be seen from the photos in Fig.4a that the shape of the stabilized drop corresponds to  $a/R \approx 2.2$ . It should be noted that the analyses of other photographs and graphical data on droplet deformation, given in works of various researcher, also indicates that the shape of droplets, stabilized in the supercritical mode at *Ca* close to  $Ca_{cr}$ , corresponds to  $a/R \approx 2.0 \div 2.3$  [2, 6, 7, 9] or  $D = 0.5 \div 0.55$  [3-5, 10].

**Drop deformation equations.** The current shape of a drop during it stretching depends both on the elongation parameter  $r_s(\tau)$  and orientation angle  $\alpha(\tau)$ . Therefore, the problem of drop evolution is expedient to solve in the polar coordinate system  $(r, \alpha)$ . The deformation of a drop in shear flows is determined by the displacement of  $r_s$  under the combined action of the above forces. The equation of motion of the center of mass of the half-drop has the form

$$m \cdot \frac{d^2 r_s}{d\tau^2} = F_\mu + F_\sigma + F_\zeta = C_\mu \cdot \frac{dr_s}{d\tau} + C_\sigma \cdot r_s + C_\zeta.$$
(6)

The values  $C_{\mu}$ ,  $C_{\sigma}$ , and  $C_{\zeta}$ , are defined, respectively, from equations (4), (5) and (2), taking into account that, in accordance with Eq. (1),  $r_s(\tau) = 3a(\tau)/8$ . The equation (6) is solved with the following initial conditions:  $r_s(0) = 3R/8$ ,  $(dr_s/d\tau)_{\tau=0} = 0$ .

The change of the orientation angle  $\alpha = f(\tau)$  is determined by the equation

$$\alpha = \arccos\left(\left(\sqrt{2} \cdot a/R\right)^{-1}\right) = \left(3\sqrt{2}R/8r_s\right),\tag{7}$$

which is solved jointly with equation (6) with the initial condition  $\alpha(0) = 45^{\circ}$ C.

Eqs. (6) and (7) are the basic equations for the mathematical model considered here. Unlike the most existing models, this rather simple model allows the prediction of behavior of deformed drops both in creeping ( $\text{Re}_{fl} \ll 1$ ) and inertial ( $\text{Re}_{fl} = 0.01 \div 100$ ) shear flows with no adjustable parameters and additional assumptions. This study is limited to modeling the droplets deformation in the region ( $\lambda \le 1$ ).



**Fig.5.** Dependences of drop deformation D (a) and orientation angle  $\alpha$  (b) on the capillary number Ca (at  $\lambda = 0.08$ ), as well as dependence  $Ca_{cr} = f(\lambda)$  for low viscosity ratios ( $\lambda < 1$ ). Quantitative comparison of the simulation results (solid curve) with experimental data of Torza at al. [10] (points) for R = 0.3 mm,  $\sigma = 4.1$  mN/m.

**Results and analysis.** Below, we discuss the results of numerical calculations for various regimes of flow around a liquid drop. A comparison is made with the known experimental data of other authors on the deformation of droplets in shear flows.

To verify the reliability of the above model a computational experiment was carried out that reproduced the conditions of the experiments, performed by Torca et. al. [10]. In those experiments, deformation of castor oil drops ( $R = 0.15 \div 0.8$  mm) in creeping shear flows (Re < 0.001,  $G = 0.1 \div 5 \text{ s}^{-1}$ ) of organic liquids was investigated. During each experiment, a change in the shape of the drop was recorded on the film, and its deformation degree was represented by the parameter D = (a - b)/(a + b). The orientation angles  $\alpha$  were also determined. When comparing those experimental regimes were picked, for which the viscosity ratio values  $\lambda < 1$  have been used.

Figs 5a,b show a comparison of the experimental and theoretical dependencies D = f(Ca) is  $\alpha = f(Ca)$  for one of the regimes. It seen that the experimental points lie reasonably close to the theoretical lines. The model with good accuracy predicts the changes both in the deformation degree D and the orientation angle  $\alpha$  with increasing the capillary number Ca in the indicated range of its variation. This range (Ca < 0.4) refers to the subcritical flow regime  $(Ca < Ca_{cr})$ , and so the values D, shown in Fig 5a, below the deformation critical value  $D_{cr} = 0.53$ .

Those studies had focused primarily on relating the critical capillary number  $Ca_{cr}$  and viscosity ratio  $\lambda$ . There was found that for all the systems studied the dependence  $Ca_{cr} = f(\lambda)$  has a minimum in the region  $0.3 \div 0.9$  [10]. In their experiments the critical number  $Ca_{cr}$  was evaluated as the arithmetic average of a highest subcritical Ca, when the drop does not still break and attains a steady state, and a lowest supercritical Ca, for which the drop breaks into fragments. In our model, the critical capillary number  $Ca_{cr} = \mu_c G_{cr} R/\sigma$  is calculated from Eq.(6) for given values of R,  $\mu_c$  and  $\sigma$ , as a some value Ca = f(G), for which the condition  $a/R = (a/R)_{cr} = 2,2$ 

is satisfied at a certain  $G = G_{cr}$ . Figure 5c shows the experimental [10] and calculated dependences  $\operatorname{Ca}_{cr} = f(\lambda)$ . The results calculated, using the model, are in good agreement with experiment in the range of values  $\lambda$ , for which the accepted assumptions  $\lambda < 1$  are valid.

The paper [10] does not give quantitative data on the time of stabilization of the deformed drop shape or the onset of the "drop burst" moment. It was previously established that in carrying out this type of research, a value  $G^{-1}$  should be used as the base temporal scale [2-4, 10]. For the interval of shear rates *G*, used in [10] (where  $0.1 \div 5 \text{ s}^{-1}$ ), this corresponds exactly to the time scale of seconds.

To assess the ability of this model to describe shear deformation in the wider range of shear rates G, special numerical investigations were performed. The objectives of these investigations were to compare the droplet behavior in the creeping and inertial shear flow regimes, as well as to verify the validity of the assumption that the parameter  $G^{-1}$  can be used as a base temporal scale not only for the Stokes flows around the droplet, but also for the inertial ones.

The choice of model systems was determined by the possibility of varying, to a certain extent, the physical parameters of both the phases ( $\rho_d$ ,  $\rho_c$ ,  $\mu_d$ ,  $\mu_c$ ,  $\sigma$ ) with keeping the condition that  $\lambda < 1$ . The following systems were investigated:

1. Water drops in motor oil ( $\rho_d = 10^3 \text{ kg/m}^3$ ,  $\rho_c = 0.8 \times 10^3 \text{ kg/m}^3$ ,  $\mu_c = 1 \text{ mPas}$ ,  $\mu_c = 200 \text{ mPas}$ ,  $\sigma = 29 \times 10^{-3} \text{ N/m}$ ,  $\lambda = 0,005$ , R = 1 mm).

2. Toluen drop in water ( $\rho_d = 0.8 \cdot 10^3 \text{ kg/m}^3$ ,  $\rho_c = 10^3 \text{ kg/m}^3$ ,  $\mu_d = 0.6 \text{ mPas}$ ,  $\mu_c = 1 \text{ mPas}$ ,  $\sigma = 35 \cdot 10^{-3} \text{ N/m}$ ,  $\lambda = 0.6$ , R = 1 mm).

For both the systems, drop deformations were considered with a gradual increasing the shear rate G, up to an irreversible "burst" extension of the droplet, leading to its destruction. The kinetics of deformation of a drop was determined by the dependences  $a/R = f(\tau)$  and  $\alpha = f(\tau)$ .

The results of the calculation, in the form of kinetic dependencies  $a/R = f(\tau)$  and  $\alpha = f(\tau)$ , are presented in Figs. 6a,b (system 1) and Figs.7a,b (system 2).

In all cases, an increase in the shear rate G necessarily leads to the burst elongation of the drop. The results of the modeling show that this is indeed a "burst" effect, since an insignificant increase in G (of the order of 0.01%) drastically changes the droplet deformation regime (Fig. 6a).

The peculiarity of drop deformation depends qualitatively on the value of Reynolds number. In Fig. 6a,b, where the continuous phase is high viscous motor oil, the hydrodynamic action of the flow on the drop surface is not the inertial, but rather viscous effect (Re < 0.4). The stretching of droplets at low shear rates ( $G < G_{cr}$ ) proceeds monotonically, so does their irreversible elongation at  $G > G_{cr}$ .

In Figures 7a,b, where continuous phase is low-viscous water, the strictly inertial flow regime (700) is realized. This mode is characterized by the presence of damped oscillations of the droplet shape as it stabilizes. Simultaneously with the oscillations of the shape, the oscillations of the drop orientation also occur.



**Fig.6.** Change in the deformation degree a/R (a) and in the orientation angle  $\alpha$  (b) during deformation of water droplets in inertial shear flow of motor oil at different values of the shear rate *G*. Operation parameters:  $\rho_d = 10^3 \text{ kr/m}^3$ ,  $\rho_c = 0.8 \cdot 10^3 \text{ kr/m}^3$ ,  $\mu_d = 1 \text{ mPa/s}$ ,  $\mu_c = 200 \text{ mPa/s}$ ,  $\sigma = 29 \text{ mN/m}$ , R = 1 mm,  $Ca_{cr} = 0.545$ ;  $\lambda = 0.005$ .



**Fig.7.** Change in the deformation degree (a) and in the orientation angle  $\alpha$  (b) during deformation of toluene droplets in inertial shear flow of water. 1 – subcritical regime  $(Ca < Ca_{cr})$ :  $G=938,0 \text{ s}^{-1}$ ; 2 – supercritical regime  $(Ca > Ca_{cr})$ :  $G=938.5 \text{ s}^{-1}$ . Operation parameters:  $R=1 \text{ mm. } \rho_d = 0.8 \cdot 10^3 \text{ kr/m}^3$ ,  $\rho_c = 10^3 \text{ kr/m}^3$ ,  $\mu_d = 0.6 \text{ mPars}$ ,  $\mu_c = 1 \text{ mPars}$ ,  $\sigma = 40 \text{ mN/m}$ ,  $Ca_{cr} = 0.011$ ;  $\lambda = 0.6$ .

Obviously, the physical nature of oscillations in the shape and orientation of lowviscosity liquid droplets in inertial shear flows requires special discussion.

It will be noted that up to present neither experiments nor simulations have been reported for the case of the time-depended drop deformation in inertial shear flows.

According to some researchers, inertia can change the initial stages of droplet deformation [9]. Besides, in the inertial regime, drop deformation occurs under the action of pressure fluctuations, created by the irregular velocity of the fluid [2, 3, 6, 8].

Consideration of the dynamic transition between the initial and final stages of droplet destruction in supercritical shear flows is not included in the tasks of this study.

**Conclusion.** The results of the analysis confirm the reliability of the model and the validity of the physical provisions used in its development. The model is able to predict the character of droplet deformation and the conditions of their destruction in

shear flows with known regime parameters with a greater degree of accuracy than existing empirical relationships. The results of the present study can find industrial applications, such as in creation and processing of emulsions and liquid-liquid dispersions. Knowledge of the conditions, when this breakup mechanism occurs, may improve the efficiency of production of monodisperse emulsion.

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## Численное моделирование деформации и дробления капель в сдвиговых течениях

#### АННОТАЦИЯ

Рассмотрены принципы построения модели, описывающей деформацию капель в сдвиговых потоках, в инерционном и в стоксовском режимах течения, в широком интервале режимных параметров процесса. Модель базируется на допущении, что деформируемая капля имеет форму вытянутого эллипсоида вращения. Обсуждается роль основных факторов, определяющих эволюцию капли в потоке под действием сдвиговых напряжений. Установлен критерий начала перехода к необратимому удлинению капли, приводящему к ее разрушению. Приведены результаты численных расчетов сдвиговой деформации капли при различных физических и режимных параметрах в сравнении с экспериментальными данными других авторов.

# Іваницький Г. К. Чисельне моделювання деформації і руйнування краплин в зсувних течіях

#### АНОТАЦІЯ

Розглянуто математичну модель, що описує деформацію крапель в зсувних потоках у ишрокому діапазоні зміни режимів течії та фізичних параметрів обох рідких фаз. Модель базується на припущенні, що деформована краплина має форму витягнутого еліпсоїда обертання. Деформація краплі розглядається як переміщення центрів маси напівкрапель, симетричних по відношенню до центру краплі. Вплив в'язких і капілярних сил на деформацію краплі розглянуто із застосуванням моделі Фойхта для в'язко пружних середовищ. Отримано простий критерій початку руйнування краплини у зсувних течіях. Наведено результати чисельних розрахунків зсувної деформації краплі при різних фізичних і режимних параметрах в порівнянні з експериментальними даними інших авторів.