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Application of Hough Transform to the Recognitions Problem of Regular Solutions of Dynamical Systems

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Применение преобразования Хафа для распознавания регулярных решений динамических систем

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Застосування перетворення Хафа для розпізнавання регулярних рішень динамічних систем

In this work the investigations of the recognitions problem of regular solutions of autonomous dynamical systems are continued. As shown in [1], this problem reduces to the recognition of three-dimensional convex closed analytic curves constructed on the Poisson sphere by means the Poincaré sections [2], [3]. In some cases, these curves are circles which lie on the surface of sphere. For recognition of such curves in this paper a new algorithm was formulated. He is extending the Circle Hough Transform to three-dimensional case and is called a Generalized Circle Hough Transform. The computational complexity of this algorithm can be reduced to the computational complexity in the two-dimensional case.
Keywords: Poincaré section, the classical and the generalized Hough transform, circle detection.

В этой работе продолжаются исследования задачи распознавания регулярных решений автономных динамических систем. Как показано в [1], эта задача сводится к распознаванию трехмерных выпуклых замкнутых аналитических кривых, построенных на сфере Пуассона с помощью сечения Пуанкаре [2], [3]. В некоторых случаях эти кривые являются окружностями, которые лежат на сфере. Для распознания таких кривых в этой работе предлагается новый алгоритм. Предложенный метод обобщает классическое преобразование Хафа на трехмерный случай и называется сферическое обобщенное преобразование Хафа. В работе показано, как вычислительную сложность этого алгоритма свести к вычислительной сложности двумерного случая.
Ключевые слова: сечение Пуанкаре, классическое и обобщённое преобразования Хафа, распознавание окружности

У цій роботі продовжуються дослідження задачі розпізнавання регулярних розв'язків автономних динамічних систем. Як показано в [1], ця задача зводиться до розпізнавання тривимірних опуклих замкнутих аналітичних кривих, побудованих на сфері Пуассона за допомогою перетину Пуанкаре [2], [3]. У деяких випадках ці криві є колами, які лежать на сфері. Для розпізнання таких кривих у цій роботі пропонується новий алгоритм. Запропонований метод узагальнює класичне перетворення Хафа на тривимірний випадок і називається сферичне узагальнене перетворення Хафа. У роботі показано, як обчислювальну складність цього алгоритму звести до обчислювальної складності двовимірного випадку.
Ключові слова: перетин Пуанкаре, класичне і узагальнене перетворення Хафа, розпізнавання окружності.

Introduction

As it is known, the computer solution of the predictions problem of regular and chaotic behavior of nonlinear dynamical systems can be obtained by several stages. At the first stage, the process of digitization solutions system by means of numerical integration is used. At the second stage, process the graphics and geometric modeling of the results is applicable. If the dimension of the system is less than three, the result of this integration is conveniently represented in graphical form on a image as a series of points forming a curve in space and characterizing the state of the system at any time. In the third stage, the analysis of the form of the trajectory and the conclusion about the behavior of the system is spent. In the case, when the dimension of the system is greater than three, the result of numerical simulation of the system in graphical form in three-dimensional space can be represented only by special mappings, which to build projections and sections. However, the computer simulation results in the fact that some information on the nature of the system is lost and the results are incorrect. The construction of sections of space curves is more relevant and accurate method, as more information about the qualitative behavior of the trajectory space of the system. One of these kind sections is a spherical Poincaré section.

In works [1], [2] with help Modeler we received the global spherical Poincaré section. This section builds a three-dimensional sphere and shows a set of points (point cloud). In regular cases, these sets of points form a three-dimensional closed curve with self-crossing or without self-crossing. A particular case of this curve is a circle, which lies on the surface of a sphere, with the center of the circle can pass or not pass through the center of the sphere. If the center of the circle through the center of the sphere, then the radius of the sphere and the circle are the same, otherwise the radius of the circle is smaller than the radius of the sphere. Therefore, detection and recognition of three-dimensional convex closed analytic curves constructed on the Poisson sphere (see Figure 1) using the method of Poincaré sections, is an important and actual problem which solution by means of the generalized spherical Hough transform [3].

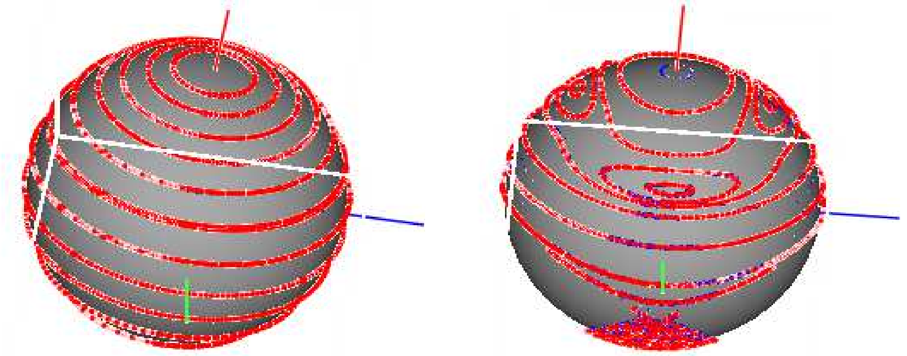

Figure 1 – The shape of the curve on the Poisson sphere

Formulation of the problem

So, in this work, it is necessary to solve the problem of the detection and recognition of the circles formed by the points and lying on a surface of sphere. To solve this problem it is proposed to use a method, which generalizes the classical Hough transform to three dimensions on a sphere - spherical generalized Hough transform. The proposed algorithm is used an accumulator array increased dimension, which coincides with the number of unknown parameters in the equation of the family of the required curves. Computational complexity of the voting process carried out in the Hough extended accumulator space depends on the sampling of the parameter space. The main limitation on the use of the method – noisy source image – must be overcome in the preliminary stage.

The analysis of the current state of the problem

In common cases the classical Hough transform [4], [5] is used for analysis of flat monochrome images. This transformation allows allocating flat two-parametrical curves set, which analytically by means of the equation of a straight line, a circle, an ellipse is built. The essence of this transformation is to translate the original image in the parameter space and the subsequent carrying out of a voting procedure. The voting procedure is applied to the parameter space, from which the allocated objects of a certain class of figures on the local maximum in the so-called, an accumulator space. We will consider existing methods of search of flat and spatial images on Hough transform and its modifications. The best known works in this area are [6-15].

In work [6] the Hough transform, as a method for the detection of curves, using two parameter points on the curve, and the parameters of the curve itself, is described. In research it is shown how to detect both analytic and non-analytic curves, which are limited by borders of a binary picture. As a result, the work was extended to detect the analytical curves in black-and-white images, including lines, circles, and parabolas. In work [7] the randomized Hough transform to search ellipses in the image is discussed. The equations to detect the ellipses and their implementation are given. The algorithm has shown good results in detection the ellipse at an angle 0 to 90 degrees to the axis of coordinates in the image. For examples, detecting and finding the ellipses in the real image after pre-processing the image data. In work [8] of the new approach to the detection of 3D objects of arbitrary shape in a given cloud of points in space is represented. He is the extended generalized Hough transform for detecting objects specified in the point cloud, which are received from the data of laser range finder. In work [9] provides a method for reconstructing 3D building models from images of two-dimensional images. The method based on the determination of levels of detail. It is considered each of the three levels of detail. Also, in this paper it is described, how Hough transform can be applied to detect the boundaries of buildings. In work [10] recognition of lines of a road marking in a mode of real time is investigated. Preliminary processing of images, a finding and tracing of lines, and as results of practical realization of algorithm is described. In work [11] describes algorithms for detecting contours on images of topological layers of integrated circuits. As the operator of transformation two-dimensional Walsh function, approximation of the allocated contours is carried out by rectilinear pieces on the basis of the modified Hough transform. In work [12] questions of application of a hydrolocator of the lateral review for inspection of underwater communications are considered. For allocation of lines of the pipeline the modified and classical Hough transform was used. In work [13] ways of detect of parametrical curves on the binary image with use of Hough transform are analyzed. In particular in details the algorithm of Hough transform for straight line search and a circle on the image is described. The program example in language JAVA is resulted and is described its applications in recognition of radius of a coin for definition of its face value. In work [14] the question of automatic definition of a corner of a deviation of a line of horizon from a horizontal line of a photo is studied. The review of existing decisions is made, the algorithm of automatic definition of an angle of rotation of a line of horizon is described, and the details of realization and possible errors and errors are specified. In work [15] features of algorithms of search of an impress of a seal on the image of the document on the basis of Hough transform are considered. Results of researches for base algorithm of Hough transform and modified Hough transform in which the gradient of brightness for faster finding of radius of the press and acceleration of its search is in addition used are resulted.

Thus, in the current literature to solve the problem of finding the two-dimensional geometry of the image and three-dimensional objects in space effectively used Hough transform.

Hough transforms to detect a circle on a spherical surface

As the purpose of the given work a method of a detecting of a circle on a spherical surface base on for the algorithm of Hough transform to detect a circle on the flat image [5]. We will consider it more in detail.

The essence of the Hough transform is to find curves that pass through a sufficient number of points of interest. Consider a family of curves in the plane defined by the parametric equation:

$$F(a_1, a_2, \ldots a_n, x, y)=0,$$

where F - some function; $a_1, a_2, \ldots a_n$ – parameters of the family of curves; x, y – coordinates on a plane. Parameters of a family of curves form a phase space, which each point corresponds to some curve. In view of step-type behavior of machine representation and the entrance data (image), it is required to translate continuous phase space in the discrete. For this purpose in phase space the grid breaking it on cells is entered, each of which corresponds to a set of curves with close values of parameters. Each cell of the phase space can be associated with a number (counter), indicating the number of points of interest on image that belong to at least one of the curves corresponding to the cell. The analysis of counters of cells allows to find curves on which the greatest quantity of points of interest lies on the image.

The base algorithm of detection of curves consists of following steps.

Step 1. A choice of a grid of digitization. At this stage it is necessary to choose a step of digitization for each parameter of a curve. Complexity will depend on this choice, and accordingly, speed and efficiency of algorithm.

Step 2. Accumulator filling (a matrix of counters). Frequently it is the longest step of algorithm as filling is made by full search. Complexity of algorithm directly depends on the first step and makes: $O(N*M)$, where N – quantity of points, M – quantity of cells of the accumulator.

Step 3. The analysis of the accumulator array and detection of peaks. In an accumulator array the counter with the maximum value is searched.

Step 4. Curve allocation. Each cell of the accumulator is value of phase space so, it sets some, required, curve. As value on a step 1 became discrete, curve specification by any other method on already found points of a curve can be demanded.

Step 5. Subtraction from the accumulator array. For points of the allocated curve it is considered the temporary accumulator it is subtracted from the core.

Step 6. Go to step 3.

The idea of a generalized spherical Hough transform is to find a circle which passes through enough of points of the interest lying on a surface of sphere.

Let's consider sphere of individual radius. We will enter system of the coordinates which center and place in the sphere center then the equation of sphere looks like

$$x^2+y^2+z^2=1 \quad (1)$$

in Cartesiancoordinates or $R = 1$ in spherical coordinates

$$\begin{aligned} x&=R\cos(\alpha)\cos(\beta)\\ y&=R\cos(\alpha)\sin(\beta)\\ z&=R\sin(\beta)\\ R&=sqrt(x^2+y^2+z^2) \end{aligned} \quad (2)$$

where x, y, z – Cartesian coordinates, and α, β – angle.

Let's consider some circle lying on a sphere which center passes through axis OZon distance z_0. Its equation is

$$x^2+y^2=r^2 \qquad (3)$$
$$z=z_0$$

Where the radius r of the circle can be found from

$$z_0^2+r^2=1 \qquad (4)$$

If $z_0 > 0$ circle is located in the top part of the sphere, if $z_0 < 0$ circle is located at the bottom of the sphere, with $z0 = 0$ – circle passes through the "equator" areas, and if $z_0 = + -1$ – circle reduce in a point. From equations (3), (4) follows

$$x^2+y^2=1-z_0^2 \qquad (5)$$
$$z=z_0$$

In parametric form this equations is a two-parameter family

$$x=(1-z_0^2)cos(\alpha)$$
$$y=(1-z_0^2)sin(\alpha) \qquad (6)$$
$$z=z_0$$

and we receive $F(z_0,x,y)=0$.

Let's consider the general case, when the circle is not parallel to the plane OXY.

The sphere surface is represented the equation:

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2, \qquad (7)$$

where x_0, y_0, z_0 – coordinates of the center of the sphere, and R is the radius of the sphere.

The sphere with the center in the beginning of coordinates will correspond to the equation:

$$x^2+y^2+z^2=R^2, \qquad (8)$$

where R – radius of the sphere.

The equation of a circle lying on the surface of the sphere can get the intersection of the main and ancillary areas and the center. It corresponds to the coordinates (x_c, y_c, z_c) and radius Rc, and who is also the radius of the desired circle.

Each point of the circle will be required from the system of equations:

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2, \qquad (9)$$
$$(x_c-a)^2+(y_c-b)^2+(z_c-c)^2=R_c^2,$$

Each point of the circle (x_c, y_c, z_c) lying on the surface of the sphere in the parameter space corresponds to a sphere with a radius of the desired circle Rc and center at point (a, b, c) (see Figure 2).

If the problem is put to find a circle on a surface of sphere of in advance known radius the phase space will be three-dimensional: $H (a, b, c)$.

If the circle radius is not known beforehand, then the phase space of the parameters will be a four-dimensional: $H (a, b, c, Rc)$, where (a, b, c) – the circle center, Rc - radius of the desired circle. Increase the dimension of the phase space will lead to a significant increase in computational complexity of the task.

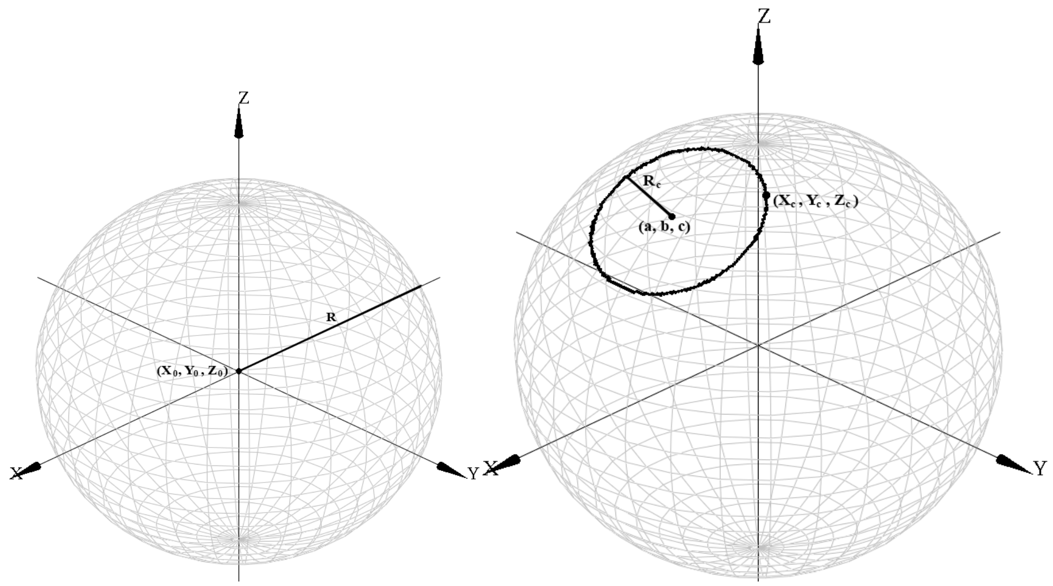

Figure 2 – The circle on the surface of sphere

Then the algorithm of detecting of a circle with the set radius on a sphere surface will be the following.

Step 1. A choice of a grid of digitization of phase space. At this stage, we will step rate for each parameter of the circle. This choice will depend on the speed and efficiency of the algorithm.

Step 2. Accumulator filling. Filling is done by exhaustive search of points of interest in view of (9). Each point of the phase space can be put a number (counter), indicating the number of points of interest on the surface of the sphere.

Step 3. The accumulator analysis. In an accumulator matrix the counter with the maximum value is searched.

Step 4. Circle search. Each cell of the accumulator array is value of phase space so, , and thus it sets a point of the center of a circle in space. Crossing of the basic sphere and sphere with the center to which there corresponds a maximum in the accumulator, will give a required circle.

If it is necessary to find more than one circle of the set radius on a sphere surface to the general algorithm the additional, fifth step is added.

Step 5. Subtraction from the accumulator. For points of the found sphere. Transition to a step 3.

The proposed algorithm will allow to find the coordinates of the circle center *(a, b, c)* of a given radius. If it is necessary to find a problem a circle of unknown radius it is necessary to use this algorithm cyclically for a range of values of possible radiuses. On each step we select for circles with the set radius. On each step of algorithm it is necessary to remember found values of coordinates of the center of a circle, its radius and the maximum value of the counter of a storage file in the separate table. After a finding of parameters of a circle for each of possible radiuses, it is necessary to analyzed the created table. Of all counters taken out in the table gets out with the maximum value. The required circle will have co-ordinates of the centre and radius of value which correspond to the maximum counter.

For the period of calculation of algorithm of search of a circle on a sphere surface, with unknown radius, the factor of scaling which sets the size of a cell of a grid of digitization of phase space will influence in a greater degree.

Algorithm testing

Results of testing of algorithm have been spent more than for 40 copies of images and shown in Figures 3.

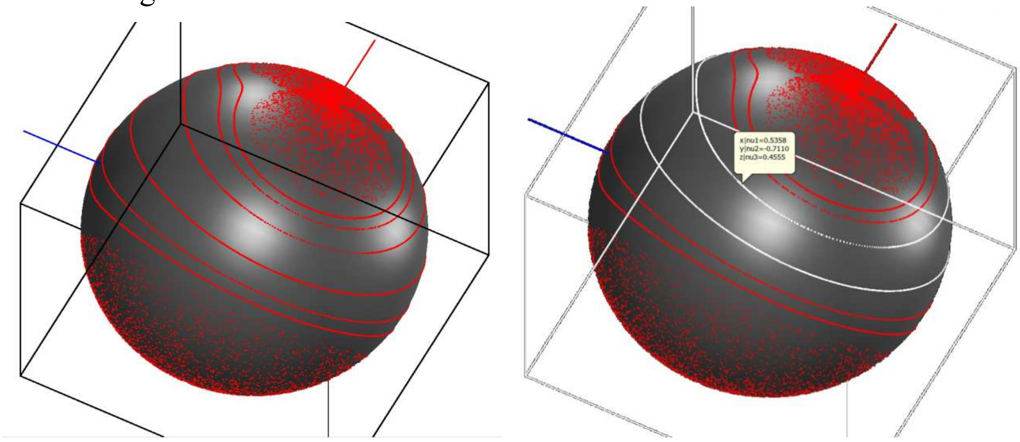

Figure 3 – The circle on the Poisson sphere

Also results of testing show that by means of this algorithm the circle is allocated on sphere well enough. However this problem dares by means of full search, and to concern to np – problems. It computing complexity is equal O (N*M).

Conclusions

In this article the algorithm of a detecting of a circle on the spherical surface, based on generalization of Hough transform, has been presented. Thus, the task of detection and recognition of three-dimensional convex closed analytic curves constructed on the Poisson sphere by means the Poincare section is solved.

Also a generalization of the Hough transform will recognize other types of investigation of closed curves is proposed and classify them, and then to draw a conclusion on character of behavior of dynamical system. This question will be considered in the further works.

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