

On an approach to solution of problems of porous bodies drying

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In the processes of drying of porous thin-walled plates even without the application of external mechanical forces problems of fracture arise because of cracks initiation losses of the form stability. It is possible to help this by heat-moisture treatment of porous material. For this purpose it is necessary to know the critical efforts arising at this or that way of drying and time of stability loss of the porous body form. To determine the forces arising at the expense of nonuniform distribution of moisture content over the thickness of a body, it is necessary to know change of relative saturation over the thickness with time on which moisture content is defined. At neglect of air-streams over pores, by the decision of corresponding Stefan problems, the basic results of the relative saturation change by moisture at natural convective and electroosmotic drying of a porous layer on the thickness in time. These results will be further used for definition of the critical relative saturation, critical forces and time of stability loss of the porous plates form.

Keywords: natural, convective, electro-osmotic drying, relative moisture saturation of the pore.

Introduction. The formalism of thermodynamics of irreversible processes, being an important tool in the study of transport processes in porous media by means of methods of continuum mechanics provides a complete type of determining relations and thermodynamic flows which are included in the system of differential equations of heterogeneous system to be modelled.

The porous body is considered to be a thermodynamical system in which solid particles of the system construct a capillary-porous body and its pores are filled with particles of moisture and air. Under the influence of external changes of the temperature, the moisture concentration and the pressure, the heat and mass transfer processes emerge accompanied by deformation processes. The gaseous phase is assumed to be homogeneous and transformations of the kind of phase passages can occur at the expense of capillary forces (i. e. of the differences of pressures in capillaries).

In real materials the properties of the mutual distribution of phases are defined both by a function of the distribution of characteristics structure of individual elements

and by parameters those reflect the relationship of these elements. The influence of porous structure is taken into account by introducing the effective coefficients of the binary interaction into the Stefan-Maxwell equation [1]. Effective coefficients of transport are determined by random geometry of porous space, by nature of microinhomogeneities, or empirical relationships that connect them with the parameters of the porous structure. The simplest model of the porous space of polydisperse porous body is a system of cylindrical capillaries of an ideal contact. In such model, if the filled pores of the radius are humidifying with fluid, then the smaller pores are filled too. It follows from this, that a critical radius of filled pores corresponds to each value of saturation, here movable coordinate of phase passage.

In such model if a radius pores r_f are filled with moistening liquid then necessarily and pores of smaller radius are filled. It follows from this, that critical radius of the filled pores $r_f(\bar{z}_m)$, where L_m — dynamic coordinate of phase transition, corresponds to each value of a saturation $\alpha_m = \bar{z}_m = L_m/L$.

In our further consideration, we assume that the transverse of pores is significantly greater than mean free path of available molecules. This allows us the expressions of air \vec{j}_a and vapor \vec{j}_v flows of two-component mixture in the area drained to record as follows:

$$\vec{j}_k = \rho_k \vec{v} - D' \vec{\nabla} \rho_k, \quad k = a; v,$$

where ρ_a, ρ_v are the air and vapor densities, respectively; D' is the effective coefficient of the binary diffusion in pores. Due to the fact that the specific volume of air is much greater than the specific volume of vapour let us neglect the air flow in pores. With these assumptions, isothermal equations of Stefan-Maxwell for binary gas mixture in the drained area are derived, namely

$$\rho_a \vec{v} - D' \vec{\nabla} \rho_a = 0, \quad \vec{\nabla} \cdot (\rho_v \vec{v} - D' \vec{\nabla} \rho_v) = 0. \quad (1)$$

Average velocity \vec{v} satisfies the Darcy equation

$$\vec{v} = -\frac{K_g}{\mu_g} \vec{\nabla} P_g, \quad (2)$$

where K_g, μ_g, P_g is the material penetrability to gas, gas viscosity and pressure.

Using the state equation for the gas mixture and the Darcy law (2), the equation of Stefan-Maxwell is recorded relative to key functions ρ_a, ρ_v . The boundary value problems are formulated for the case of natural convective and electro-osmotic dryings [2-4].

1. Natural drying of porous body

Based on the model of the natural drying, the dependences of the relative saturation of vaporizing fluid on time are constructed:

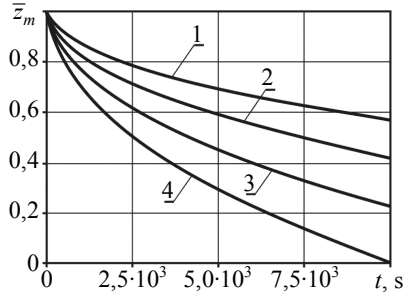


Fig. 1. Dependence of the relative humidity on drying time for different values of temperature when $\delta = 0,0001$ m

$$t = \frac{1}{\Omega} \left[(0,5\Gamma_0 - B) z_m^* \right], \quad (3)$$

where

$$\Omega = \left\{ b \left[1 + \frac{b}{2(1+a)} \right] - (C_0 - 1) \left[1 + \frac{C_0 - 1}{2(1+a)} \right] \right\} \frac{D'_1 M_v P}{RT\delta},$$

$$C_0 = \frac{P_{g1}}{RT} \frac{M_a}{\rho_{a1}} = 1 + \frac{\rho_{v1}}{M_v} \frac{M_a}{\rho_{a1}}, \quad B = 1 + \frac{C_0 - 1}{1+a} + \Gamma_0,$$

$$\Gamma_0 = C_0 \frac{L_0}{\delta} \frac{D'_1}{D'}, \quad a = \frac{D' M_a \mu_g}{K_g \rho_{a1} RT}, \quad b = \frac{\rho_n M_a}{\rho_{a1} M_v};$$

D'_1, D' are effective diffusion coefficients in adjacent layer and pores; R, T are the absolute gas constant and absolute temperature; M_a, M_v are the molecular weight of air and vapour; ρ_a, ρ_v are densities of air and vapour; $z_m^* = 1 - z_m$ is relative moisture saturation being vaporizing, or $\bar{z}_m^* = \alpha_m^*$ is the change of the relative humidity during the drying process; z_m is relative saturation with fluid.

The distributions of vapour density and vapour flow in the pores are determined depending on parameters of the problem, namely the thickness of the adjacent layer δ and the body temperature, diffusion coefficients, filtration and viscosity of the vapour-air mixture in the pores

In Fig. 1 the curves 1-4 depict the change of the relative humidity in time at temperature 300, 310, 320, and 330 K, respectively for $L_0 = 0,5$ m.

2. Convective drying

For the model of convective drying, where the influence of the adjacent layer is taken into account in the convective mass transfer conditions

$$\rho_v \frac{K_g}{\mu_g} \frac{\partial P}{\partial z} + D' \frac{\partial \rho_v}{\partial z} = -j, \quad \rho_a = \rho_{a0}$$

at the surface $z = L_0$, $j = \beta'(\rho_v - \rho_0)$; at the surface $z = L_m$, $\rho_v = \rho_n$; β' is the mass transfer coefficient; ρ_n is the density of the saturated vapour at a certain temperature, ρ_0 is the vapour density outside the porous layer, we obtain the following relationship between the relative saturation of the vaporizing fluid and time:

$$t = \frac{1}{2HB} \left\{ \ln \left| \frac{2B\chi - S}{2B\sqrt{U} - S} \right| + \frac{1}{2} [S^2 - 4B^2U] \left[\frac{1}{(2B\chi - S)^2} - \frac{1}{(2B\sqrt{\chi}U - S)^2} \right] \right\}. \quad (4)$$

Here $\sqrt{U + S\bar{z}_m^* + B^2\bar{z}_m^{*2}} = \chi + \bar{z}_m^*B$, $A = \frac{(1+a)}{b}$, $B = \beta^* A$, $\beta^* = \frac{L_0\beta'}{D'}$, $U = (A+1)^2$, $S = 2B(A + \eta_0)$, $a_1 = A + \eta_0$, $H = \beta'\rho_n$, β' is coefficient of mass transfer; η_0 is relative saturation of the external medium.

Computational results based on the model of convective drying are shown in Fig. 2, 3. The results show that with the increase of the mass transfer coefficient and temperature and with the decrease of vapour density of the medium, the relative humidity of the porous layer decreases.

Since the mass transfer coefficients are connected with the blow-off velocity by the Reynolds criterion, which is included in the parameter B , then the total drying time is minimized in terms of the parameter B and the humidity of the drying medium η_0 . Meanwhile, two equations to determine B and η_0 for the case of the stationary temperature distribution are obtained

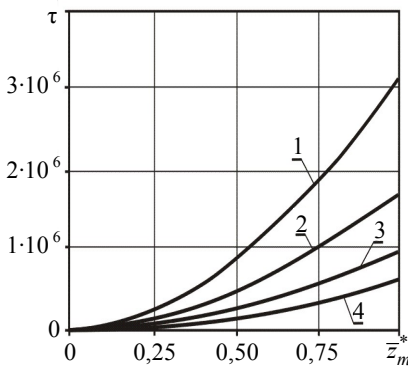


Fig. 2. The change of in time \bar{z}_m^* . Curves 1-4 correspond to temperatures 300; 310; 320; 330 K

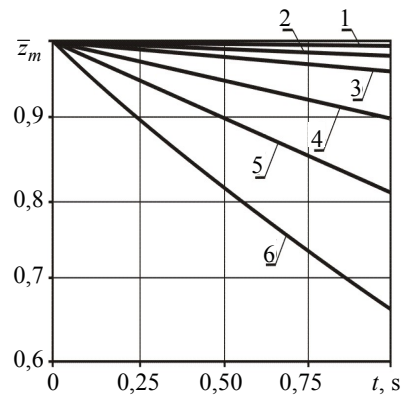


Fig. 3. Dependence $\bar{z}_m = \alpha_m$ on $\beta = 0,0001$; 0,0005; 0,001; 0,0025; 0,005; 0,01

$$\begin{aligned}
 & \left\{ \ln \frac{\chi_0 - (A + \eta_0)}{(1 - \eta_0)} + \frac{1}{2} \left[(A + \eta_0)^2 - (A + 1)^2 \right] \left[\frac{1}{[\chi_0 - (A + \eta_0)]^2} - \frac{1}{(1 - \eta_0)^2} \right] \right\} - \\
 & - \frac{B}{2} \left\{ (1 - \eta_0) - \frac{[(A + \eta_0)^2 - (A + 1)^2]}{[\chi_0 - (A + \eta_0)]^2} \right\} \frac{1}{(\chi_0 + B)} = 0; \\
 & - \left\{ \frac{1}{[\chi_0 - (A + \eta_0)]} - \frac{[(A + \eta_0)^2 - (A + 1)^2]}{[\chi_0 - (A + \eta_0)]^3} \right\} \left(\frac{\chi_0}{\chi_0 + B} \right) + \\
 & + \left\{ \frac{1}{\eta_0} + (A + \eta_0) \left[\frac{1}{[\chi_0 - (A + \eta_0)]^2} - \frac{1}{(1 - \eta_0)^2} \right] - \right. \\
 & \left. - \left[(A + \eta_0)^2 - (A + 1)^2 \right] \frac{1}{(1 - \eta_0)^3} \right\} = 0. \tag{5}
 \end{aligned}$$

By the parameters B and η_0 , taking into account $B = \beta * A = \frac{L\beta'}{D} A$, we find

$$\beta' = \frac{BD}{LA} = \frac{Nu'D}{L} = \frac{D}{L} 0,63 \text{Re}^{0,52} = \frac{D}{L} 0,63 \left(\frac{\nu L}{\nu} \right)^{0,53} \quad \text{or} \quad \frac{B}{A} = 0,63 \left(\frac{\nu L}{\nu} \right)^{0,53}.$$

From this equation, the velocity of the drying medium can be found. It minimizes the drying time and, therefore, the proper parameters of fans, by means of those this velocity is realized.

3. Electro-osmotic drying

Drying of a porous body can be also performed with the help of electroosmose. It was considered unilateral and bilateral dryings of the porous layer [3], in particular, strengthening draining of porous layer by electroosmose, at the one surface of this layer the free evaporation into the medium occurs, and the second surface is replenished by moisture.

Let us consider a porous layer initially saturated with the moisture, one of the surfaces of whose (surface 1) contacts with the medium, which is a mixture of air and vapour; surface 2 contacts with the well penetrable humid medium. The temperature of the air and the layer are the same.

Since the vapour at the surface of the fluid in the pores is saturated and in the medium surrounding the porous layer is unsaturated, then from the surface 1 the vapour flows outwards. As a result, the area of drained pores filled with a mixture of air and vapour emerges. These pores are to be considered as individual components of the

filling gas. During drying, this area extends into the depth of the body. To intensify the drying process by electro-osmotic removal of moisture from the porous layer, the constant difference of potentials between the surfaces of the layer 1 and 2 is provided.

Caused by the electric field, the directional movement of electric charges of the diffusive part of electrical double layer emerges. Besides, the movement of the fluid layer along the surface of the pores (electroosmos) occurs [3].

Due to the available well penetrable humid medium, from the side of the surface 2 the great amount of the moisture is absorbed by the porous layer in way of capillary absorption. Capillary absorption caused by the capillary pressure gradient is the cause of the emergence of filtering flow j_2 . As a result of the total flows forces arising during such drying, the change of the relative humidity in the porous layer occurs. Note, that if in the porous body model to neglect by the the dispersion of pore sizes, then the relative humidity α_m defined as above will coincide with the phase interface $\bar{z}_m = L_m/L_0$. Based on the electroosmose flow model, the corresponding Cauchy problem is formulated in order to determine the law of the relative humidity change in the body with time taking into account the convection-diffusive, capillary and electro-osmotic flows and the influence of the electric field intensity on the change of the relative saturation with moisture in the body is shown. The dependences between time and relative saturation with moisture in the body are defined. For unilateral drying it is obtained the following

$$\frac{1}{2} \frac{a_{32}}{a_{21}} (\bar{z}_m^2 - 1) + \frac{a_{42}}{a_{21}} (\bar{z}_m - 1) + \left(-\frac{a_{42}a_{11}}{2a_{21}^2} + \frac{a_{41}}{2a_{21}} \right) \ln |f_2(\bar{z}_m)| + f(\bar{z}_m) = t, \quad (6)$$

where

$$f(\bar{z}_m) = \left(\frac{a_{42}(a_{11}^2 - 2a_{21}a_{01})}{2a_{21}^2} - \frac{a_{41}a_{11}}{2a_{21}} \right) f_1(\bar{z}_m),$$

$$f_1(\bar{z}_m) = \frac{2}{\sqrt{4a_{21}a_{01} - a_{11}^2}} \left(\operatorname{arctg} \frac{2a_{21}\bar{z}_m + a_{11}}{\sqrt{4a_{21}a_{01} - a_{11}^2}} - \operatorname{arctg} \frac{2a_{21} + a_{11}}{\sqrt{4a_{21}a_{01} - a_{11}^2}} \right),$$

$$|f_2(\bar{z}_m)| = \left| \frac{a_{21}\bar{z}_m^2 + a_{11}\bar{z}_m + a_{01}}{a_{21} + a_{11} + a_{01}} \right|,$$

if

$$4a_{21}a_{01} > a_{11}^2; \quad f_1(\bar{z}_m) = \frac{1}{\sqrt{a_{11}^2 - 4a_{21}a_{01}}} \ln |f_3(\bar{z}_m)|,$$

where

$$f_3(\bar{z}_m) = \frac{2a_{21}\bar{z}_m + a_{11} - \sqrt{a_{11}^2 - 4a_{21}a_{01}}}{2a_{21}\bar{z}_m + a_{11} + \sqrt{a_{11}^2 - 4a_{21}a_{01}}} \frac{2a_{21} + a_{11} + \sqrt{a_{11}^2 - 4a_{21}a_{01}}}{2a_{21} + a_{11} - \sqrt{a_{11}^2 - 4a_{21}a_{01}}},$$

if

$$a_{11}^2 > 4a_{21}a_{01}.$$

Here $\tilde{K} = \frac{K_L}{\mu_L \bar{R}} \frac{2\sigma_{Lg}}{L_0^2}$, $\tilde{K}_1 = \frac{K_L}{\mu_L} \frac{2\bar{\eta}\varepsilon_p E}{\bar{R}L_0}$, $a_{01} = \tilde{K}B$, $a_{12} = B$, $a_{21} = \Omega(1 - K_2) + \tilde{K}\Gamma_0(1 - K_2) - \tilde{K}_1\Gamma_0$, $a_{11} = -\Omega - \tilde{K}B(1 - K_2) - \tilde{K}\Gamma_0 + \tilde{K}_1B$, $a_{32} = \Gamma_0(1 - K_2)$, $a_{22} = -B(1 - K_2) - \Gamma_0$, $a_{42} = a_{22} - \frac{a_{32}a_{11}}{a_{21}}$, $a_{41} = a_{12} - \frac{a_{32}a_{01}}{a_{21}}$, that is reflected in the

tables 1 and 2.

Table 1

The account of capillary accumulate ($t = 10^4$ s, $L_0 = 0,1$ m)

	δ	$T_1 = 300$ K	$T_2 = 310$ K	$T_3 = 320$ K	$T_4 = 330$ K
$\alpha_m = \bar{z}_m$	0,0001	0,573	0,428	0,254	0,082
	0,0010	0,591	0,445	0,270	0,091
	0,0100	0,727	0,590	0,416	0,209
	0,1000	0,967	0,933	0,875	0,784

Table 2

Influence of intensity of electric field on relative humidity at $E = 200$ V/m

	δ	$T_1 = 300$ K	$T_2 = 310$ K	$T_3 = 320$ K	$T_4 = 330$ K
$\alpha_m = \bar{z}_m$	0,0001	0,561	0,415	0,242	0,075
	0,0010	0,577	0,432	0,257	0,083
	0,0100	0,705	0,569	0,396	0,193
	0,1000	0,914	0,873	0,810	0,718

In Fig. 4 and 5, the dynamics of the relative humidity dependence on time for $\delta = 0,001$ m and $\delta = 0,1$ m are reflected (the curves 1-4 correspond to $T = 300, 310, 320, 330$ K respectively).

Thus, the intensity of drying rather depends on the thickness of the boundary layer. With the 100-times increase of the thickness of the boundary layer the relative humidity can increased in 10 or more times. For the thickness of the boundary layer $\delta = 0,1$ m and for temperature 300 K, 310 K, 320 K, the relative humidity during approximately 3 hours decreases no more than by 10%. The electric fields of the intensity $E = 200$ V/m and $E = 400$ V/m (which are not of the great magnitude) oppose of the fluid leakage through the surface 2 and lead to the resultant decrease in relative humidity till 40%.

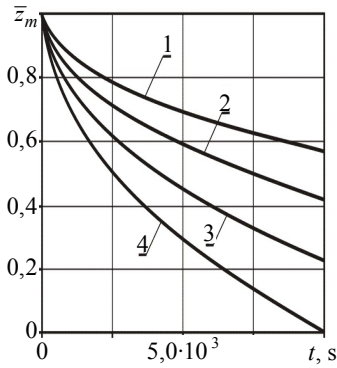


Fig. 4. Dependence of the relative humidity on time at the various temperatures for $\delta = 0,001$ m

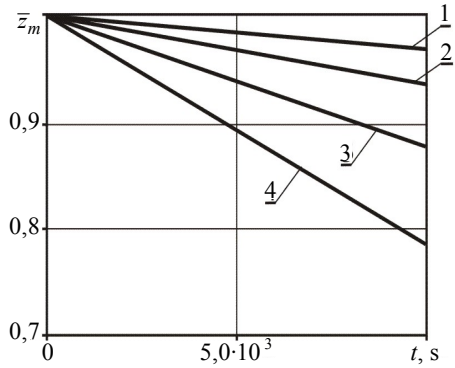


Fig. 5. Dependence of the relative humidity on time at the various temperatures for $\delta = 0,1$ m

By analogy, the dependence of the relative humidity on time during the both-side electro-osmotic drying is obtained [3].

Further, these results are used to study the stability of the forms of porous bodies during rapid heating process. Moisture content is defined as the ratio of fluid to the mass of absolutely dry material. In drained pores ($\bar{z}_m < |\bar{z}| < 1$) at any moment of time the moisture content is equal to the ratio of vapour mass to the mass of dry material and is determined by the dimensionless coordinate of moving interface of phases as follows $W_v = \rho_v \Pi (1 - \bar{z}_m) / [\rho_s (1 - \Pi)]$ in the fluid zone ($0 < \bar{z} < \bar{z}_m$) $W_L = \rho_L \Pi \bar{z}_m / [\rho_s (1 - \Pi)]$.

Here $\rho_L = \frac{m_L}{V_L}$, $\rho_s = \frac{m_s}{V_s}$ true densities of the fluid and dry skeleton, respectively.

When the external power loads are absent, due to changes in mass transfer, over the thickness of the porous layer the one-dimensional fields of moisture content are implemented

$$\eta_L(z, t) = (W_L - W_0), \quad \eta_v(z, t) = (W_v - W_0), \quad (7)$$

where $W_0 = \frac{\rho_L \Pi}{\rho_s (1 - \Pi)}$ — moisture content at the initial moment of time.

During the drying, due to the irregular distributions of temperature and moisture content, without the application of external forces along the thickness and in the middle plane of the plate, compressive and tensile stresses, which equalize compressive ones, emerge. In areas of compression, a loss of stability can occur [5, 6].

Consider an example. We assume that the plate is heated from both sides and a thin wet plate is initially at temperature $T = 0$ and at time $\tau = 0$ ambient temperature rises up to $T = \bar{T}$. Between the surfaces of the plate and medium a heat transfer occur with the heat transfer coefficient K . The edges of the plate are heat-insulated. Since the

plate is thin and the heating is carried out from both sides, we assume that it is uniformly heated over the thickness.

Then, for the temperature distribution in time over the thickness of the plate we get the following equation

$$\frac{dT_0}{d\tau} + \frac{K}{C\rho L_0} T_0 = \frac{K}{C\rho L_0} \bar{T}.$$

Here C, ρ — normalised heat capacity and the density of the porous material.

Let us determine the general solution of this equation and take into account inertial forces only, which correspond to transverse movements w of plate elements. To determine the stability of the plate form let us take into account in the equation of compatibility of deformations and in the equation of bending, the distortion caused by irregular distribution of moisture content over the thickness. Then the problem is reduced to determining heat-moisture efforts and transverse displacements.

Conclusions. Having defined heat-moisture efforts caused by the distortion connected with changes in the moisture content, taking into account the movement of the boundary of the phase passage, by means of the function of stress, and having built the solution of the equation of bending, the characteristic equation can be constructed for each considered case of mass transfer for determining the critical saturation taking into account the stiffness and geometric characteristics of the body, followed by the critical time of the loss of the plate stability in the drying process, which are necessary to know for the timely moisturing to equalize the moisture content in the pores.

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Про один із підходів до розв'язування задач сушіння пористих тіл

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В процесах сушіння пористих тонкостінних пластин навіть без прикладення зовнішніх механічних зусиль виникають проблеми руйнування через тріщинотворення та втрати стійкості форми. Зарадити цьому можна з допомогою тепловлагообробки пористого матеріалу. Для цього необхідно знати критичні зусилля, які виникають при тому чи іншому способі осушення та час втрати стійкості форми пористого тіла. Для визначення зусиль, які виникають за рахунок нерівномірного розподілу вологовмісту за товщиною тіла, необхідно знати зміну відносної насиченості за товщиною в часі, через яку визначається вологовміст. При нехтуванні потоками повітря в пори, розв'язанням відповідних задач Стефана, приведено основні результати зміни відносної насиченості вологою при природному, конвективному та електроосмотичному сушінні пористого шару за товщиною в часі. Ці результати буде використано надалі для визначення критичної відносної насиченості, критичних зусиль та часу втрати стійкості форми пористих пластин.

Об одном из подходов к решению задач сушки пористых тел

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В процессах сушки пористых пластин даже без приложения внешних механических усилий возникают проблемы повреждаемости через трещинообразование и потери устойчивости формы. Избежать их можно с помощью тепловлагообработки пористого материала. Для этого необходимо знать критические усилия, которые возникают при том или ином способе сушки и время потери устойчивости формы пористого тела. Для определения усилий, которые возникают за счет неравномерного распределения влагосодержания по толщине тела, необходимо знать изменение относительной насыщенности по толщине во времени, через которую определяется влагосодержание. При пренебрежении потоками воздуха в поры, решением соответствующих задач Стефана, приведено основные результаты изменения относительной насыщенности влагой при природной, конвективной и электроосмотической сушке пористого слоя по толщине во времени. Эти результаты будут использованы в дальнейшем для определения критической относительной насыщенности, критических усилий и времени потери устойчивости формы пористых пластин.

Отримано 20.11.12