УДК 532.595

Bifurcation dynamics of pipeline with liquid

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We investigate dynamics of pipeline with liquid with one fixed end and free another one in a vicinity of the so-called critical velocity of liquid flow. It was shown that classical representation about critical velocity should be supplemented by new interpretations. We investigate bifurcation modes of pipeline behavior and show that there are new alternative positions of dynamic equilibrium of pipeline conveying liquid, which enables new stable modes of pipeline behavior in a vicinity of critical velocities and when liquid velocity exceeds its critical values.

Keywords: pipeline, critical velocity, bifurcation, alternative equilibrium positions, motion for velocities, which exceeds critical value.

Introduction. Objective of the present article consists in investigation of loss of stability of the first normal mode of oscillations of the fixed-end pipeline with liquid in a vicinity of the first critical velocity and system behavior for velocity of flows, which exceeds critical value of velocity. It is necessary to note that well-known result, obtained by V. Feodosiev [1], was based on one-mode problem statement, which neglects the Coriolis forces. Expression for critical velocity was derived from the criterion that frequency degenerates to zero. In this case centrifugal forces are in balance with elastic forces. Really this critical velocity corresponds to loss of stability of rectilinear shape of pipeline. Investigation shows that there is new dynamical equilibrium state for pipeline, which we call alternative one, in vicinity of which further development of oscillations occurs. If we consider two-modes model, eigenfrequency of oscillations of the first normal mode dies not coincide with its partial frequency, on the basis of which critical velocity was specified in [1]. For correction of the first critical velocity and investigation of system behavior for velocities, which exceed the critical one, we shall consider three systems, namely, the one-mode system (similar to [1]), the two-modes system without the Coriolis forces and the two-modes system with considering the Coriolis forces.

1. Investigated object

We consider pipeline with conveying liquid. Pipeline is considered within the model of a beam with one fixed end and another end is free. We consider denotations of the article [2], where ρ and μ are linear densities of liquid and beam material, *EJ* is beam stiffness, F is square of beam cross-section, P is internal pressure of liquid, V is liquid velocity, which is supposed to be given. If we use the of method modal decomposition for displacements of pipeline points $u(x,t)=\sum_{i=1}^{N} A_i(x)c_i(t)$ (here $A_i(x)$ are normal modes of oscillations) we obtain the following discrete model for pipeline with liquid [2, 3]

$$\ddot{c}_r N_r + \frac{1}{2}\sum_{ijk}\ddot{c}_i c_j c_k\, d^2_{jkir} + \sum_{ijk}\dot{c}_i\dot{c}_j c_k\left(d^2_{jkir} - \frac{1}{2}d^2_{krij}\right) + \frac{EJ}{\rho+\mu}c_r N_r \kappa_r^4 +$$
$$+\frac{EJ}{\rho+\mu}\sum_{ijk}c_i c_j c_k\, d^6_{ijkr} + \frac{EF}{2(\rho+\mu)}\sum_{ijk}c_i c_j c_k d^4_{ijkr} + \frac{13}{4}\frac{\rho V^2}{\rho+\mu}\sum_{ijk}c_i c_j c_k d^4_{ijkr} -$$
$$-\frac{7}{2}\frac{\rho V^2}{\rho+\mu}\sum_i c_i\beta^2_{ir} + \frac{2\rho V}{\rho+\mu}\sum_i \dot{c}_i\left(\beta^1_{ir} - \beta^1_{ri}\right) + \frac{PF}{\rho+\mu}\sum_i c_i\beta^2_{ir} + \frac{2\rho\dot{V}}{\rho+\mu}\sum_i c_i\beta^1_{ir} = 0. \quad (1)$$

Here $N_r, d^2_{jkir}, d^6_{ijkr}, d^4_{ijkr}, \beta^2_{ir}, \beta^1_{ir}$ are quadratures of $A_i(x)$, κ_r^4 is frequency parameter. This system was used further for numerical investigation of processes, and its linear approximation was used for stability analysis.

Fig. 1 shows dependency of the first eigenvalue λ_1 on ratio of liquid velocity to its critical value (dimensionless velocity) for one-mode model (analog of the result [1]).

For dimensionless velocity of liquid less than 1 the first frequency (here it is partial frequency) λ_1 has only imagine part (dashed line), i. e., only oscillatory process occurs. After passing the critical value 1 ($V = V^1_{kp}$) bifurcation of oscillations occurs, λ_1 becomes real, so motion of pipeline becomes aperiodic with respect to the first normal mode of oscillations, rectilinear state of pipeline becomes to be unstable.

If we consider two-modes models of the pipeline and neglect nonlinear part of equations and Coriolis forces, then two first equations can be reduced to the matrix form $A\cdot\begin{bmatrix}c_1''\\ c_2''\end{bmatrix} = B\cdot\begin{bmatrix}c_1\\ c_2\end{bmatrix}$. Eigenvalues are determined for the matrix $A^{-1}\cdot B$. In this

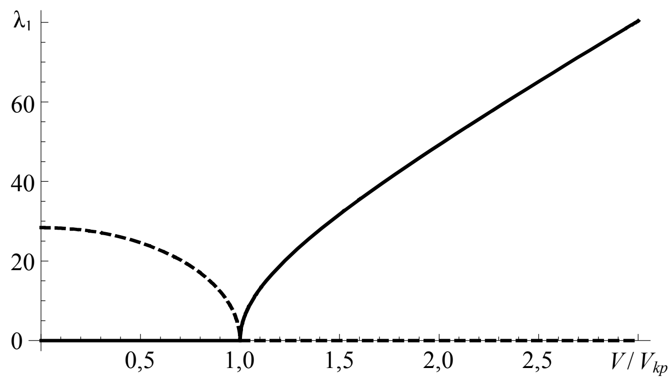

Fig. 1. Dependency of λ_1 on dimensionless velocity of liquid

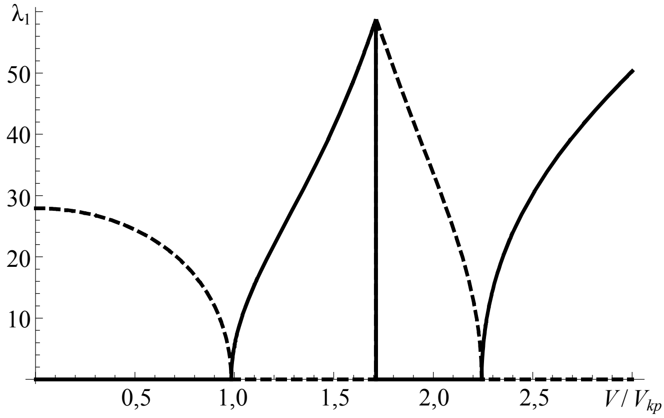

Fig. 2. Dependency of eugenvalues λ_i on dimension velocity
for two-modes model without Coriolis forces

case dependency of eigenvalues λ_1 and λ_2 on dimensionless velocity of liquid is shown in Fig. 2.

Figure shows that the first bifurcation occurs for insignificantly lower velocity $V = 0,993 V_{kp}^{1}$. However two additional bifurcation points appear, for $V = 1,7 V_{kp}^{1}$, after which process again becomes periodic, and for $V = 2,23 V_{kp}^{1}$, which corresponds loss of stability with respect to the second normal mode (the second critical velocity).

For two-modes model of pipeline with liquid, which consider the Coriolis forces we reduce the motion equations to the matrix form $A \cdot \begin{bmatrix} c_1' \\ c_2' \\ v_1' \\ v_2' \end{bmatrix} = B \cdot \begin{bmatrix} c_1 \\ c_2 \\ v_1 \\ v_2 \end{bmatrix}$ and find eigenvalues for the matrixes $A^{-1} \cdot B$. Fig. 3 shows dependence of λ_1 on dimensionless velocity

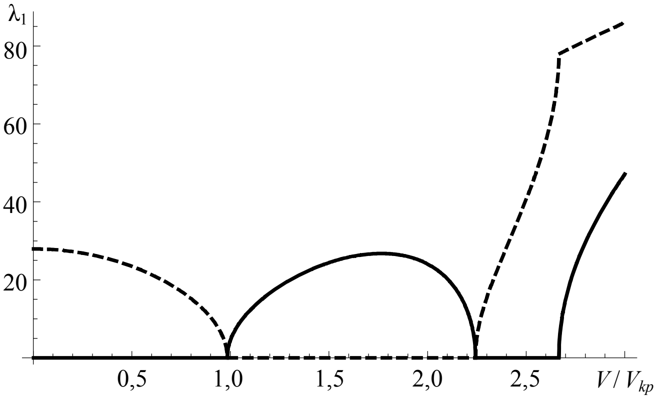

Fig. 3. Dependency of eugenvalues λ_i on dimension velocity
for two-modes model with considering the Coriolis forces

of liquid. The presence of the Coriolis forces results in considerable displacement to the right of two bifurcation points from their positions, shown in Fig. 2. Moreover, even after the second critical velocity $V = 2{,}66V_{kp}^{1}$ imaginary part of λ_1 is nonzero, i. e., process has oscillatory character. So, in the interval $V = 2{,}25 \div 2{,}66V_{kp}^{1}$ we have pure oscillatory process for both modes of motion, which was not predicted before using models without the Coriolis forces.

For validation of these qualitative results we consider numerical solution of the problem of pipeline dynamics within the framework of the nonlinear model (1), which takes into account 12 normal modes of oscillations as well as the Coriolis forces. In Fig. 3 $V = 0{,}9V_{kp}^{1}$ corresponds to *a*, $V = 1{,}5V_{kp}^{1}$ corresponds to *b*, $V = 2{,}56V_{kp}^{1}$ corresponds to *c*, $V = 2{,}7V_{kp}^{1}$ corresponds to *d*.

After passing the first bifurcation point the initial rectilinear equilibrium state of pipeline becomes unstable. However owing to system nonlinearity dynamical system passes to alternative stable equilibrium position and performs oscillations in its vicinity (Fig. 4*b*). Here the fist normal mode performs oscillations relative to alternative equilibrium position, while the second normal mode performs oscillation in a vicinity of the rectilinear state. Since bifurcation happened relative to the first normal mode, therefore we analyze further only amplitude of the first normal mode. In this case we specify the shape of alternative equilibrium state as $u(x) = A_1(x)\cdot C_{eq}(V)$, where normal mode is determined as $A_1(x) = U(k_1x) - \dfrac{S(k_1l)}{T(k_1l)}V(k_1x)$ for cantilever beam; here *S*, *T*, *U*, *V* are the Krylov functions) [3], $C_{eq}(V)$ amplitude of the first normal mode of pipeline in the position of alternative equilibrium depending on liquid velocity. We look for $C_{eq}(V)$ on the basis of one-mode system. So, we obtain the following nonlinear equation, which neglects the Coriolis forces

$$c_1''(t) + c_1(t)\left(-\frac{7\rho V^2}{2}\beta_{1,1}^2 + EJ\,\beta_{1,1}^3 + PF\beta_{1,1}^2 \right) +$$
$$+c_1^3(t)\left(EJ\cdot d_{1,1,1,1}^6 + \frac{1}{2}EFd_{1,1,1,1}^4 + \frac{13}{4}\rho V^2 d_{1,1,1,1}^4 \right) = 0\,.$$

For alternative equilibrium state $c_1''(t) \equiv 0$, so we get

$$c_1(t)\left(-\frac{7\rho V^2}{2}\beta_{1,1}^2 + EJ\,\beta_{1,1}^3 + PF\,\beta_{1,1}^2 \right) +$$
$$+c_1^3(t)\left(EJd_{1,1,1,1}^6 + \frac{1}{2}EFd_{1,1,1,1}^4 + \frac{13}{4}\rho V^2 d_{1,1,1,1}^4 \right) = 0\,.$$

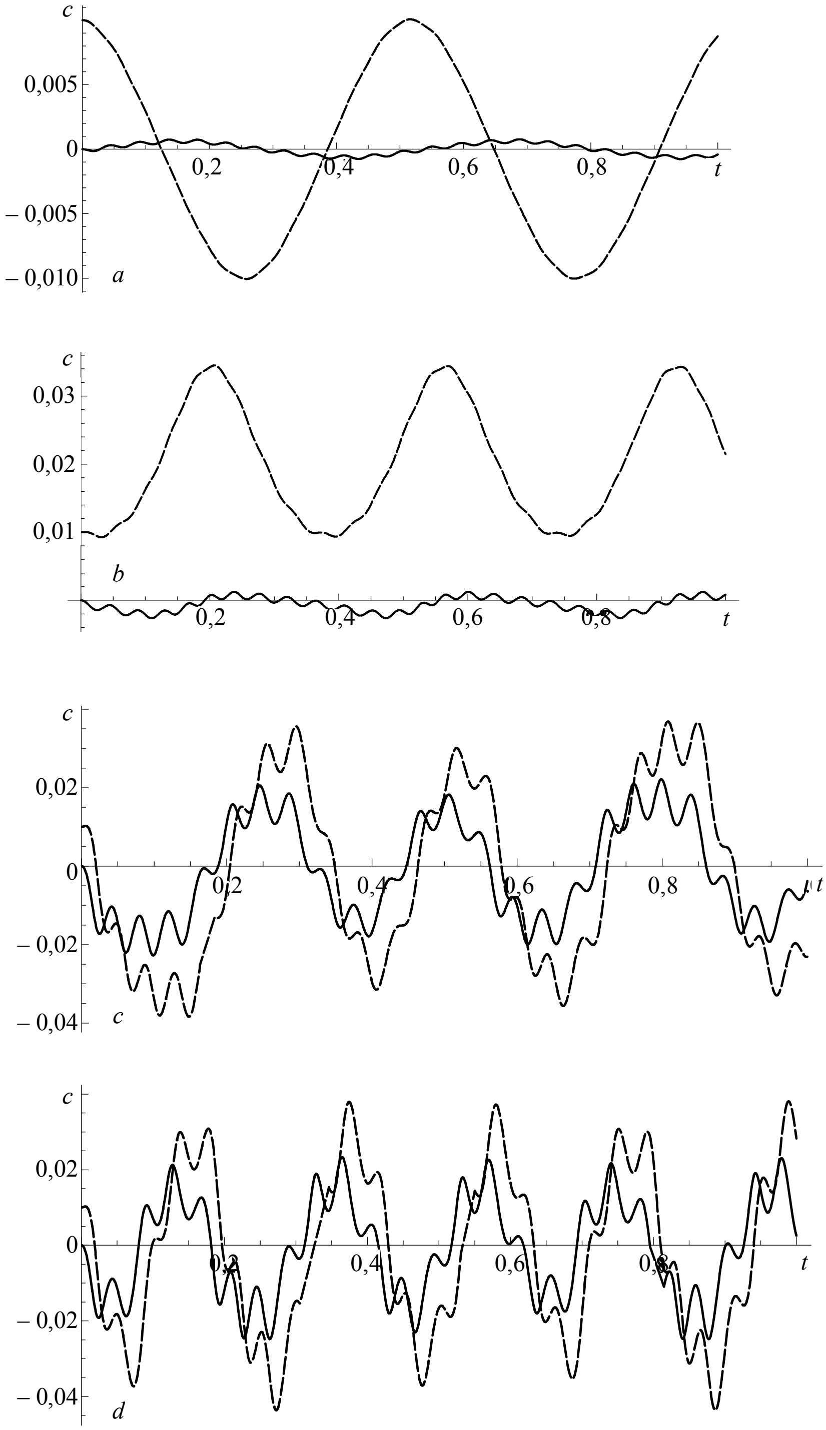

Fig. 4. Amplitudes of two first normal modes for multidimensional nonlinear model, $c_1(t)$ — dashed line, $c_2(t)$ — solid line

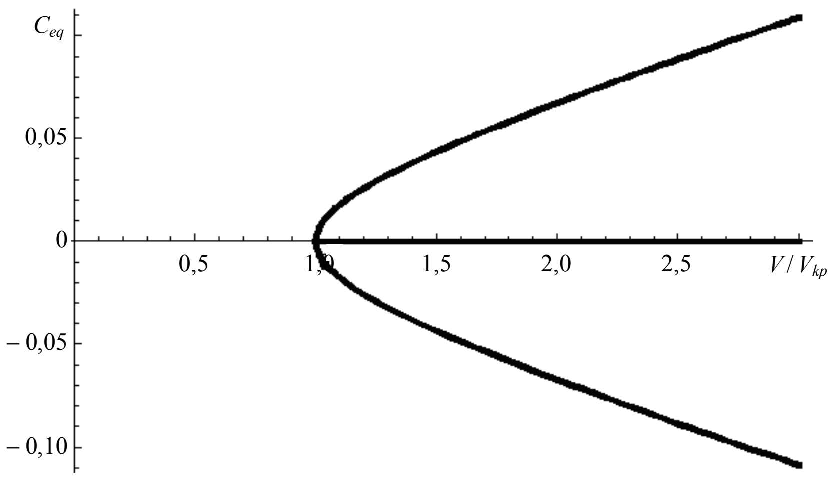

Рис. 5. Dependence of three values of amplitudes
for equilibrium positions on dimensionless velocity

Roots of this cubic equation gives us values of $C_{eq}(V)$. Fig. 5 shows graphs of three roots $C_{eq}(V)$ depending on dimensionless liquid velocity. Upper and lower branches are symmetrical, middle position corresponds to rectilinear shape, which is unstable in the case when liquid velocity exceeds value of the critical velocity of flow.

Fig. 6 shows dependence of dynamic equilibrium states of pipeline for different dimensionless velocities of liquid flows, which values are shown from the right of every curvilinear shapes of pipelines. So, this investigation shows that stable flow of liquid in pipeline can be realized for velocities, which exceed critical velocity of liquid flow, determined according to [1], but in this case pipeline will perform oscillations in a vicinity of new alternative dynamical position.

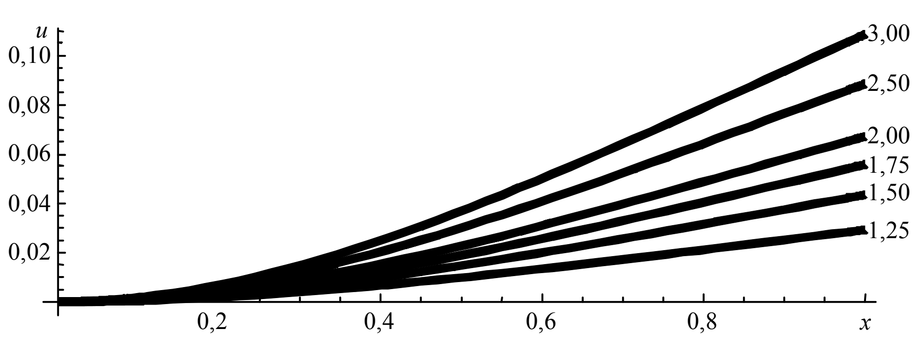

Fig. 6. Alternative dynamical equilibrium positions of pipeline
for different dimensionless velocities of liquid flow

Conclusion. We investigate behavior of pipeline conveying liquid in a vicinity of critical velocity of liquid flow and for velocities, which exceed critical velocity. Investigation showed that influence of the Coriolis forces is decisive for formation of the bifurcation modes of motion of the system. We found alternative positions of dynamic equilibrium states, in vicinity of which pipeline will perform stable oscillations for velocities exceeding critical value of liquid velocity. Results of qualitative analysis were confirmed by numerical examples, obtained on the basis of nonlinear multidimensional model. The obtained results proved that for liquid velocities greater than its critical value, determined in [2], stable motion is possible, but it will occur not in a vicinity of rectilinear shape of pipeline, but with respect to new alternative equilibrium position, which depends on liquid velocity.

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Біфуркаційна динаміка трубопроводу з рідиною

Олег Лимарченко, Олексій Тімохін

Досліджується динаміка трубопроводу з рідиною з одним закріпленим та іншим вільним кінцями в околі, так званої, критичної швидкості течії рідини. Показано, що класичне уявлення про критичну швидкість має бути доповнене новою інтерпретацією. Досліджено біфуркаційні режими поведінки трубопроводу та показано існування нових альтернативних положень динамічної рівноваги трубопроводу з рідиною, що тече, які допускають нові стійкі режими поведінки трубопроводу в околі критичних швидкостей і для швидкостей, які перевершують критичні значення.

Бифуркационная динамика трубопровода с жидкостью

Олег Лимарченко, Алексей Тимохин

Исследуется динамика трубопровода с жидкостью с одним закрепленным и другим свободным концом в окрестности так называемой критической скорости течения жидкости. Показано, что классическое представление о критической скорости должно быть дополнено новой интерпретацией. Исследованы бифуркационные режимы поведения трубопровода и показано существование новых альтернативных положений динамического равновесия трубопровода с протекающей жидкостью, которые допускают новые устойчивые режимы поведения трубопровода в окрестности критических скоростей и для скоростей, превосходящих критические значения.

Представлено доктором технічних наук Я. П'янилом Отримано 22.11.12