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THE DETERMINATION OF DISTRIBUTION PARAMETERS OF ONE-DIMENSIONAL CONTINUOUS RANDOM VARIABLE ACCORDING TO ITS INITIAL CHARACTERISTICS BY FINANCIAL RISKS MODELLING

Abstracts. The conception of direct and inverse problem of random variable modelling is introduced. The direct problem is a problem for getting value of continuous random variable, which is contributed according to the given distribution law, which parameters are known. The inverse problem is a problem for defining the distribution law parameters, which are necessary for modelling of continuous one-dimensional random variable, for which the distribution law, mathematical expectation and dispersion are known. For its solution by known type of distribution it is necessary to find the parameter dependence of simulated distribution on set initial characteristics – ensemble average and standard deviation. The assigned problem is solved in explicit form for the following cases: normal distribution, exponential distribution, Laplace distribution, extreme value minimum distribution, extreme value maximum distribution, double exponential distribution, logistic distribution, gamma distribution, Erlang distribution of n-th order, Rayleigh distribution, Maxwellian distribution, parabolic distribution, Simpson distribution, arc sine distribution, inverse Gaussian distribution , Cauchy distribution, one-parameter distribution of n-dimansional random value, hyperexponential distribution, beta distribution, common- beta distribution, Birnbaum-Sanders distribution.
For random variables, which are distributed according to the laws: Erlang second order, beta-distribution of second order, logarithmic normal distribution, it is described the interactive procedure to solve the modelling inverse problem, which realizes the Newton's method for solution of linear equation system. The expressions for elements of matrix solution are received. The solution procedure of assigned task for Weibull and Nakagami distribution is set, which is based on construction of regressive equations, which interpolate the table values to determine links of distribution law parameters and initial characteristics of random variable, which is distributed according to the given law.

Key words: Monte Carlo method, statistical modeling, probability density function, n, inverse problem of statistic simulation, normal distribution, exponential distribution, Laplace distribution, minimum distribution, maximum distribution, double distribution, logistic distribution, gamma distribution, Erlang distribution of n-th order, Rayleigh distribution, Maxwellian distribution, parabolic distribution, Simpson distribution, inverse sine distribution, inverse Gaussian distribution, Cauchy distribution, one-parameter distribution of n-dimensional random value, hyperexponential distribution, beta distribution, common- beta distribution, Birnbaum-Sanders distribution.

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ВИЗНАЧЕННЯ ПАРАМЕТРІВ РОЗПОДІЛУ ОДНОВИМІРНОЇ БЕЗПЕРЕРВНОЇ ВИПАДКОВОЇ ВЕЛИЧИНИ ПО ЇЇ ПОЧАТКОВИХ ХАРАКТЕРИСТИКАХ ПРИ МОДЕЛЮВАННІ ФІНАНСОВИХ РИЗИКІВ

Анотація. Сформульовано зворотне завдання моделювання безперервної одновимірної випадкової величини. Для її вирішення при відомому типі розподілу необхідно знайти залежність параметрів модельованого розподілу від заданих початкових характеристик - математичного очікування і середнєквадратичного відхилення. Поставлена задача вирішена в явному вигляді для наступних випадків: нормального розподілу, показового розподілу, розподілу Лапласа, розподілу мінімального значення, розподілу максимального значення, подвійного показового розподілу, логістичного розподіл, гамма-розподілу, розподілу Ерланга n-го порядку, розподілу Рэлея, розподілу Максвела, параболічного розподілу, розподіл Симпсона, розподіл арксинуса, зворотного розподілу Гауса, однопараметричного розподілу модуля n-мірної випадкової величини, бета-розподілу, розподілу Бирнбаума-Сандерса. Описано процедура отримання рішення зворотної задачі моделювання для випадкових величин, розподілених по законах: Ерланга другого порядку, бета-розподілу другого роду, логарифмічно нормального розподілу. Запропонована процедура рішення поставленої задачі для розподілів Вейбулла і Накагамі.

Ключові слова: метод Монте-Карло, статистичне моделювання, щільність розподілу безперервної випадкової величини, зворотне завдання статистичного моделювання, нормальний розподіл, показовий розподіл, розподіл Лапласа, розподіл мінімального значення, розподіл максимального значення, подвійний показовий розподіл, логістичний розподіл, гамма-розподіл, розподіл Ерланга n-го порядку, розподіл Рэлея, розподіл Максвела, параболічний розподіл, розподіл Симпсона, розподіл арксинуса, зворотний Гаус-розподілі, розподіл Коші, однопараметричні розподіли модуля n-мірної випадкової величини, гіперекспоненціальний розподіл, бета-розподіл, узагальнений бета-розподіл, розподіл Бирнбаума-Сандерса.

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ОПРЕДЕЛЕНИЕ ПАРАМЕТРОВ РАСПРЕДЕЛЕНИЯ ОДНОМЕРНОЙ НЕПРЕРЫВНОЙ СЛУЧАЙНОЙ ВЕЛИЧИНЫ ПО ЕЁ НАЧАЛЬНЫМ ХАРАКТЕРИСТИКАМ ПРИ МОДЕЛИРОВАНИИ ФИНАНСОВЫХ РИСКОВ

Аннотация. Введено понятие прямой и обратной задачи моделирования случайной величины. Для её решения при известном типе распределения необходимо найти зависимость параметров моделируемого распределения от заданных начальных характеристик - математического ожидания и среднеквадратического отклонения. Поставленная задача решена в явном виде для следующих случаев: нормального распределения, показательного распределения, распределения Лапласа, распределения минимального значения, распределения максимального значения, двойного показательного распределения, логистического распределение, гамма-распределения, распределения Эрланга n-го порядка, распределения аналитические Рэлея, распределения Максвелла, параболического распределения, распределение Симпсона, распределение арксинуса, обратного Гауссовского распределения, однопараметрического распределения модуля n-мерной случайной величины, бета-распределения, распределения Бирнбаума-Сандерса. Описана процедура получения решения обратной задачи моделирования для для случайных величин, распределенных по законам: Эрланга второго порядка, бета-распределения второго рода, логарифмически нормального распределения. Предложена процедура решения поставленной задачи для распределений Вейбулла и Накагами .

Ключевые слова: метод Монте-Карло, статистическое моделирование, плотность распределения непрерывной случайной величины, обратная задача статистического моделирования, нормальное распределение, показательное распределение, распределение Лапласа, распределение минимального значения, распределение максимального значения, двойное показательное распределение, логистическое распределение, гамма-распределение, распределение Эрланга n-го порядка, распределение Рэлея, распределение Максвелла, параболическое распределение, распределение Симпсона, распределение арксинуса, обратное Гауссовское распределении, распределение Коши, однопараметрическое распределения модуля n-мерной случайной величины, гиперэкспоненциальное распределение, бета-распределение, обобщённое бета-распределение, распределение Бирнбаума-Сандерса.

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Introduction. For many years the statistical modeling is one of the most routine methods of determination of financial risks effects. In detail its application for this goal is described in works [1…11]. Statistical modeling technique in different degree of details is described in works [12…15]. Its main and integral part is the procedure of reception of random quantity with designated partition law. In this research the problem will be considered, which arises through the necessity to obtain the finite set of one-dimensional pseudorandom variables, which imitate the order of random variables with designated partition. Let us accept that one-dimensional random variable X is assigned by its density $f(x;\Theta)$, where Θ-is vector of parameters. Accept that this vector length does not rank over two. For example for the random variable X which is distributed according to normal law, we get that:

$$f(x;\theta_1,\theta_2) = \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-\mu)^2}{2\sigma^2}),\ -\infty < x < \infty. \quad (1)$$

In situation (1) parameter $\theta_1 = \mu$ - is location parameter and parameter $\theta_2 = \sigma$ - is scale parameter. As is known from work [16], that $\mu = m$ and $\sigma = s$, where m - is expectation of relative X, s is its standard deviation. To get the normal distribution of random variable X with parameters μ,σ - $N_x(\mu,\sigma)$, using the correlation, which is given at work [16], we obtain:

$$N_x(\mu,\sigma) = \mu + \sigma N_x(0,1).\ . \quad (2)$$

Taking account of the convention of getting the value $N_x(0,1)$, which is given at work [16] and the connection between parameters of this distribution μ,σ and initial characteristics, we get, that:

$$N_x(\mu,\sigma) = m + s\sqrt{\frac{12}{n}}(\sum_{i=1}^{n} r_i - \frac{n}{2}),\ n > 6. \quad (3)$$

In situation (3) it is set, that r_i is i-ic realization of quasirandom, uniformly distributed quantity on the interval [0,1]. Methods of its obtaining are described in detail at works [12…15].

Let us consider a density of model distribution like:

$$f(x) = \lambda e^{-\lambda x}. \quad (4)$$

From work [16] follows that for this distribution $\theta = \lambda = 1/m$. Modeling correlation used for obtaining quasirandom quantity x_i, distributed according to demonstrative law with parameter λ will assume the form:

$$\mathrm{EP}(x_i) = -\frac{1}{\lambda}\ln r_i = -m\ln r_i. \quad (5)$$

The given examples show, that for a start with the modeling process of continuous random quantity it is necessary to choose its distribution law and its parameters, at the same time, usually, at the research beginning only desired values of average (mods, medians) and standard deviations are known.

Problem definition. Within the framework of this work two-parameter distribution will be considered, in other cases it will be mentioned. It is supposed that such dependencies are established:

$$\begin{cases} \theta_1 = g_1(m,s) \\ \theta_2 = g_2(m,s) \end{cases} \quad (6)$$

It is required to get such dependencies:

$$\begin{cases} m = v_1(\theta_1, \theta_2) \\ s = v_2(\theta_1, \theta_2) \end{cases}. \qquad (7)$$

In convention (7) value of a quantity m, s are known before the modeling beginning. At the beginning we examine distributions of random continuous quantity, which admit problem (7) explicit solution.

Publications analysis. In available to authors of this report publications the similar problem statement wasn't discovered. At work [17] this task solution is got by carrying out actuarial calculations.

Received results. At tab. 1 are showed the results of solution of the given problem for distributions, which allow to receive result in explicit form. By compiling this table for the first, second and third columns we used data given at work [16].

Table 1.

The results of solution of inverse problem of statistical modeling for distributions, which allow to receive result in explicit form.

Type of distribution:	Density of distribution	Dependence of distribution parameters on its initial characteristics	Dependence of initial characteristics on distribution parameters
Laplace	$f(x) = \frac{1}{2}\lambda \exp\left(-\lambda\lvert x-\mu\rvert\right),$ $-\infty < x < \infty.$	$\mu = m,$ $s = \frac{\sqrt{2}}{\lambda}.$	$\mu = m,$ $\lambda = \frac{\sqrt{2}}{s}.$
Minimum value distribution	$f(x) = \frac{1}{\lambda}\exp(\frac{x-\mu}{\lambda} - \exp(\frac{x-\mu}{\lambda}))$ $-\infty < x < \infty.$	$m = \mu - \lambda\gamma^{*)},$ $s = \frac{\pi}{\sqrt{6}}\lambda.$	$\lambda = \frac{s\sqrt{6}}{\pi},$ $\mu = m + \gamma\lambda.$
Maximum value distribution	$f(x) = \frac{1}{\lambda}\exp(-\frac{x-\mu}{\lambda} - \exp(\frac{x-\mu}{\lambda})),$ $-\infty < x < \infty.$	$m = \mu + \lambda\gamma,$ $s = \frac{\pi}{\sqrt{6}}\lambda.$	$\lambda = \frac{s\sqrt{6}}{\pi},$ $\mu = m - \gamma\lambda.$
Double distribution	$f(x) = \lambda\mu\exp(-\lambda\mu - \mu e^{-\lambda \cdot x}),$ $-\infty < x < \infty.$	$m = \frac{1}{\lambda}\left(\ln\mu + \gamma\right),$ $s = \frac{\pi}{\lambda\sqrt{6}} = \frac{1{,}2825}{\lambda}.$	$\lambda = \frac{\pi}{s\sqrt{6}},$ $\hat{\mu} = \exp\left(\frac{1{,}2825}{\sigma} - \gamma\right).$
Logistic distribution	$f(x) = \frac{\exp\left(\frac{x-\mu}{\lambda}\right)}{\lambda\left[1+\exp\left(\frac{x-\mu}{\lambda}\right)\right]},$ $-\infty < x < \infty.$	$\mu = m,$ $s = \frac{\lambda\pi}{\sqrt{3}.}$	$\mu = m,$ $\lambda = \frac{\sqrt{3}}{\pi}s = 0{,}5513s.$
Gamma distribution	$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x},$ $x>0$	$m = \frac{\alpha}{\lambda},$ $s = \frac{\sqrt{\alpha}}{\lambda}.$	$\lambda = \frac{m}{s^2},$ $\alpha = \frac{m^2}{s^2}.$

Erlang distribution of n-th order	$f(x0=\frac{\lambda}{(n-1)!}x^{n-1}e^{-\lambda x}$, $x>0.$	$m=n^{**)}/\lambda$	$\lambda=n/m.$
Rayleigh distribution	$f(x)=\frac{x}{a^2}\exp\left(-\frac{x^2}{2a^2}\right)$, $a>0,\ x>0.$	$m=a\sqrt{\frac{\pi}{2}}.$	$a=m\Big/\sqrt{\frac{\pi}{2}}=0,79$
Maxwellian distribution	$f(x)=\frac{2x^2}{a^3\sqrt{2\pi}}\exp(-x^2/(2a^2))$, $x\geq 0.$	$m=2a\sqrt{\frac{2}{\pi}}.$	$a=\frac{m}{2\sqrt{\frac{2}{\pi}}}=1,595$
Parabolic distribution	$f(x)=\frac{6(x-\alpha)(\beta-x)}{(\beta-\alpha)^3}$, $\alpha\leq x\leq\beta.$	$m=\frac{\alpha+\beta}{2}$, $s=\frac{\beta-\alpha}{2\sqrt{5}}.$	$\alpha=m-s\sqrt{5},$ $\beta=m+s\sqrt{5}.$
Simpson distribution	$f(x)=\frac{2}{\beta-\alpha}\left(1+\frac{\lvert\alpha+\beta-2x\rvert}{\beta-\alpha}\right)$, $\alpha\leq x\leq\beta.$	$m=\frac{\alpha+\beta}{2}$, $s=\frac{\beta-\alpha}{\sqrt{24}}.$	$\alpha=m-2.4498s,$ $\beta=m=2.4498s.$
Arcsine distribution	$f(x)=1/\left[\pi\sqrt{\lambda^2-(x-\mu)}^2\right]$, $\mu-\lambda<x<\mu+\lambda.$	$\mu=m,$ $s=\lambda/\sqrt{2}.$	$\mu=m,$ $\lambda=s\sqrt{2}.$
Inverse Gaussian distribution	$f(x)=\sqrt{\frac{c\mu}{2\pi x^3}}\exp\left[\frac{c(x-\mu)^2}{2\mu x}\right]$, $x>0.$	$m=\mu,$ $s=\frac{\mu}{\sqrt{c}}.$	$m=\mu,$ $c=\frac{m}{s^2}.$

Note: $^{*)}\gamma$ - Euler constant, γ=0,5772; $^{**)}$ n- order of simulated system, that supposed to be known before the start of modeling.

Let us consider the solving of the given problem for one-parameter distribution module of n-dimensional random quantity, which view and numerical characteristics are given in work [16]. Density of this distribution has form:

$$f(x)=\left[2^{(n/2)-1}a^n\Gamma\left(\frac{n}{2}-1\right)\right]^{-1}x^{n-1}\exp(-x^2/(2a^2)),\ x>0. \tag{8}$$

In convention (8) is accepted that quantity n - is positive, integer and known before the beginning of modeling, parameter a>0. Mathematical expectation of this distribution is:

$$m=a\cdot B, \tag{9}$$

therefore:

$$a=m/B.. \quad (10$$

In turn:

$$B=\begin{cases}\sqrt{\frac{\pi}{2}}\cdot\frac{(n-1)!!}{(n-2)!!}, & n=2k\\ \sqrt{\frac{2}{\pi}}\cdot\frac{(n-1)!!}{(n-2)!!}, & n=2k-1\end{cases}\quad k=1,2,...n \tag{11}$$

Let us consider the solving of the given problem for modeling of continuous random quantity, which is distributed in concordance with beta – distribution, which view and numerical characteristics are given in work [16].

$$f(x) = \frac{x^{u-1}(1-x)^{v-1}}{B(u,v)} = \frac{\Gamma(u+v)}{\Gamma(u)\Gamma(v)} x^{u-1}(1-x)^{v-1}, 0 < x < 1; u > 0, v > 0. \quad (12)$$

Numerical characteristics of this distribution are connected with its parameters form requirements [16]:

$$m = \frac{u}{u+v}, \quad (13)$$

$$s^2 = \frac{uv}{(u+v)^2(u+v+1)}. \quad (14)$$

Solving (13) and (14) relative to u and v we will receive, that:

$$u = -\frac{m(m^2 - m + s^2)}{s^2}, \quad (15)$$

$$v = \frac{(m-1)(m^2 - m + s^2)}{s^2}. \quad (16)$$

The cited above distributions can be applied to more or less famous distributions. Then let us consider less common Birnbaum-Sanders distribution, which is used by modeling insurance risks, connected with life assurance. Work [18] contains information about it. Density of this distribution has the form:

$$f(x) = \frac{\sqrt{\frac{x}{\alpha}} + \sqrt{\frac{\alpha}{x}}}{2\beta x} \cdot \phi\left(\frac{1}{\beta}\left(\sqrt{\frac{x}{\alpha}} - \sqrt{\frac{\alpha}{x}}\right)\right), \quad \alpha > 0, \beta > 0, x > 0. \quad (17)$$

Given (21) that:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), \quad (18)$$

so it is density of standard normal distribution. At the system definition STATGRAPHICS v. XV.I [19] is given modeling ratio to get random variables, which are distributed according to Birnbaum-Sanders law with parameters α, β:

$$X_{b-s} = \frac{\left((N_x(0,1) \cdot \beta + \sqrt{4N_x(0,1)\beta^2}\right)\alpha}{4}. \quad (19)$$

Numerical characteristics of Birnbaum-Sanders distribution, given at system STATGRAPHICS v. XV.I definition are:

$$m = \alpha\left(1 + \frac{\beta^2}{2}\right), \quad (20)$$

$$s^2 = (\alpha\beta)^2\left(1 + \frac{5\beta^2}{4}\right). \quad (21)$$

Considering conventions (20) and (21) as a system of equations concerning variables α and β and accepting, that:

$$A = m^2 + 3s^2; \quad (22)$$

we will get the solution like:

$$\alpha_1 = \frac{\sqrt{A} + 4m}{3}; \ \beta_1 = \frac{\sqrt{2} \cdot \sqrt{A + m^2 - 5s^2}}{\sqrt{s^2 - 5m^2}}; \quad (23)$$

$$\alpha_2 = \frac{\sqrt{A}+4m}{3}; \qquad \beta_2 = -\beta_1; \tag{24}$$

$$\alpha_3 = \frac{4m-\sqrt{A}}{3}; \quad \beta_3 = \beta_1; \tag{25}$$

$$\alpha_4 = \frac{4m-\sqrt{A}}{3}. \qquad \beta_4 = -\beta_1 \tag{26}$$

Then let us consider distribution of random continuous quantities, for which problem solution under desired conditions (7) in explicit form is impossible.

In this situation for solution of given problem Newton method for system solution of two nonlinear equations is used, in the view, which is described at work [20]. Let us represent the system (7) like:

$$\begin{cases} F = F(\lambda,\mu) = & m - q_1(\lambda,\mu) = 0; \\ G = G(\lambda,\mu) = & s - q_2(\lambda,\mu) = 0. \end{cases}. \tag{27}$$

Jacobian for system (31) assumes the form:

$$J(\lambda,\mu) = \begin{vmatrix} \frac{\partial F}{\partial \lambda} & \frac{\partial F}{\partial \mu} \\ \frac{\partial G}{\partial \lambda} & \frac{\partial G}{\partial \mu} \end{vmatrix}. \tag{28}$$

Determinant:

$$\Delta_\lambda = \begin{vmatrix} F & \frac{\partial F}{\partial \mu} \\ G & \frac{\partial G}{\partial \mu} \end{vmatrix}. \tag{29}$$

Determinant:

$$\Delta_\mu = \begin{vmatrix} \frac{\partial F}{\partial \lambda} & F \\ \frac{\partial G}{\partial \lambda} & G \end{vmatrix}. \tag{30}$$

Successive values of roots of this system are adduced:

$$\begin{cases} \lambda_{n+1} = \lambda_n - \frac{\Delta_\lambda^{(n)}}{J(\lambda_n,\mu_n)}, \\ \mu_{n+1} = \mu_n - \frac{\Delta_\mu^{(n)}}{J(\lambda_n,\mu_n)}. \end{cases} \tag{31}$$

Hereinafter, where it will not rise misunderstanding index number of iteration n is skipped.

Let us consider the solving of the given problem as applied to general Erlang distribution of the second order, which features are described at work [16].

Density of this distribution has such form:

$$f(x) = \frac{\lambda\eta}{\lambda-\mu}\left(e^{-\mu x} - e^{-\lambda x}\right), \; x \ge 0. \tag{32}$$

Parameters of this distribution $\lambda > 0, \eta > 0$ are connected with mathematical expectation m, dispersion D and standard deviation s equalities:

$$m = \frac{\lambda+\mu}{\lambda\mu}, \tag{33}$$

$$D = \frac{\lambda^2+\mu^2}{(\lambda\mu)^2}, \tag{34}$$

$$s = \frac{\sqrt{\lambda^2 + \eta^2}}{\lambda\mu}. \tag{35}$$

Using conventions (35) instead convention (34) simplifies problem solving and does not increase difficulties by choosing initial data for the solving of the given problem.
Convention (27) will have in such case the form:

$$\begin{cases} F = F(\lambda,\mu) = \quad m - q_1(\lambda,\mu) = m - \dfrac{\lambda+\mu}{\lambda\mu} = 0; \\ G = G(\lambda,\mu) = \quad D - q_2(\lambda,\mu) = D - \dfrac{\lambda^2+\mu^2}{(\lambda\mu)^2} = 0. \end{cases} \tag{36}$$

Jacobian (28) for system (31) assumes the form:

$$J(\lambda,\mu) = \begin{vmatrix} \dfrac{\partial F}{\partial\lambda} & \dfrac{\partial F}{\partial\mu} \\ \dfrac{\partial G}{\partial\lambda} & \dfrac{\partial G}{\partial\mu} \end{vmatrix} = \begin{vmatrix} \dfrac{1}{\lambda^2} & \dfrac{1}{\mu^2} \\ \dfrac{2}{\lambda^3} & \dfrac{2}{\mu^3} \end{vmatrix} = \begin{vmatrix} A & B \\ C & E \end{vmatrix} = \frac{2(\lambda-\mu)}{(\lambda\mu)^3}. \tag{37}$$

Let us note form determinant (29) for system (36), taking into account the convention (37) like:

$$\Delta_\lambda = \begin{vmatrix} F & B \\ G & E \end{vmatrix} = \frac{\lambda^3(2m\mu-1) - 2\lambda^2\mu + \lambda\mu^2 - 2\mu^2}{\lambda^3\mu^4}. \tag{38}$$

Let us note form determinant (30) for system (36), taking into account the condition (37) like:

$$\Delta_\mu = \begin{vmatrix} A & F \\ C & G \end{vmatrix} = \frac{\mu^2(d\lambda^2 - 2m\lambda + 1) + 2\lambda\mu - \lambda^2}{\lambda^4\mu^2}. \tag{39}$$

Let us take that:

$$U_n = \left(\frac{\Delta_\lambda}{J(\lambda,\mu)}\right)_n = \left(\frac{\lambda^3(2m\mu-1) - 2\lambda^2\mu + \lambda\mu^2 - 2\mu^2}{\lambda^3\mu^4}\right)_n; \tag{40}$$

$$W_n = \left(\frac{\Delta_\mu}{J(\lambda,\mu)}\right)_n = \left(\frac{\mu\left[\lambda^3 + 2\lambda^2\mu(m\mu-1) - \lambda\mu^2 - 2\mu^2\right]}{2\lambda^2(\mu-\lambda)}\right)_n. \tag{41}$$

Then iterative procedure for inverse solution for general Erlang distribution of the second order will have the form:

$$\lambda_{n+1} = \lambda_n - U_n; \tag{42}$$

$$\mu_{n+1} = \mu_n - W_n. \tag{43}$$

Initial value λ_0, μ_0, using the references given at work [16] and accepted, that :

$$Z = \sqrt{2D - m^2}, \tag{44}$$

we will get as convention:

$$\lambda_0 = \frac{m \pm Z}{D \pm mZ}, \tag{45}$$

$$\mu_0 = \frac{m \pm Z}{m^2 - D}. \tag{46}$$

Let us consider the solving of the given problem as applied to beta-disrtibution of the second kind, which features are described at work [16].
Density of this distribution has the form:

$$f(x) = \frac{1}{B(u,v)} \cdot \frac{x^{u-1}}{(1+x)^{u+v}} = \frac{\Gamma(u+v)}{\Gamma(u)\Gamma(v)} x^{(u-1)}(1+x)^{-(u+v)},\ x > 0; u > 0, v > 0. \tag{47}$$

Mathematical expectation of the considered distribution has the form:

$$m = \frac{u}{v-1}, v > 1. \quad (48)$$

Dispersion of beta-disrtibution of the second kind has the form:

$$D = \frac{u(u+v-1)}{(v-1)^2 (v-2)}, v > 2. \quad (49)$$

Conditions (27) will have in this case the form:

$$\begin{cases} F = F(u,v) = m - \dfrac{u}{v-1} = 0 & ; \\ G = G(u,v) = D - \dfrac{u(u+v-1)}{(v-1)^2 (v-2)} = 0 & . \end{cases} \quad (50)$$

Jacobian (28) for system (27) will assume the form:

$$J(u,v) = \begin{vmatrix} \dfrac{\partial F}{\partial u} & \dfrac{\partial F}{\partial v} \\ \dfrac{\partial G}{\partial u} & \dfrac{\partial G}{\partial v} \end{vmatrix} = \begin{vmatrix} A & B \\ C & E \end{vmatrix} = \frac{u(u+v-1}{(1-v)^3 (v-2)^2}. \quad (51)$$

Jacobian value is received under condition:

$$A = \frac{\partial F}{\partial u} = \frac{1}{1-v}; \quad B = \frac{\partial F}{\partial v} = \frac{u}{(v-1)^2}; \quad C = \frac{\partial G}{\partial u} = \frac{2u+v-1}{(2-v)(v-1^2)}; \quad (52)$$

$$E = \frac{\partial G}{\partial v} = \frac{u\left[u(3v-5) + 2v^2 - 5v + 3\right]}{(v-2)^2 (v-1)^3}. \quad (53)$$

Let us note form determinant (29) for system (27), taking into account the condition (28)- (30) like:

$$\Delta_u = \begin{vmatrix} F & B \\ G & E \end{vmatrix} = \frac{u^3(3-2v)}{(v-2)^2 (v-1)^4} + \frac{u^2[m(3v-5)+v-1]}{(v-2)^2 (v-1)^3} - \frac{u[D(v-2)^2 + m(3-2v)]}{(v-1)^2 (v-2)^2}. \quad (54)$$

Let us note form determinant (30) for system (27), taking into account the condition (28 -30), like:

$$\Delta v = \begin{vmatrix} A & F \\ C & G \end{vmatrix} = \frac{u^2 + 2mu(v-1) - (v-1)^2 (Dv - 2D - m)}{(v-2)(v-1)^3}. \quad (55)$$

Taking into account the condition (28), we will get, that:

$$U_n = \left(\frac{\Delta_u}{J(u,v)} \right)_n = \left(\frac{(v-1)(v-2)[D(v-2)+m-1]}{u+v-1} + \frac{u(2v-3)}{1-v} + m(5-3v) + v - 2 \right)_n ; \quad (56)$$

$$W_n = \left(\frac{\Delta_v}{J(u,v)} \right)_n = \left(\frac{(v-2)[D(v-2)(v-1)^2 + m(1-v)(2u+u-1) - u^2]}{u(u+v-1)} \right)_n . \quad (57)$$

Then iterative procedure of search of inverse solution for beta-distribution of the second kind will have the form:

$$u_{n+1} = u_n - U_n; \quad (58)$$

$$v_{n+1} = v_n - W_n. \quad (59)$$

Initial value u_0, v_0, using the references given at work [16] we will get as conventions :

$$v_0 = \frac{m(m+1)}{D} + 2 ; \quad (60)$$

$$u_0 = m\left(\frac{\sqrt{D}}{m} - 1 \right) . \quad (61)$$

Let us consider the solving of the given problem for normal logarithmic distribution, which characteristics are described at work [16].

Density of this distribution has the form:

$$f(x) = \frac{1}{ux\sqrt{2\pi}} \exp\left\{ -\frac{[\ln(x/v)]^2}{2u^2} \right\}, x > 0. \quad (62)$$

Mathematical expectation of the considered distribution has the form:

$$m = v \exp\left(\frac{u^2}{2} \right). \quad (63)$$

Variance of logarithmically normal distribution has the form:

$$D = v^2 \exp(u^2)\left[\exp(u^2) - 1\right]. \quad (64)$$

Conventions (27) in this case assume the form:

$$\begin{cases} F = F(u,v) = m - v\exp\left(\frac{u^2}{2} \right) = 0 & ; \\ G = G(u,v) = D - v^2 \exp(u^2)\left[\exp(u^2) - 1\right] = 0 & . \end{cases} \quad (65)$$

Jacobian (28) for system (65) assumes the form:

$$J(u,v) = \begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} \end{vmatrix} = \begin{vmatrix} A & B \\ C & E \end{vmatrix} = -2uv^2 \exp\left(\frac{5u^2}{2} \right). \quad (66)$$

Jacobian value (66) is provided that:

$$A = \frac{\partial F}{\partial u} = -vu\exp\left(\frac{-u^2}{2} \right); \quad B = \frac{\partial F}{\partial v} = \exp\left(\frac{-u^2}{2} \right); \quad (67)$$

$$C = \frac{\partial G}{\partial u} = 2uv^2 \exp(u^2)(1 - 2\exp(u^2)); \quad E = \frac{\partial G}{\partial v} = 2v\exp(u^2)(1 - \exp(u^2)). \quad (68)$$

Let us note form determinant (29) for system (65), taking into account the condition (66)- (68) like:

$$\Delta_u = \begin{vmatrix} F & B \\ G & E \end{vmatrix} = \exp\left(\frac{u^2}{2} \right)\left[v^2 \exp(2u^2) - 2mv\exp\left(\frac{3u^2}{2} \right) - v^2 \exp(u^2) + 2mv\exp\left(\frac{u^2}{2} \right) + d \right]. \quad (69)$$

Let us note form determinant (30) for system (65), taking into account the condition (66)-(68) like:

$$\Delta v = \begin{vmatrix} A & F \\ C & G \end{vmatrix} = -uv\exp\left(\frac{u^2}{2} \right)\left[3v^2 - 4mv\exp\left(\frac{3u^2}{2} \right) - v^2 \exp(u^2) 2mv\exp\left(\frac{u^2}{2} \right) + d \right]. \quad (70)$$

Taking into account the condition (31), we will get, that:

$$U_n = \left(\frac{\Delta_u}{J(u,v)} \right)_n = \left[\frac{m\exp\left(-\frac{u^2}{2} \right)}{uv} + \frac{\exp(-u^2) - 1}{2u} - \frac{m\exp\left(-\frac{3u^2}{2} \right)}{uv} - \frac{d\exp(-2u^2)}{2uv^2} \right]_n ; \quad (71)$$

$$W_n = \left(\frac{\Delta_v}{J(u,v)} \right)_n = (T_n - K_n), \ . \quad (72)$$

where:

$$T_n = \left[\frac{3v}{2} + \frac{d\exp(-2u^2)}{2v} + m\exp\left(-\frac{3u^2}{2} \right) \right]_n ; \quad (73)$$

$$K_n = \left[\frac{v\exp(-u^2)}{2} - 2\exp\left(-\frac{u^2}{2} \right) \right]_n . \quad (74)$$

Then iterative procedure of search of inverse solution for logarithmically normal distribution will have the form:

$$u_{n+1} = u_n - U_n; \tag{75}$$

$$v_{n+1} = v_n - W_n. \tag{76}$$

Initial value u_0, v_0, using the references given at work [16] and accepted, that coefficient of variation:

$$\vartheta = \frac{\sqrt{D}}{m} = \sqrt{\exp(u^2 - 1)} \ : \tag{77}$$

we will get in the form of conditions:

$$v_0 = \frac{m}{\sqrt{1+\vartheta^2}} \ ; \tag{78}$$

$$u_0 = \sqrt{\ln\left(1+\vartheta^2\right)} \ . \tag{79}$$

The considered examples are representative because of combined equations (27), which solve the given problem, contain forms, which components are elementary functions. In many cases for dependence expression of initial characteristics of random variable on its distribution parameters are used the expressions, which contain special functions. In this case solving of the given problem for each type of distribution turns into independent task. The problem solving is greatly simplified by using methods based on construction of interpolational dependences. Let us consider the possible methods of its solution in terms of Weibull and Nakagami distributions.

Solution of the given problem for Weibull distribution, which features are described at work [16]. Density of this distribution has the form:

$$f(x) = \frac{v}{u}\left(\frac{x}{u}\right)^{v-1} \exp\left[-\left(\frac{x}{u}\right)^{v}\right] . \tag{80}$$

Mathematical expectation m of random variable, which is distributed by Weibull law, has the form:

$$m = u\Gamma\left(\frac{1}{v}+1\right). \tag{81}$$

For problem solving of parametrization of Weibull distribution, which correspondent to predetermined numerical, the next procedure is used.

1. Initial data were mathematical expectation m and standard deviation s as in previous problems.
2. Using convention (50) we defined a quantity of expected coefficient of variation ϑ.
3. According to data from tab. 3.1, given at [6, p.151], we got interpolation equations $v = \psi(\vartheta)$.
4. Through convention (54) we defined quantity

$$u = m \cdot \left[\Gamma\left(\frac{1}{v}+1\right)\right]^{-1} . \tag{82}$$

Table 2

Interpolation equations for defining a quantity of form of v Weibull distribution according to coefficient of variation ϑ.

Form of a model	Boundary applicability of a model	Model measurement *)		
		Rad	Pv	MAE
$v = (0{,}8641 + 1{,}4767\ln\vartheta)$	$1{,}053 \le \vartheta \le 15{,}83$	0,9983	$< 1\cdot 10^{-4}$	0,040
$C = (-0.049 + 1.0523\vartheta)^{-1}$	$0{,}363 \le \vartheta \le 1$	0,9999	$< 1\cdot 10^{-4}$	0.001
$C = \exp\left(3{,}7915 - 4{,}5365\sqrt{\vartheta}\right)$	$0{,}12 \le \vartheta \le 0.316$	0,9963	$< 1\cdot 10^{-4}$	0,013
$C = \exp(0{,}0741 - 1{,}051\ln\vartheta)$	$0{,}06 \le \vartheta < 0{,}12$	0,9998	$< 1\cdot 10^{-4}$	0,002

Note: Rad- adjusted R-squared; Pv- quantity of error of first kind, MAE – average absolute error of interpolation.

It should be especially considered the procedure of function evaluation

$$g(v) = \Gamma\left(\frac{1}{v} + 1\right). \quad (83)$$

To solve the problem of parametrization of Nakagami distribution, which correspondents to predetermined numeric, we used the next procedure. Calculation of value of form function (83) is easy using almost all basic mathematical packets. The problem will become complicated if the user tries to solve it by using such distributed pack EXCEL. Calculation of gamma-function as wired subprogram is provided by version of EXCEL-2013, this procedure is absent at previous versions. In such case for calculation of convention numerical value (56), it should be used the expression given at work [21, p.151]:

$$g_1(v) = \Gamma\left(\frac{1}{v} + 1\right) \approx 1 - 0{,}427(v-1)v^{-1{,}9}. \quad (84)$$

To check expression 5 (57) applicability for function calculation, which is specified by condition (57), are made calculations, which results are given at tab.3.

Table 3

Comparison of values of function $\Gamma\left(\frac{1}{v} + 1\right)$, which are determined by specialized subprogram (function g(v)) and by approximation formula (function g(v))

ϑ	0,2	0,24	0,28	0,32	0,36	0,40	0,44	0,48	0,50	0,55	0,60
v	15,834	9,248	6,304	4,727	3,771	3,141	2,697	2,370	2,236	1,965	1,758
$g(v)$	0,967	0,948	0,930	0,915	0,903	0,895	0,889	0,886	0,886	0,886	0,890
$g_1(v)$	0,966	0,948	0,931	0,916	0,905	0,896	0,889	0,886	0,885	0,885	0,889

From the given results follows, that the convention (84) at the basic range of changing operation of variation coefficient ϑ gives good value approximation, which is indicated at convention (83).
Let us consider the solving of the given problem for Nakagami distribution, which characteristics are described at work [16]. Density of this distribution has the form:

$$f(x) = \frac{2}{\Gamma(\alpha)}\left(\frac{\alpha}{\beta}\right)^{\alpha} x^{2\alpha-1} \exp\left(-\frac{\alpha x^2}{\beta}\right), \quad \mathrm{x} > 0, \beta > 0. \quad (85)$$

Mathematical expectation is defined through:

$$m = \frac{\Gamma\left(\alpha + \frac{1}{2}\right)}{\Gamma(\alpha)}\sqrt{\frac{\beta}{\alpha}}. \quad (86)$$

The procedure by solving the given problem is following.

1. Initial data were mathematical expectation m and standard deviation *s* as in previous problems.
2. Using convention (50) we defined a quantity of expected coefficient of variation ϑ.
3. According to data from tab. 3.3, given at [16, p.179], we got interpolation equations

$$\alpha(\vartheta) = \left(0.0515 + \frac{0.4904}{\vartheta}\right)^2, Rad = 0{,}9990, \ Pv < 1 \cdot 10^{-4}, \ \mathrm{MAE}=0.003 \quad (87)$$

4. Substituting expression (87) in (86) and resolving it according to parameter *u* we get that:

$$\beta = \left[\frac{m\Gamma(\alpha^*) \cdot \sqrt{\alpha^*}}{\Gamma\left(\alpha^* + \frac{1}{2}\right)}\right]^2. \quad (88)$$

In convention (88) upper character $(*)$ stands for numerical value of α parameter is defined according to convention (87).

Conclusions.

1. The explicit solution of inverse problem of continuous one-dimensional random variable modeling is defined. For its solution by known type of distribution it is necessary to find the parameter dependence of simulated distribution on set initial characteristics – ensemble average and standard deviation.
2. The assigned problem is solved in explicit form for the following cases: normal distribution, exponential distribution, Laplace distribution, extreme value minimum distribution, extreme value maximum distribution, double exponential distribution, logistic distribution, gamma distribution, Erlang distribution of n-th order, Rayleigh distribution, Maxwellian distribution, parabolic distribution, Simpson distribution, arc sine distribution, inverse Gaussian distribution, Cauchy distribution, one-parameter distribution of n-dimansional random value, hyperexponential distribution, beta distribution, common- beta distribution, Birnbaum-Sanders distribution.
3. The solution procedure of modeling inverse problem of random variables, which are distributed according to the laws: Erlang second order, beta-distribution of second order, logarithmic normal distribution, is described.
4. The solution procedure of assigned task for Weibull and Nakagami distribution is suggested.
5. Received results may be used by numerical simulation of random variable estimation of financial risk effects.

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