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THE STRESS STATE OF THE CONICAL LAYERED MEDIUM

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An infinite conic layered cone which represents sequence of conical adjacent funnels, inserted one into another, is considered. Each of these funnels differs by their shear modulus. Conditions of an ideal contact are fulfilled between the adjacent conical surfaces. An external conical surface is loaded by tangent torsion stress. An exact solution of corresponding one-dimensional boundary problem is constructed in domain of Mellin's transformation, which is applied directly to the torsion equation. A formulating of boundary conditions in the matrix form allows to obtain recurrent type relations for unknown constants of a general solution for each layer. The proposed solving method leads to the exact solution which is independent of the layers' number. Mellin's transformation is inversed to finish the construction of the formulas for displacements and stress.

Keywords: conical layered medium, integral transformation, vector boundary problem

Introduction

An interest to the stress state of conical layered medium investigations is due to development of corresponded mathematical methods for such a problems' solving, and to a wide range of practical applications of the investigation's results in an engineering.

A lot of present day works are dedicated to a numerical approach to the solving the problems on a stress state of layered mediums ([1–3]), but numerical methods are not very effective in this case, because during layered mediums' modeling it is necessary to change a distance between the grid's points. That is why working up of an analytical solving methods is actual. Last time in the papers [4, 5] a new approach to the solving of inhomogeneous medium problems based on the further development of the factorization method was proposed for an investigation and application of block structures. For the analytical solution's construction the multilayered step model is used during the stress state analysis of the layered materials in three-dimensional case [6]. A new effective method of the axisymmetrical mixed problems' solving for stratified mediums is developed in [7]. An exponential model reducing the problem to the integral equation is worked out for functionally-gradient materials by [8, 9]. Dynamic problems for the inhomogeneous mediums are also actively investigated during past time [10–12].

The torsion of a conical body under static and dynamic loadings is investigated [13–16].

A new analytical method of a conical layered medium investigation was proposed by G. Popov in [17, 18]. An initial problem in terms of Lame's equations is reduced to the one-dimensional problem. The integral transformation with respect to space coordinates is applied directly to the equilibrium equations but not to their representations through the harmonic or other functions as they do it traditionally. In the transformations' domain the exact solution of the problem is constructed with a help of matrix differential calculus, and besides it is constructed independently of the layers' quantity. This approach is used for a solving of the torsion problem for a conical layered elastic cone.

The formulation of the initial boundary value problem

The torsion of the elastic circular infinite cone $0<r<\infty, 0<\theta<\omega_n, -\pi\le\varphi<\pi$ (r, θ, φ — the spherical system of coordinates) is considered. The cone's shear module G stepwise changes on the conical surfaces $\theta=\omega_j$, $j=\overline{1,n-1}$. The external surface $\theta=\omega_n$ is loaded by the tangent torsion stresses. Let's set the layer $\omega_{j-1}<\theta<\omega_j$ and input the designation for its shear modulus G_j and the displacement $u_\varphi^j(r,\theta)=w_j(r,\theta)$, $j=\overline{1,n-1}$. At the problem's formulation Lame's equations are reduced to one equation relative to the displacement $w_j(r,\theta)$ [19]

$$\left[r^2 w_j'(r,\theta)\right]' + \frac{\left[\sin\theta\, w_j^\bullet(r,\theta)\right]^\bullet}{\sin\theta} - \frac{w_j(r,\theta)}{\sin^2\theta} = 0, \qquad (1)$$
$$0<r<\infty, \quad \omega_{j-1}<\theta<\omega_j, \quad j=\overline{1,n}$$

Here and further the stroke denotes the derivative with respect to the variable r and the dot – the derivative with respect to the variable θ. The nonzero stress are defined by the formulas [19]

$$r\tau_{r\varphi}^{(j)}(r,\theta) = G_j\left[r w_j'(r,\theta) - w_j(r,\theta)\right],$$
$$r\tau_{\theta\varphi}^{(j)}(r,\theta) = G_j\left[w_j^\bullet(r,\theta) - ctg\theta\, w_j(r,\theta)\right]. \qquad (2)$$

The boundary condition is defined by the equality

$$\tau_{\theta\varphi}^{(n)}(r,\omega_n) = q(r) \quad \text{or} \quad G_n\left[w_n^\bullet(r,\omega_n) - ctg\omega_n w_n(r,\omega_n)\right], \qquad (3)$$
$$0<r<\infty.$$

The conditions of the ideal contact are fulfilled between the two adjacent layers with the different shear moduli

$$w_j(r,\omega_j) = w_{j+1}(r,\omega_j), \quad r\tau_{\theta\varphi}^{(j)}(r,\omega_j) = r\tau_{\theta\varphi}^{(j+1)}(r,\omega_j), \qquad (4)$$
$$j=\overline{1,n-1}.$$

The last equality can be written in the form, taking into consideration the relations (2)

$$G_j\left[w_j^\bullet(r,\omega_j) - ctg\omega_j w_j(r,\omega_j)\right] = G_{j+1}\left[w_{j+1}^\bullet(r,\omega_j) - ctg\omega_j w_{j+1}(r,\omega_j)\right], \qquad (5)$$
$$j=\overline{1,n-1}.$$

It is necessary to estimate and to investigate the cone's stress state.

The scheme of the problem solution

The Mellin's integral transformation is applied to the relations (1), (3)-(5)

$$w_{j,s}(\theta)=\int_0^\infty w_j(r,\theta)r^{s-1}dr,$$
$$\operatorname{Re}s>0,\quad \omega_{j-1}<\theta<\omega_j,\quad j=\overline{1,n}. \tag{6}$$

As a result, one obtains the one-dimensional boundary problem

$$\frac{\left[\sin\theta w^{\bullet}_{j,s}(r,\theta)\right]^{\bullet}}{\sin\theta}+s(s-1)w_{j,s}(\theta)-\frac{w_{j,s}(\theta)}{\sin^2\theta}=0,\quad \omega_{j-1}<\theta<\omega_j,\quad j=\overline{1,n}; \tag{7}$$

$$G_n\left[w^{\bullet}_{n,s}(\omega_n)-ctg\omega_n w_{n,s}(\omega_n)\right]=q_s,\quad q_s=\int_0^\infty q(r)r^{s-1}dr; \tag{8}$$

$$w_{j,s}(\omega_j)=w_{j+1,s}(\omega_j),$$
$$G_j\left[w^{\bullet}_{j,s}(\omega_j)-ctg\omega_j w_{j,s}(\omega_j)\right]=G_{j+1}\left[w^{\bullet}_{j+1,s}(\omega_j)-ctg\omega_j w_{j+1,s}(\omega_j)\right],\quad j=\overline{1,n-1}. \tag{9}$$

One may be sure that the general solution of the equation (7) is the linear combination of Legendre's functions [20]

$$w_{j,s}(\theta)=C_j^0(s)P_{s-1}^1(\cos\theta)+C_j^1(s)Q_{s-1}^1(\cos\theta),$$
$$j=\overline{2,n},\quad \omega_{j-1}<\theta<\omega_j, \tag{10}$$

where $C_j^0(s)$, $C_j^1(s)$ ($j=\overline{2,n}$) are unknown constants, $P_{s-1}^1(\cos\theta)$, $Q_{s-1}^1(\cos\theta)$ are Legendre's functions. For the case $j=1$ the general solution of the equation (7) is given by the formula

$$w_{1,s}(\theta)=C_1^0(s)P_{s-1}^1(\cos\theta),$$
$$0<\theta<\omega_1. \tag{11}$$

The substitution of the equalities (10) in the conditions (9) leads to the relations

$$C_j^0(s)P_{s-1}^1(\cos\omega_1)+C_j^1(s)Q_{s-1}^1(\cos\omega_j)=C_{j+1}^0(s)P_{s-1}^1(\cos\omega_1)+C_{j+1}^1(s)Q_{s-1}^1(\cos\omega_j),$$
$$G_j\left[C_j^0(s)P_{s-1}^*(\omega_j)+C_j^1(s)Q_{s-1}^*(\omega_j)\right]=G_{j+1}\left[C_{j+1}^0(s)P_{s-1}^*(\omega_j)+C_{j+1}^1(s)Q_{s-1}^*(\omega_j)\right], \tag{12}$$
$$j=\overline{2,n-1}.$$

One must understand here that $P_s^*(\omega_j)$, $Q_s^*(\omega_j)$ are the correspondences

$$P_{s-1}^*(\theta)=P_{s-1}^2(\cos\theta)-ctg\theta P_{s-1}^1(\cos\theta),$$

$$Q^*_{s-1}(\theta) = Q^2_{s-1}(\cos\theta) - ctg\theta Q^1_{s-1}(\cos\theta)$$

Instead of the relation (12) for the case $j=1$ the conjugation's formulas (9) because of the equality (11) take the form

$$\begin{gathered} C^0_1(s)P^1_{s-1}(\cos\omega_1) = C^0_2(s)P^1_{s-1}(\cos\omega_1) + C^1_2(s)Q^1_{s-1}(\cos\omega_1), \\ G_1C^0_1(s)P^*_{s-1}(\omega_1) = G_2\left[C^0_2(s)P^*_{s-1}(\omega_1) + C^1_2(s)Q^*_{s-1}(\omega_1)\right]. \end{gathered} \tag{13}$$

One must proceed from the conjugations' condition (9). Let's input into consideration the matrices and the vectors

$$\mathbf{C}_j(s) = \begin{pmatrix} C^0_j(s) \\ C^1_j(s) \end{pmatrix}, \quad j = \overline{2,n},$$

$$\mathbf{C}_1(s) = \begin{pmatrix} C^0_1(s) \\ 0 \end{pmatrix}, \tag{14}$$

$$\mathbf{g}_j = \begin{pmatrix} 1 & 0 \\ 0 & G_j \end{pmatrix}, \quad j = \overline{1,n};$$

$$\begin{gathered} \mathbf{R}_j(s) = \begin{pmatrix} P^1_{s-1}(\cos\omega_j) & Q^1_{s-1}(\cos\omega_j) \\ P^*_{s-1}(\omega_j) & Q^*_{s-1}(\omega_j) \end{pmatrix}, \quad j = \overline{1,n-1}, \\ \mathbf{R}^0_1(s) = \begin{pmatrix} P^1_{s-1}(\cos\omega_j) & 0 \\ P^*_{s-1}(\omega_j) & 0 \end{pmatrix}. \end{gathered} \tag{15}$$

One can write the conjugation's conditions (12) with regard of these matrices and vectors in the form

$$\begin{gathered} \mathbf{g}_j\mathbf{R}_j(s)\mathbf{C}_j(s) = \mathbf{g}_{j+1}\mathbf{R}_j(s)\mathbf{C}_{j+1}(s), \\ j = \overline{2,n-1}. \end{gathered} \tag{16}$$

For the case $j=1$ the conjugation's condition correspondently to the relation (13) is written

$$\mathbf{g}_1\mathbf{R}^0_1(s)\mathbf{C}_1(s) = \mathbf{g}_2\mathbf{R}_1(s)\mathbf{C}_2(s). \tag{17}$$

One can obtain from the formulas (16), (17) the relations that connect the coefficients on the two adjacent layers

$$\begin{gathered} \mathbf{C}_{j+1}(s) = \mathbf{R}^{-1}_j(s)\mathbf{g}^{-1}_{j+1}\mathbf{g}_j\mathbf{R}_j(s)\mathbf{C}_j(s), \\ j = \overline{2,n-1}; \end{gathered} \tag{18}$$

$$\mathbf{C}_2(s) = \mathbf{R}^{-1}_1(s)\mathbf{g}^{-1}_2\mathbf{g}_1\mathbf{R}^0_1(s)\mathbf{C}_1(s). \tag{19}$$

The matrix $\mathbf{\Omega}_1^0(s)=\mathbf{R}_1^{-1}(s)\mathbf{g}_2^{-1}\mathbf{g}_1$ after calculations of the inverse matrices in it can be represented in the form

$$\mathbf{\Omega}_1^0(s)=\begin{pmatrix}\Phi_{11}^0(s) & \Phi_{12}^0(s)\\ \Phi_{21}^0(s) & \Phi_{22}^0(s)\end{pmatrix}=-\sin\omega_1\begin{pmatrix}Q_{s-1}^*(\omega_1) & -G_1^*Q_{s-1}^1(\cos\omega_1)\\ -P_{s-1}^*(\omega_1) & G_1^*P_{s-1}^1(\cos\omega_1)\end{pmatrix},$$
$$G_1^*=\frac{G_1}{G_2}. \tag{20}$$

One must use the relation, that defines Wronsky's determinant for Legendre's functions (3.4(25), [20]), during obtaining of this formula. Taking into consideration the equalities (15), (20) one can rewrite the formula (19)

$$\mathbf{C}_2(s)=\mathbf{\Omega}_1^0(s)\begin{pmatrix}P_{s-1}^1(\cos\omega_1)\\ P_{s-1}^*(\omega_1)\end{pmatrix}\mathbf{C}_1^0(s)=$$
$$=-\begin{pmatrix}Q_{s-1}^*(\omega_1) & -G_1^*Q_{s-1}^1(\cos\omega_1)\\ -P_{s-1}^*(\omega_1) & G_1^*P_{s-1}^1(\cos\omega_1)\end{pmatrix}\begin{pmatrix}P_{s-1}^1(\cos\omega_1)\\ P_{s-1}^*(\omega_1)\end{pmatrix}\frac{\mathbf{C}_1^0(s)}{(\sin\omega_1)^{-1}}. \tag{21}$$

Introducing the designation

$$\mathbf{\Omega}_j(s)=\mathbf{R}_j^{-1}(s)\mathbf{g}_{j+1}^{-1}\mathbf{g}_j\mathbf{R}_j(s), \tag{22}$$

formula (18) can be represented in the form

$$\mathbf{C}_{j+1}(s)=\mathbf{\Omega}_j(s)\mathbf{C}_j(s),$$
$$j=\overline{2,n-1}. \tag{23}$$

The equality $\mathbf{C}_n(s)=\mathbf{\Omega}_{n-1}(s)\mathbf{C}_{n-1}(s)$ leads from the formula (23), which is why one can obtain

$$\mathbf{C}_n(s)=\mathbf{\Omega}_{n-1}(s)\mathbf{\Omega}_{n-2}(s)\ldots\mathbf{\Omega}_2(s)\mathbf{C}_2(s),\quad n\ge 3, \tag{24}$$

(when *n=2* the formula (21) is correct). Introducing the matrix

$$\mathbf{\Phi}_n(s)=\mathbf{\Omega}_{n-1}(s)\mathbf{\Omega}_{n-2}(s)\ldots\mathbf{\Omega}_2(s)\mathbf{\Omega}_1^0(s)=\begin{pmatrix}\Phi_{11}(s) & \Phi_{12}(s)\\ \Phi_{21}(s) & \Phi_{22}(s)\end{pmatrix}, \tag{25}$$

with the help of formulas (24), (19) and (25) one obtains the correspondence

$$\mathbf{C}_n(s)=\begin{pmatrix}C_n^0(s)\\ C_n^1(s)\end{pmatrix}=\mathbf{\Phi}_n(s)\begin{pmatrix}P_{s-1}^1(\cos\omega_1) & C_1^0(s)\\ P_{s-1}^*(\omega_1) & C_1^0(s)\end{pmatrix}. \tag{26}$$

Let's satisfy the boundary condition (3), that can be represented with regard of (10) in the form

$$G_n\left[C_n^0(s)P_{s-1}^*(\omega_n)+C_n^1(s)Q_{s-1}^*(\omega_n)\right]=q_s \tag{27}$$

As a result of the formula (26), one can obtain

$$C_1^0(s)[G_n\Delta_n(s)]^{-1}q_s,$$

$$\Delta_n = P_{s-1}^*(\omega_n)[\Phi_{11}(s)P_{s-1}^1(\cos\omega_1) + \Phi_{12}(s)P_{s-1}^*(\omega_1)] + \tag{28}$$

$$+ Q_{s-1}^*(\omega_n)[\Phi_{21}(s)P_{s-1}^1(\cos\omega_1) + \Phi_{22}(s)P_{s-1}^*(\omega_1)].$$

With the help of the formula (21) one constructs $\mathbf{C}_2(s)$, and the rest of the coefficients must be found with the relation (23). For example, $\mathbf{C}_3(s) = \Omega_2(s)\mathbf{C}_2(s)$ and so on. Mellin's transformations of the displacements can be written by the formulas (10), (11). The required displacements are obtained from the formulas

$$w_j(r,\theta) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}[C_j^0(s)P_{s-1}^1(\cos\theta) + C_j^1(s)Q_{s-1}^1(\cos\theta)]r^{-s}ds,$$

$$w_1(r,\theta) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}C_1^0(s)P_{s-1}^1(\cos\theta)r^{-s}ds, \quad j=\overline{2,n}. \tag{29}$$

where γ is the small positive quantity.

Conclusions

1) The problem of the conical layered medium torsion is solved by a new approach. The core of the proposed method is in a direct application of the integral transformation to the equilibrium equations (to the torsion equation in this case), instead of an applying it to the well known different representations of these equations through the harmonic or any other functions as it is done traditionally. The further reduction of initial problem's boundary conditions in the transformation space to the vector form allows the construction of the exact solution, where coefficients are calculated by the recurrent formula.

2) The exact solution of the torsion problem for the conical layered elastic cone is obtained when on the external layer the torsion stress are given. The conditions of the ideal contact are fulfilled on the adjacent layers.

3) The proposed approach allows to obtain the solution of the axisymmetrical problem for the elastic conical layered medium when the conditions of the ideal contact are fulfilled on the adjacent layers.

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НАПРУЖЕНИЙ СТАН КЛИНУВАТО-ШАРУВАТОГО СЕРЕДОВИЩА

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Розглядається нескінченний шаруватий конус, що заповнює область, що представляє собою послідовність воронок-шарів, вставлених одна в іншу. Модуль зсуву конуса стрибкоподібно змінюється на конічних поверхнях. Між сусідніми шарами з різними модулями зсуву виконуються умови ідеального контакту. До зовнішньої поверхні конуса прикладені дотичні напруження. Потрібно визначити зміщення конуса, що задовольняють рівнянню кручення при крайових умовах, а також дослідити напруги, що виникають на поверхні конічних шарів. Для розв'язання задачі застосовується інтегральне перетворення по радіальній координаті. Отримана одновимірна задача розв'язана точно за допомогою побудованих рекурентних формул, що пов'язують константи загальних рішень кожного шару. Застосування зворотного перетворення завершує побудову точного рішення. Задача деталізована для окремих випадків шаруватості.

Ключові слова: клинувато-шарувате середовище, інтегральне перетворення, векторна крайова задача

НАПРЯЖЕННОЕ СОСТОЯНИЕ КЛИНОВИДНО-СЛОИСТОЙ СРЕДЫ

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Рассматривается бесконечный слоистый конус, заполняющий область, представляющую собой последовательность воронок-слоев, вставленных одна в другую. Модуль сдвига конуса скачкообразно меняется на конических поверхностях. Между соседними слоями с различными модулями сдвига выполняются условия идеального контакта. К внешней поверхности конуса приложены касательные напряжения. Требуется определить смещения конуса, удовлетворяющие уравнению кручения при краевых условиях, а также исследовать напряжения, возникающие на поверхности конических слоев. Для решения задачи применяется интегральное преобразование по радиальной координате. Полученная одномерная задача решена точно с помощью построенных рекуррентных формул, связывающих константы общих решений каждого слоя. Применение обратного преобразования завершает построение точного решения. Задача детализирована для частных случаев слоистости.

Ключевые слова: клиновидно-слоистая среда, интегральное преобразование, векторная краевая задача