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Refined estimations of strength of structures from carbon-carbonic materials subjected to intense power and thermal loading

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The results of computer simulation of the deformation processes of non-homogeneous structural element, which is manufactured using of carbon-carbonic composite materials, under intensive non-stationary thermal and power loading are presented. The spatial structure and dependence of characteristics of materials on temperature are taken into account. The computational experiment was conducted using the finite element software developed on the basis of non-isothermal thermo-elastic-plasticity. A convergence of numerical solutions has been tested. It is shown that the carbon-carbonic composite material in the structural element ensures its operational reliability when the element is subjected to the specified internal pressure and intense heating to the temperatures at which traditional materials cannot longer functioning in proper way.

Keywords: computer simulation, finite element method, composites.

Introduction. Structural elements operate often under conditions of intense thermal-power load. They can be heated up to temperatures at which traditional materials are no longer able to function, hold their shape, not to melt etc. To ensure the operational reliability of structures in such cases, the structural members are being manufactured from several different materials including composites. The correct choice of materials can be done on the basis of information about the stress state in structural elements during operation. This makes the problem of determination of the stress state of structures under operational conditions as one of the most important.

The accuracy of estimation of the possible safe operation of structural members depends essentially on accuracy and data reliability about maximum stresses in them during operation. Expert estimation of the strength of structural members according to existing now normative documents is realized usually using simple engineering formulas obtained for the bodies of canonical shape under corresponding boundary conditions or within simplified calculation models of bars, beams, plates, shells (see, e. g., [1-3]). The obtained maximal stresses are being compared with admissible ones, and possible errors of such approach are compensated by safety factor, the excessive values of which cause the increase of structure mass, overspending of materials etc. At the same time the structural members are the spatial, frequently structurally non-homogeneous bodies of complex shape, and they can be in volumetric, spatially inhomogeneous stress-strain state under load. At high temperature load Essential factor is also the temperature dependence of material characteristics [4, 5]. Therefore determination of stresses in structural members according to the simple engineer formulas can cause errors not only quantitative but also qualitative ones [6].

Refined calculation can be realized on the basis of general equations of nonlinear thermomechanics. The spatially 3D approach allows us to take into account the real shape of structural elements, and, hence, more accurately to estimate (in comparison with existing methods of theoretical or structural mechanics and strength of materials) their strength, service life and to develop recommendations as to optimization of the spatial structure of the material, weight reduction etc.

This work is devoted to the calculation of the stress-strain state and determination of the estimation of strength of structural members made from carbon-carbonic materials operating under conditions of non-uniform intensive heating and power loading on the basis of proposed earlier approach [5] and corresponding finite-element software to simulate the processes of deformation of elastic-plastic thermosensitive bodies.

1. The problem statement

In order to describe the deformation process of structural members under intensive thermal-power load according to the approach proposed in [5] we use the relations of non-isothermal elastic-plastic yield written in increments. In this case the physical and geometrical relations and balance equations with corresponding boundary conditions are written in the matrix-vector form convenient for the numerical realization by the finite element method:

$$\{d\sigma\} = \mathbf{C}^{t+\Delta t}\left(\{d\varepsilon\} - \{d\varepsilon_T\}\right) + d\mathbf{C}\left(\{\varepsilon\} - \{\varepsilon_T\} - \{\varepsilon_p\}\right) + \mathbf{Z}\,dT, \quad \mathbf{x} \in V; \tag{1}$$

$$\{\varepsilon\} = \mathbf{A}\mathbf{u}; \tag{2}$$

$$\mathbf{A}^T\{\sigma\} + \mathbf{F} = 0; \tag{3}$$

$$\mathbf{N}^T\{\sigma\} = \mathbf{p}, \quad \mathbf{x} \in S_\sigma; \quad \mathbf{u} = \mathbf{u}^{(0)}, \quad \mathbf{x} \in S_u, \tag{4}$$

where $\{\varepsilon\}, \{\varepsilon_T\}, \{\varepsilon_p\}$ are the vectors of total, temperature and plastic deformation; $\{\sigma\}$ is the stress vector; V is the space region occupied by the structure member; S_σ, S_u are the parts of the region V surface where the power load (characterized by the vector **p**) and displacements $\mathbf{u}^{(0)}$ are given; **A**, **N** are, respectively, the matrix of the operator of geometric relations of the theory of elasticity and the matrix of direction cosines of unit normal to the surface; **F** are volumetric forces;

$$\mathbf{C}^{t+dt} = \mathbf{G}^{t+dt} - \frac{\mathbf{G}^{t+dt}\left\{\dfrac{\partial\Phi}{\partial\sigma}\right\}\left\{\dfrac{\partial\Phi}{\partial\sigma}\right\}^T\mathbf{G}^{t+dt}}{\dfrac{2}{3}H^t\left\{\dfrac{\partial\Phi}{\partial\sigma}\right\}^T\left\{\dfrac{\partial\Phi}{\partial\sigma}\right\} + \left\{\dfrac{\partial\Phi}{\partial\sigma}\right\}^T\mathbf{G}^{t+dt}\left\{\dfrac{\partial\Phi}{\partial\sigma}\right\}};$$

$$d\mathbf{C}=d\mathbf{G}-\frac{\mathbf{G}^{t+dt}\left\{\frac{\partial\Phi}{\partial\sigma}\right\}\left\{\frac{\partial\Phi}{\partial\sigma}\right\}^{T}d\mathbf{G}}{\frac{2}{3}H^{t}\left\{\frac{\partial\Phi}{\partial\sigma}\right\}^{T}\left\{\frac{\partial\Phi}{\partial\sigma}\right\}+\left\{\frac{\partial\Phi}{\partial\sigma}\right\}^{T}\mathbf{G}^{t+dt}\left\{\frac{\partial\Phi}{\partial\sigma}\right\}};$$

$$\mathbf{Z}=\frac{\sqrt{\left\{\frac{\partial\Phi}{\partial\sigma}\right\}^{T}\left\{\frac{\partial\Phi}{\partial\sigma}\right\}}\mathbf{G}^{t+dt}\left\{\frac{\partial\Phi}{\partial\sigma}\right\}\frac{\partial\bar{\sigma}}{\partial T}}{\frac{2}{3}H^{t}\left\{\frac{\partial\Phi}{\partial\sigma}\right\}^{T}\left\{\frac{\partial\Phi}{\partial\sigma}\right\}+\left\{\frac{\partial\Phi}{\partial\sigma}\right\}^{T}\mathbf{G}^{t+dt}\left\{\frac{\partial\Phi}{\partial\sigma}\right\}};$$

$\mathbf{G}$ is the matrix of elastic constants; $\{\partial\Phi/\partial\sigma\}$ is the vector of derivatives of the yield function Φ in the space of stresses; H is inclination angle of the curve "equivalent stress $\bar{\sigma}$ — equivalent plastic strain $\bar{\varepsilon}_p$"; the index $^{t+dt}$ means that the value is calculated at the moment $t+dt$, i. e. at the end of the current step of loading (the rest values are calculated at the moment t).

The relations (1)-(4) form a complete system of equations to determine the stress-strain state of the body [5]. It solved using finite element method.

2. Input data for computational experiment

According to the method concept [5] at first we form a finite elements model of the problem, construct division of the space region, occupied by structural member, by finite elements, assign the initial and boundary conditions on a discrete level, approximate the temperature dependences of material characteristics etc. We apply this approach to the determination stress-strain state of an insert of critical section of typical nozzle of rocket engine [3] during its operation (region of this most important structural member of the nozzle is given on Fig. 1).

The considered structural element operates under internal pressure and inhomogeneous high temperature heating up to 3000 K, at which traditional materials can not realize their functions. Therefore it is made of three different materials - erosion resistant carbon-carbonic composite material (region *ABCH*, see Fig. 1), pressed glass-reinforced plastic (region *CDGH*) and titanium alloy (region *DEFG*).

On the insert surface, formed by rotating generatrix *ALJB* about the axis *z*, the pressure $p=c(r)p_{kp}(t)$, where $p_{kp}(t)$ is determined from the diagram of the pressure change in the combustion chamber of the engine [3], and $c(r)$ for each point on the surface we interpolate according to the linear law between the specified reference values of the coefficient at the points, where $(r/r_A)^2=1(c=0{,}575)$; $(r/r_A)^2=1{,}16\,(c=0{,}334)$; $(r/r_A)^2=1{,}3431(c=0{,}24877)$. Hence the pressure on the internal surface changes smoothly from point to point from the value of p_A at the point *A* to p_B at point *B*; on the surface formed by rotation of the generatrix *AH* (see Fig. 1) the pressure p_A is given, and on the surface formed by the rotation of the generatrix *BC*, the pressure p_B is given.

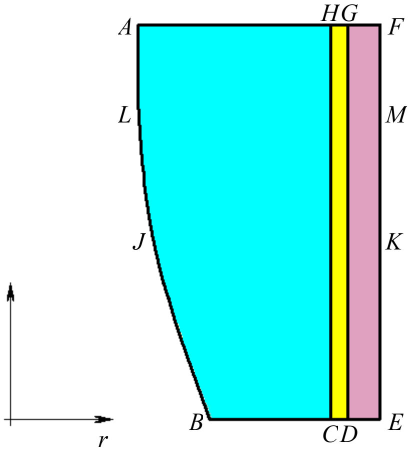

Fig. 1. The calculation region of insert

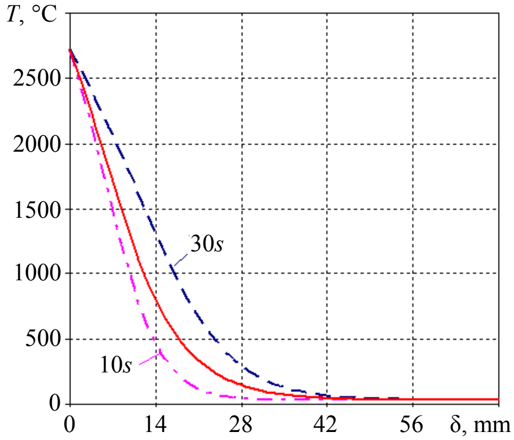

Fig. 2. Temperature distributions in section *JK*

Non-stationary temperature load is assigned according to the obtained values of temperature distributions at different moments in the sections *AF*, *LM*, *JK*, *BE* (for example in Fig. 2 we show the temperature distribution in the section *JK* at $t = 20, 36, 60$ s, where dimensionless distance *s* was determined for the section by referring the real distance of point from the surface to the entire range of change of the radial coordinate for the section). The temperature values at other points of the body at other moments we obtain by linear interpolation.

The characteristics of materials depend essentially on the temperature [3]. According to the developed approach [5] the temperature dependence of materials characteristics we approximate by spline interpolation correspondingly to the above reference points.

We carry out the computer simulation using isoparametric biquadratic axially symmetric finite elements with eight nodes [5]. Research of convergence of numerical solutions we realize comparing the numerical solutions on different finite element meshes, which essentially differ by element sizes.

3. The results of computational experiments

Fig. 3 and 4 show the distributions of stresses in sections *AF*, *JK* and *BE* at moment of time $t = 18$ s (dimensionless distance *s* was determined for each section by referring the real distance of point from the surface to the entire range of change of the radial coordinate for the section considered).

Fig. 5 gives the stresses on the internal surface of the insert (along generatrix *BA*) and on the surface of its contact with the substrate layer (along generatrix *CH*).

It can be seen that power and temperature loading compensate mutually each other in layers near internal surface of the insert (see Fig. 4). High temperature gradients cause compressive circumferential stresses $\sigma_{\varphi\varphi}$ in these layers. These stresses imposed here on the tensile circumferential stresses from the pressure. At the same time the temperature stresses increase tensile power stresses in the central part of the insert and in the zone of the body of titanium alloy.

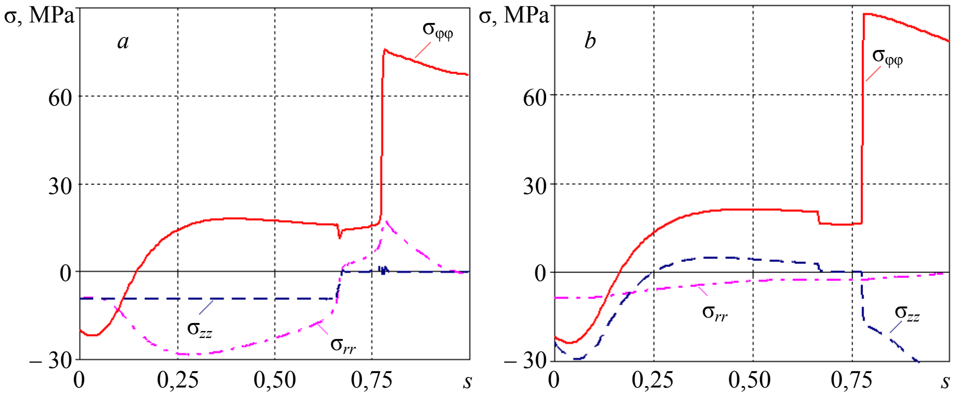

Fig. 3. The distributions of radial σ_{rr}, axial σ_{zz} and circumferential $\sigma_{\varphi\varphi}$ stresses in insert-substrate layer-titanium body in sections *AF* (*a*) and *JK* (*b*) respectively

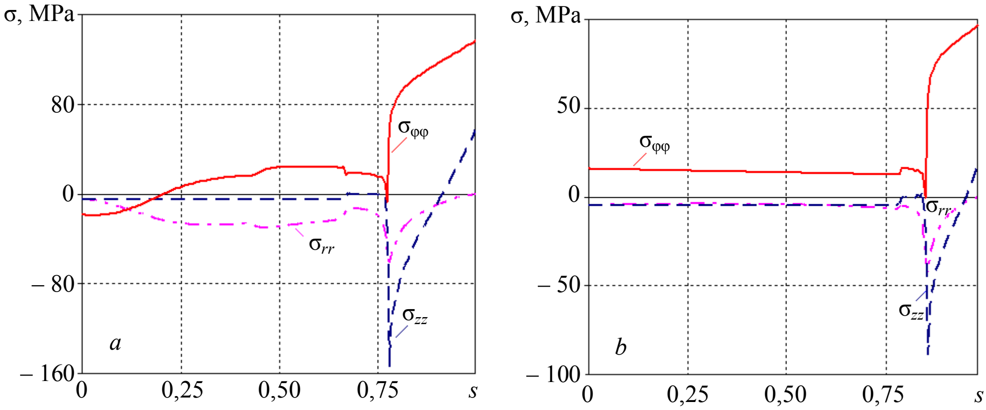

Fig. 4. The distributions of radial σ_{rr}, axial σ_{zz} and circumferential $\sigma_{\varphi\varphi}$ stresses in section *BE* at the thermo-power loading (*a*) and only from internal pressure (*b*)

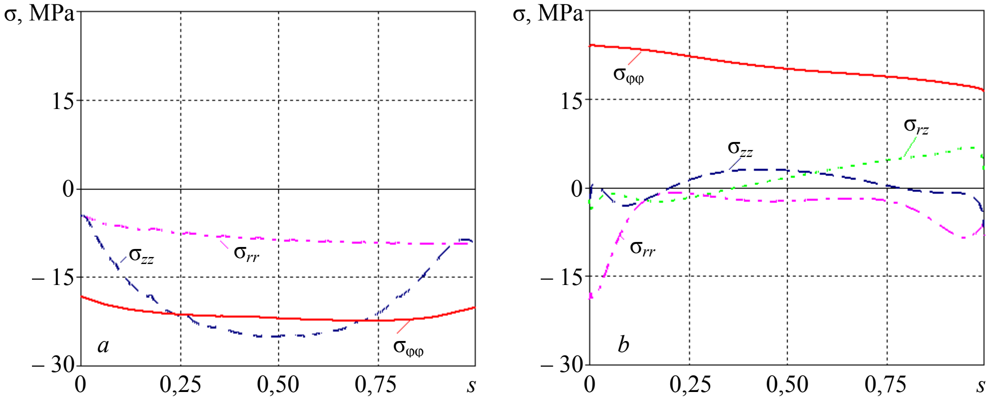

Fig. 5. The distributions of stresses on internal surface of the insert (*a*) and on the surface of its contact with the substrate layer made of pressed glass-reinforced plastic (*b*)

From the above results it follows that maximal tensile circumferential stresses in the insert during operation arise in the neighborhood of the section *BE* and are equal about 24 MPa (see Fig. 4*a*). It should be noted that they occur in the insert regions far from the internal surface, the temperature of which is not higher than 1000°C and, therefore, it is approximately one and a half times less than circumferential stresses of fracture for considered erosion resistant carbon-carbonic composite in condition of tensile (35 MPa at $T \leq 1000$°C). In the zone of high temperatures (higher than 2500°C) in layers near internal surface of the insert we have a strongly expressed zone of compression. In this zone the maximal compressive stresses are equal –23 MPa (in section *JK*; see Fig. 3*b*) at fracture stresses –55 MPa for considered carbon-carbonic composite at such conditions. Maximal axial compressive stresses in layers near the internal surface of the insert in absolute value are yet larger (up to – 30 MPa), however, absolute value of the axial compressive fracture stresses for considered composite are essentially larger (about 60 MPa).

Maximal stresses in the glass-plastic substrate layer are not higher than the corresponding stresses in the insert and at least three times lower than the corresponding fracture stresses for this material.

The stresses in titanium body are essentially higher, however in comparison with admissible stresses for this material maximal operational stresses are not more than 15% of those causing destruction.

Note that the research of convergence were carried out on the bases of comparison of numerical solutions obtained at different meshes with maximal size of finite elements in the range from 0.5 to 3 mm. All solutions thus obtained agree very well between themselves. Insignificant deviations were noted only near surface of contact of heterogeneous materials. In the rest parts the solutions for such diameters of finite elements coincided with accuracy to 1 %.

We see that the existence of efficient software, created on the basis of refined models quantitatively description of the deformation processes of structural elements, parts, machines and mechanisms enables to carry out the whole process of simulation in virtual space and to obtain non-conservative estimation of the strength of that or another member. From another side the computer simulation enables to get a rational design of geometrical configuration of structure or to do the right choice of materials for producing of needed structure members. This can reduce essentially the cost of full-scale experiments since only rational or optimal projects obtained in the process of computer simulation can pass the experimental verification.

Conclusions. We can state that the use of carbon-carbonic composite material when manufacturing an insert of critical section of nozzle guarantees its operational strength and reliability under conditions of the internal pressure and intense inhomogeneous heating to high temperatures at which traditional materials cannot operate.

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Уточнені оцінки міцності конструкцій із вуглець-вуглецевих матеріалів за інтенсивного термосилового навантаження

Богдан Дробенко, Олександр Бурик, Володимир Харченко

Подано результати комп'ютерного моделювання процесів деформування неоднорідного конструктивного елементу, виготовленого з використанням вуглець-вуглецевих композиційних матеріалів, за дії інтенсивного термосилового навантаження з урахуванням просторової структури та термочутливості матеріалів. Обчислювальний експеримент виконано за допомогою програмного забезпечення, розробленого на основі теорії неізотермічної термопружнопластичності. Досліджено збіжність отриманих розв'язків. Показано, що використання вуглець-вуглецевого композиційного матеріалу забезпечує експлуатаційну надійність конструктивного елементу за дії внутрішнього тиску та інтенсивного нагріву до температур, за яких традиційні матеріали вже не здатні виконувати свої функції.

Уточненные оценки прочности конструкций из углерод-углеродистых материалов в условиях интенсивной температурной и силовой нагрузки

Богдан Дробенко, Александр Бурык, Владимир Харченко

Представлены результаты компьютерного моделирования процессов деформирования неоднородного конструктивного элемента, изготовленного с использованием углерод-углеродистых композиционных материалов, находящегося под воздействием интенсивной термосиловой нагрузки с учетом пространственной структуры и термочувствительности материалов. Вычислительный эксперимент проведен с использованием конечно-элементного программного обеспечения, разработанного на основе теории неизотермической термоупругопластичности. Исследовано сходимость полученных решений. Показано, что использование углерод-углеродистого композиционного материала обеспечивает эксплуатационную надежность рассматриваемого конструктивного элемента при воздействии внутреннего давления и интенсивного нагрева до температур, при которых традиционные материалы уже не способны исполнять свои функции.

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