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THE OPTIMIZATION OF THE STRUCTURE OF FINANCIAL ASSETS PORTFOLIO. CASE STUDY: THE MANUFACTURING INDUSTRY IN ROMANIA

Abstract. The objective of this paper is to apply the Markowitz model to a financial assets portfolio in order to identify the optimal portfolio of risky assets. The case study is based on 33 companies of the manufacturing industry in Romania, listed on the Bucharest Stock Exchange, at the first and second category. The research was conducted over the period of 6 months, based on the weekly closing prices of the titles. In order to determine the structure of the efficient portfolio, we used several scenarios regarding the portfolio expected return, i.e. the return that we supppose that we will obtain from the portfolio. We identified the optimal portfolio of risky assets, that offers a maximum expected return for the risk that investors are willing to assume. Also, we determined the X matrices, that represent the percentage that will be invested in each company in order to obtain the expected return, and the portfolio with minimum absolute variance. After applying the Markowitz model, we obtained an illegitimate portfolio. Therefore, it is necessary to sell some of the securities in absence, the procedure of which is not regulated by the Romanian capital market.

Keywords: portfolio with absolute minimum variance, efficient portfolio, risk, rentability, efficient frontier, variance-covariance matrix, portfolio selection.

JEL classification: G11, G17.

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ОПТИМІЗАЦІЯ СТРУКТУРИ ПОРТФЕЛЯ ФІНАНСОВИХ АКТИВІВ НА ПРИКЛАДІ ПІДПРИЄМСТВ ОБРОБНОЇ ПРОМИСЛОВОСТІ РУМУНІЇ

Анотація. У статті описано застосування моделі Марковіца при формуванні портфеля фінансових активів задля встановлення оптимального співвідношення дохідності й ризику. Дослідження базується на даних 33 компаній обробної промисловості Румунії, що котируються на Бухарестській фондовій біржі в першій і другій категоріях, та проводилося протягом шести місяців на підставі щотижневих цін на момент закриття торгів. Для побудови структури ефективного портфеля використ-

тано кілька сценаріїв дохідності, очікуваної від нього, тобто прибутку, який передбачалось отримати від цього портфеля. Розроблено оптимальний портфель ризикових активів, що пропонує інвесторам максимальну очікувану дохідність за ризик, котрий вони готові прийняти. Крім того, визначено Х-матриці, які являють собою відсоток, що буде вкладений у кожну компанію для отримання очікуваної дохідності, а також портфель із мінімальною абсолютною дисперсією. За допомогою моделі Марковіца сформовано портфель, що виходив за рамки закону. Тому необхідно продати частину цінних паперів, а ця процедура не регулюється на румунському ринку капіталу.

Ключові слова: портфель з абсолютною мінімальною дисперсією, ефективний портфель, ризик, рентабельність, ефективний бар'єр, коваріаційна матриця, вибір портфеля.

Рис. 3. **Табл.** 4. **Форм.** 8. **Літ.** 13.

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ОПТИМИЗАЦИЯ СТРУКТУРЫ ПОРТФЕЛЯ ФИНАНСОВЫХ АКТИВОВ НА ПРИМЕРЕ ПРЕДПРИЯТИЙ ОБРАБАТЫВАЮЩЕЙ ПРОМЫШЛЕННОСТИ РУМЫНИИ

Аннотация. В статье описано применение модели Марковица при формировании портфеля финансовых активов для установления оптимального соотношения доходности и риска. Исследование базируется на данных 33 компаний обрабатывающей промышленности Румынии, котирующихся на Бухарестской фондовой бирже, в первой и второй категориях, и проводилось в течение шести месяцев на основании еженедельных цен на момент закрытия торгов. Для построения эффективного портфеля использовано несколько сценариев доходности, ожидаемой от него, то есть прибыли, которую предполагалось получить от этого портфеля. Разработан оптимальный портфель рисковых активов, предлагающий инвесторам максимальную ожидаемую доходность за риск, который они готовы принять. Кроме того, определены Х-матрицы, что представляют собой процент, который будет вложен в каждую компанию для получения ожидаемой доходности, а также портфель с минимальной абсолютной дисперсией. С помощью модели Марковица сформирован портфель, который выходил за рамки закона. Поэтому необходимо продать часть ценных бумаг, а эта процедура не регулируется на румынском рынке капитала.

Ключевые слова: портфель с абсолютной минимальной дисперсией, эффективный портфель, риск, рентабельность, эффективный барьер, ковариационная матрица, выбор портфеля.

The American professors Harry Markowitz and William Sharpe created the financial theory regarding the analysis of the portfolio of securities by developing the modern portfolio theory (Stanciu, 2004). The selection of portfolio securities includes two steps. The first implies observing and making a forecast regarding the future performance of those securities. The second step is the selection of the

portfolio for which the capital market investors expect to obtain a maximum future return (Markowitz, 1952). The portfolio with minimum absolute variance (PVMA) represents the combination of titles with minimum risk and maximum return, desired by the investors with the highest risk aversion (Anghelache *et al.* 2013). The optimal portfolios of securities are crossed by a curve named the efficient frontier (Markowitz, 1952). The portfolios that can be found on this efficient frontier have the following characteristics: for a certain return the portfolio has a minimum risk, or at a certain risk, the portfolio has the highest expected return (Anghelache *et al.* 2013). The investors have to diversify their securities portfolio in order to maximize the return. The application of Markowitz and Sharpe theories according to Romanian manufacturing enterprises is presented in the article.

The theories regarding the management of a portfolio of securities are considering minimizing the portfolio's risk and maximizing its diversification. This fact highlights the utility of the original portfolio theory developed by Markowitz in 1952. Through its model, Harry Markowitz discussed the possibility to build a portfolio taking into consideration both the expected return that it is preferable to be as higher as possible, and the risk that the investors are willing to assume, that it is preferable to be minimum (Steinbach, 2001). Until Markowitz approaches the importance of the risk, this concept was not defined in detail (Sharpe, 1967). Markowitz considers that an optimal portfolio represents a set of profitable securities that protect the investor. Therefore, it is recommendable for the investors to create a portfolio according to their needs (Markowitz, 1959). The theory of portfolio selection, developed by Harry Markowitz, is based on the following assumptions (Lintner, 1965): there are no transaction costs and any other costs; each investor has the possibility to invest in any financial title; the prices at the market are competitive and can not be influenced by the investors; by using the median-variance criterion, the investors in the capital market can maximize their expected utility.

Harry Markowitz demonstrated that the selection of the optimal portfolios of financial titles is based on the analysis of the portfolio expected return and variance (Anghelache *et al*. 2013). The variance of the return of a series of data from the past is represented by the average of the deviations from the mean (Markowitz, 1959).

Markowitz's main contribution consisted in the development of a theory concerning the portfolio selection in uncertainty conditions (The Sveriges Riksbank (Bank of Sweden) Prize in Economic Sciences in Memory of Alfred Nobel, 1990).

Through the Markowitz model it is possible to determine the portfolio with minimum absolute variance (PVMA) that represents the combination of titles with the lowest risk. Also, the model allows us to identify the efficient frontier where can be found the optimal portfolios. The efficient frontier implies identifying the portfolio with minimum absolute variance expected by the investors with the highest risk aversion from the capital market (Badea, 2006).

The efficient alternatives, also known as the non-dominant alternatives, compose the efficient frontier (Levy *et al.* 1972). A portfolio is inefficient if it is legitim and could obtain a superior expected return at the same level of the risk, or a lower risk at the same level of the return (Markowitz, 1959).

Harry Markowitz believed that *"the portfolios that can be found above on the efficient frontier will have a higher return, but also a higher risk".* Also, there is more volatility for those portfolios and a lower volatility for the portfolios that can be found below on the frontier (Buttell, 2010).

The capital market investors must diversify their securities portfolio in order to maximize the rate of expected return. The portfolio diversification does not eliminate entirely the risk and it is not mandatory that the portfolio with the highest expected return to have also a lower risk (Markowitz, 1952). In order to determine the efficient portfolios, the covariances are extremely important because they highlight the relationships between the financial titles. When the correlation coefficient (ρ_{i*j}) between two titles is zero, the titles are not correlated (Markowitz, 1959). If the value of the correlation coefficient is (+1), the titles are perfectly and positively correlated and the risk of the portfolio is the highest. Therefore, changing the weight of the titles in the portfolio, does not considerably reduce its risk. Contrarily if the correlation coefficient is (–1), the titles are perfectly and negatively correlated. In this situation, it can be achieved through a certain combination of the title and the complete elimination of the portfolio's risk (Anghelache *et al.* 2013).

The objective of the paper is to apply the Markowitz model to a financial assets portfolio in order to identify the optimal portfolio of risky assets with regard to Romanian manufacturing industry enterprises.

The research was based on the database provided by the financial situations of 33 companies of the manufacturing industry in Romania, listed on the Bucharest Stock Excahnge (www.bvb.ro) at the first and second category, over the period 01.01.2016 – 30.06.2016 (27 weeks), respectively: ALRO S.A. (ALR), ALTUR S.A. (ALT), AEROSTAR S.A. (ARS), ARTEGO S.A. (ARTE), ANTIBIOTICE S.A. (ATB), BIOFARM S.A. (BIO), CEMACON S.A. (CEON), COMELF S.A. (CMF), COMPA S.A. (CMP), CONTED S.A. (CNTE), ELECTROARGES S.A. (ELGS), ELECTRO-MAGNETICA S.A (ELMA), ELECTROPUTERE S.A. (EPT), ROMCARBON S.A. (ROCE), ROMPETROL RAFINARE S.A. (RRC), ZENTIVA S.A. (SCD), BOROMIR PROD S.A. (SPCU), TURBOMECANICA S.A. (TBM), TERAPLAST S.A. (TRP), VRANCART S.A. (VNC), TMK – ARTROM S.A. (ART), BERMAS S.A. (BRM), CARBOCHIM S.A. (CBC), GRUPUL INDUSTRIAL ELECTROCONTACT S.A. (ECT), MECANICA CEAHLAU (MECF), PRODPLAST S.A. (PPL), PREFAB S.A. (PREH), SANTIERUL NAVAL ORSOVA S.A. (SNO), STIROM S.A. (STIB), SINTEZA S.A. (STZ), UAMT S.A. (UAM), UZTEL S.A. (UZT), VES S.A. (VESY).

The steps of the research are presented below:

- conducting a technical analysis through the moving average of the shares of the 33 companies over the period of 01.01.2016 – 30.06.2016. The study aims to obtain a forecast regarding the evolution of these companies shares returns;

• the calculation of weekly returns of the shares (*Rsm*) for each company, based on the weekly closing prices. Therefore, it resulted for each company, a total of 27 returns (corresponding to a total of 27 weeks) for the period 01.01.2016 – 30.06.2016, according to the model (Badea, 2006):

$$Rsm = \frac{P_{i1} - P_{i0}}{P_{i0} \cdot n},$$

where: P_{i1} – the closing price at the moment t_1;
P_{i0} – the closing price at the moment t_0;
n – the number of the transactions made during the studied period;

• determining the Markowitz matrix (*W*), also known as the variance-covariance matrix, using the COVAR function from the EXCEL program, according to the model (Altar, 2002):

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1n} & E_1 & 1 \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2n} & E_2 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_n^2 & E_n & 1 \\ E_1 & E_2 & \dots & E_n & 0 & 0 \\ 1 & 1 & \dots & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ E_p^* \\ 1 \end{bmatrix} \Leftrightarrow X = W^{-1} \times K,$$

$$W \cdot X = K,$$

where: σ_{12} – the covariance between title 1 and title 2;
σ_{1n} – the covariance between title 1 and title *n*;
$E_1, E_2, \dots, E_n$ – the weekly average returns of the titles (1, …, *n*).

The covariance can be determined according to the model (Wallingford, 1967):

$$\sigma_{ij} = \sigma_i \cdot \sigma_j \cdot \rho_{i*j},$$

where: σ_{ij} – the covariances of the returns of the titles from the portfolio;
σ_i, σ_j – the standard deviation of the title *i*, respectively of the title *j*;
ρ_{i*j} – the correlation coefficient between the two titles, according to the model (Markowitz, 1959):

$$\rho_{i*j} = \frac{\sigma_{ij}}{\sigma_i \times \sigma_j};$$

• in order to determine the structure of the portfolio, respectively the percentage that it is recommendable to invest in each society for obtaining an efficient portfolio, we determined the X matrix by multiplying the inverse matrix with the K matrix, that received random values of the return expected by the investors (E_p^*). For this calculations, we used the MMULT function from the program EXCEL;

• in order to identify the optimal portfolio for the investors (the portfolio of minimum absolute variance), we determined the inverse matrix PVMA that multiplied with the K matrix, generates the X_{PVMA} matrix (Altar, 2002):

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1n} & E_1 & 1 \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2n} & E_2 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_n^2 & E_n & 1 \\ 1 & 1 & \dots & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 1 \end{bmatrix},$$

$$W_{PVMA} \cdot X_{PVMA} = K_{PVMA}.$$

The inverse matrix PVMA, multiplied with the K matrix will generate the X_{PVMA} matrix:

$$W_{PVMA}^{-1} \cdot K = X_{PVMA}.$$

Determining the Markowitz efficient frontier through the steps below:

• Including in the sample the following companies: TRP, ART, CMF, ELMA, SNO, ALR, UAM and CNTE, that are the first eight companies that recorded the highest average weekly returns.

• Determining the variance-covariance matrix for those eight companies through the COVAR function from the program EXCEL. The calculation model is the same as the one used in oder to determine the variance-covariance matrix for the efficient portfolio.

• Establishing target rentability of the portfolio, with values between the minimum and maximum weekly return of the eight companies and running Excel Solver in order to obtain the results regarding the portfolio's risks that correspond to the target rentabilities.

• Repeating the previous step until we obtain a proper graphic of the efficient frontier.

All the portfolios from the Markowitz efficient frontier have a minimum risk for a certain value of the rentability or for a certain value of the risk, their rentability is maximum (Anghelache *et al.* 2013).

The technical analysis highlights the trend of the shares of the financial titles. The purchasing signal is issued when the price of the title is higher than the moving average, and the sell signal is issued when the price of the title is below the moving average. We believe that the studied period of time of 27 weeks, allows us to observe an ascendent or descendent trend of the title's share.

The weekly average returns of the shares of the 33 companies studied in our research are determined on the base of the evolution of the weekly closing prices, represented in the figure 1.

The companies ALR, CMF, ELMA, TRP, ART and UAM recorded an increase in the *Rsm* between 1.01 % and 2.70 %. On the contrary, ARS, ARTE, ELGS, EPT,

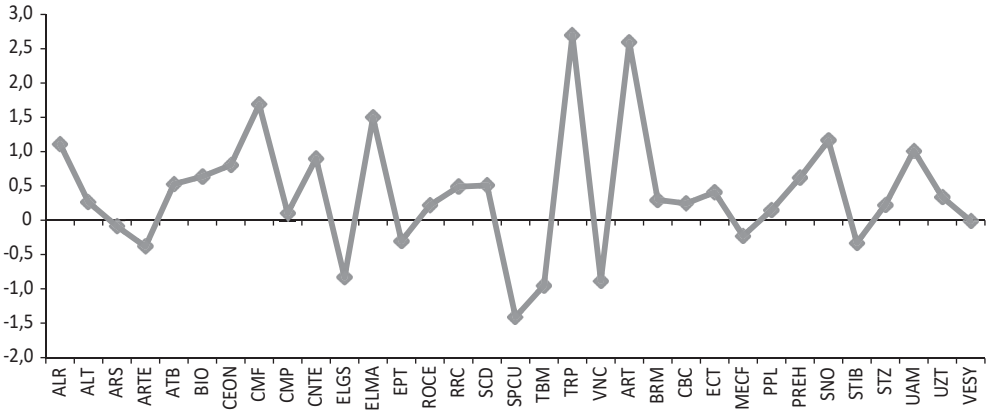

Figure 1. **The weekly average returns (*Rsm*) over the period 01.01.2016 – 30.06.2016 (%)**

Note: Own processing of the authors on the base of the data provided by the portal www.bvb.ro.

SPCU, TBM, VNC, MECF, STIB and VESY recorded decreases between 0.01 % and 1.41 %. In order to determine the X matrix that highlights the weights of the titles that are recommended to be invested in the portfolio for obtaining the expected returns (E_p), we formed the K matrix, by giving random values for the expected returns: 5 % (X_1), 7.5 % (X_2), 10 % (X_3) and 15 % (X_4).

Table 1 highlights the weights of the titles that are recommended to be invested in each of the 33 companies (X matrix), based on the expected returns.

All 4 scenarios warn us of the fact that it is necessary to sell the shares of such companies as: ALT, ARS, ARTE, BIO, CEON, TBM, TRP, VNC, ART, MECF, UAM, UZT by short sell (procedure not regulated in the Romanian capital market) and to purchase the titles of the rest of the companies of the manufacturing industry, that have positive returns. The companies that would bring the higher return in the portfolio are: ATB, ELMA, SCD, CBC and VESY.

In order to highlight the efficient portfolios with the lowest risk, in table 2, we determined the matrix of the portfolio with the minimum absolute variance (X_{PVMA}). Therefore, we used the covariance of the closing price for the shares of the 33 companies.

The information provided by table 2 leads to the conclusion that it is recommendable to invest in the shares of the companies ALR, ATB, CMF, CMP, CNTE, ELGS, ELMA, EPT, RRC, SCD, SPCU, TBM, ART, BRM, CBC, ECT, PPL, PREH, SNO, STIB, STZ, VESY. The shares of the companies ATB, ELMA, SCD, CBC and VESY are less risky and would bring the highest return to the investors.

The last step of our study consisted in determining the efficient frontier for the companies: TRP, ART, CMF, ELMA, SNO, ALR, UAM and CNTE, that are the first eight companies that recorded the highest average weekly return (figure 2).

Also, we determined the variance-covariance matrix for the eight studied companies (table 3).

Table 1. **The matrix X_1, X_2, X_3, X_4**

Companies	The expected returns			
	5 % (X_1)	7.5 % (X_2)	10 % (X_3)	15 % (X_4)
ALR	0.267684	0.3000	0.273084	0.278484
ALT	–0.53069	–0.51846	–0.49300	–0.57120
ARS	–0.8089	–0.9890	–0.972	–0.99205
ARTE	–0.36434	–0.356	–0.3783	–0.47097
ATB	0.905483	0.910107	0.914731	0.523979
BIO	–0.8011	–0.9810	–0.985	–0.9860
CEON	–0.25569	–0.29754	–0.27939	–0.43308
CMF	0.183101	0.184787	0.186473	0.189844
CMP	0.292466	0.298134	0.303801	0.315135
CNTE	0.031618	0.034799	0.037979	0.044339
ELGS	0.345465	0.333179	0.320892	0.29632
ELMA	0.9921000	0.992647	0.984666	0.968704
EPT	0.064461	0.06837	0.072278	0.080095
ROCE	0.206788	0.183005	0.159222	0.111656
RRC	0.218864	0.211934	0.205003	0.191141
SCD	0.81511	0.814855	0.814599	0.814087
SPCU	0.333047	0.325038	0.31703	0.301013
TBM	–0.05176	–0.04853	–0.04531	–0.03887
TRP	–0.18322	–0.18251	–0.1818	–0.18038
VNC	–0.38767	–0.39199	–0.3963	–0.40494
ART	–0.04826	–0.04584	–0.04342	–0.03857
BRM	0.133155	0.129359	0.125563	0.11797
CBC	0.970966	0.965092	0.959218	0.94747
ECT	0.137328	0.136476	0.135625	0.133922
MECF	–0.2798	–0.27513	–0.27047	–0.26113
PPL	0.18541	0.186245	0.18708	0.18875
PREH	0.022763	0.023431	0.024098	0.025432
SNO	0.11791	0.116897	0.115883	0.113855
STIB	0.096747	0.101852	0.106957	0.117167
STZ	0.325508	0.316117	0.306726	0.287943
UAM	–0.6530	–1.58564	–0.988	–0.9756
UZT	–0.65397	–0.64944	–0.64491	–0.63586
VESY	0.997763	0.989388	0.981013	0.964262

Note: Own processing of the authors on the base of the data provided by the portal www.bvb.ro.

Table 2. **The X_{PVMA} matrix**

Companies	The expected returns	Companies	The expected returns	Companies	The expected returns
ALR	0.322812	ELMA	0.837677	CBC	0.851033
ALT	–0.48112	EPT	0.144265	ECT	0.119942
ARS	–0.9733	ROCE	–0.27881	MECF	–0.18447
ARTE	–0.19401	RRC	0.077353	PPL	0.20246
ATB	0.999895	SCD	0.809888	PREH	0.036386
BIO	–0.9095	SPCU	0.169532	SNO	0.09721
CEON	–0.49759	TBM	0.014026	STIB	0.200978
CMF	0.217524	TRP	–0.16871	STZ	0.133759
CMP	0.408177	VNC	–0.47581	UAM	–0.99100
CNTE	0.096549	ART	0.001192	UZT	–0.5615
ELGS	0.094604	BRM	0.055648	VESY	0.826761

Note: Own processing of the authors on the base of the data provided by the portal www.bvb.ro.

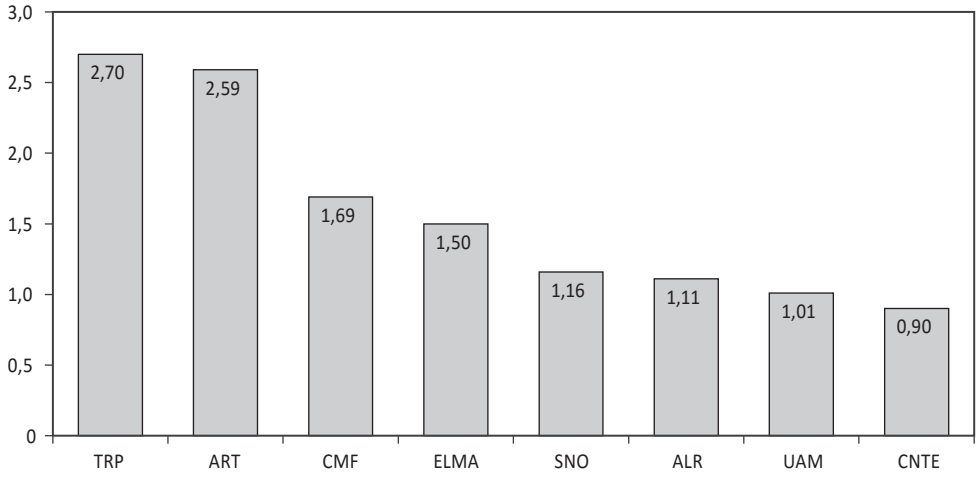

Figure 2. **The weekly average returns of the studied companies (%)**

Note: Own processing of the authors on the base of the data provided by the portal www.bvb.ro.

Table 3. **Variance-covariance matrix**

	ALR	ART	CMF	CNTE	ELMA	SNO	TRP	UAM
ALR	0.003259							
ART	0.001114	0.006003						
CMF	–0.001330	0.000513	0.005061					
CNTE	0.000368	0.001474	–0.000160	0.002061				
ELMA	0.001723	0.001111	0.000410	0.001208	0.003320			
SNO	0.000499	0.000241	0.000616	0.000764	0.001464	0.002598		
TRP	0.003183	0.000337	0.000973	–0.000260	0.002131	0.000656	0.008796	
UAM	0.000410	0.000713	–0.000050	–0.000420	0.000075	–0.000250	0.000312	0.001232

Note: Own processing of the authors on the base of the data provided by the portal www.bvb.ro.

It can be noticed the fact that between the titles of the companies there are positive and negative covariances. The positive covariances highlight the fact that the returns of the titles taken by pairs of two are evolving in the same direction. Contrarily, negative covariances highlight the returns tendency to evolve in the opposite direction. In order to establish a set of efficient portfolios, taking into consideration the shares of the companies included in the research, we established target rentabilities of the portfolio with values between the minimum and maximum weekly returns of the studied companies.

The risks of the sets of portfolios were calculated according to expected returns. Table 4 presents the minimum risks for the expected returns. The combinations of risk and rentability represent the Markowitz efficient frontier (figure 3). The efficient portfolios from the frontier are situated on the superior part of the hyperbole (http://www.ase.ro/upcpr/profesori/977/Curs%205.pdf). Also, the portfolio with absolute minimum variance is situated on the top of the hyperbole.

Table 4. **The efficient portfolios composed of risky (%)**

X axis	Y axis	Weights							
St.Dev	E[r]	wALR	wART	wCMF	wCNTE	wELMA	wSNO	wTRP	wUAM
5.88	2.59	0.00	54.93	4.93	0.00	0.00	0.00	40.14	0.00
4.54	0.90	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00
3.41	1.80	0.00	23.90	13.80	4.50	0.00	14.43	17.80	25.56
2.42	1.35	0.00	6.38	11.46	20.93	0.00	11.39	9.80	40.03
4.68	2.25	0.00	40.58	16.59	0.00	0.00	9.59	27.91	5.33
5.40	2.48	0.00	48.87	16.84	0.00	0.00	0.06	34.23	0.00
4.02	2.02	0.00	32.10	16.41	0.74	0.00	8.30	22.72	19.73
2.88	1.58	0.00	15.34	12.66	12.53	0.00	12.94	13.89	32.63
2.12	1.12	6.17	0.00	11.68	26.07	0.00	9.51	2.50	44.07
2.19	1.02	2.84	0.00	3.35	33.27	0.00	9.15	0.00	51.39
9.38	2.70	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
6.14	2.64	0.00	54.86	0.00	0.00	0.00	0.00	45.14	0.00
2.44	0.96	0.00	0.00	0.00	47.57	0.00	0.00	0.00	52.43

Note: Own processing of the authors on the base of the data provided by the portal www.bvb.ro.

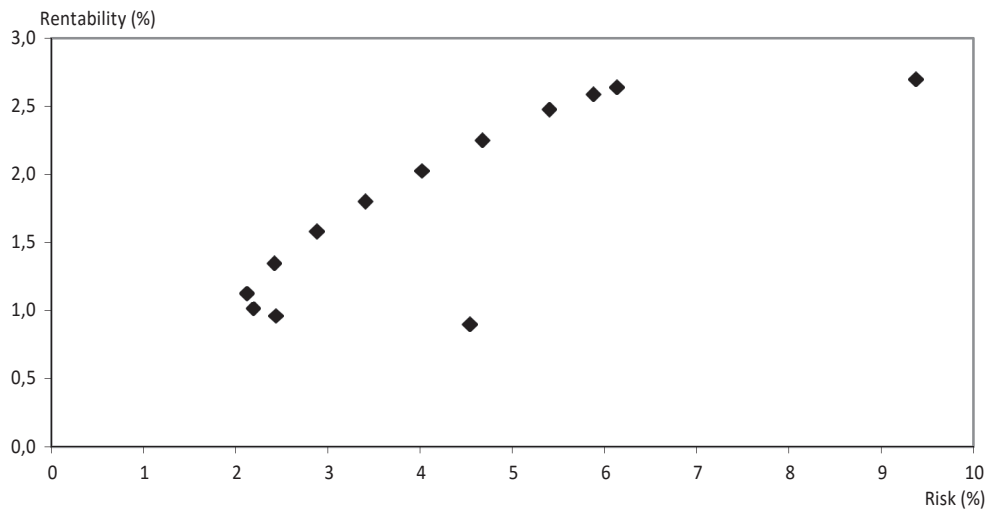

Figure 3. **Markowitz efficient frontier**

Note: Own processing of the authors on the base of the data provided by the portal www.bvb.ro.

From all the studied portfolios, the minimum risk is 2.12 % and it corresponds to a portfolio composed by 6.17 % ALR, 11.68 % CMF, 26.07 % CNTE, 9.51 % SNO, 2.50 % TRP and 44.07 % UAM. The rentability of this portfolio is 1.12 %. The portfolios which items are situated under the top of the curve are inefficient because there is the possibility to compose other portfolios with the same risk but with a higher rentability. These portfolios are called efficient and are situated after the top of the curve. Thus, it is recommendable to invest only in this type of portfolios. Capital market investors with strong aversion to risk will prefer a portfolio situated on the top of the curve, or a portfolio situated near the top of the curve. On the contrary, for the investors who are willing to assume risks in order to

obtain a higher average return, it is recommendable to invest in the risky portfolios that are situated as far as possible from the top of the curve. In the case of the portfolios composed of risky assets, if the expected return increases, the portfolio risk will also increase.

The investors who have risk aversion will prefer a portfolio with a low risk and therefore, a low return. Contrary, the investors that want to achieve a high return, will prefer the securities with a high risk. A good portfolio management is important because an apropriate structure of the portfolio determines a superior return. In consequence, the equilibrium between the return and the risk as well as the portfolio diversification represents useful instruments for investors in order to obtain a maximum return for the risk that they are willing to assume. Both the optimal portfolios structure and the one based on the expected returns, tell us that it is necessary to sell a part of the shares of the companies ALT, ARS, ARTE, BIO, CEON, ROCE, TRP, VNC, MECF, UAM, UZT. The research conducted resulted in the assumption that the highest return from the portfolio could be generated by investments in the shares of the companies ATB, ELMA, SCD, CBC and VESY.

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