УДК 621.391

Informatics and Mathematical Methods in Simulation
Vol. 5 (2015), No. 4, pp. 318-324

STRUCTURAL PROPERTIES OF THINNED MATRICES OF THE EQUIVALENCE CLASS PERFECT BINARY ARRAYS

N.I. Kushnirenko, V.Y. Chechelnytskiy

Odessa National Polytechnic University,
1, Shevchenko Ave., Odessa, 65044, Ukraine; e-mail: natalka_kni@ukr.net

The paper discusses the properties of perfect binary arrays and their thinned matrices when the cyclic shifts of rows and/or columns are performed. The relationship between the cyclic shifts parameters of generator array and its thinned matrices found, the location of thinned matrices when performing operation of interleaving revealed. The discovered properties of equivalence class thinned matrices will allow design of the cost-effective schemes of generators for perfect binary arrays of different orders. Such schemes can be used for cryptographic transmission of information, for broadband signals construction and other purposes.

Keywords: perfect binary array, thinned matrices, generator PBA, cyclic shift parameters, equivalence class PBA generator

Introduction

In recent years, a lot of domestic and foreign scientific papers are dedicated to the issues of Perfect Binary Arrays (PBA) application. Such structures are widely used to solve various problems of radio engineering, for example: synthesis of antenna aperture, perfect time-frequency codes design, new classes of block error-correcting codes design, new classes of orthogonal, biorthogonal and minimax noise-like signals with the multiloop cyclic shift property design, encryption tasks and so forth [1, 2]. However, the theory of the PBA requires further research. In particular, the task of an efficient PBA generator design, which takes into consideration structural properties of the PBA and their thinned matrices, must be solved.

The PBA is two-dimensional matrix of size $N \times N$ [2]

$$P(N) = \left\| p_{i,j} \right\|, \tag{1}$$

where $p_{i,j} \in \{+1, -1\}$ — elements of PBA;

$i = \overline{0, N-1}$ — numbers of rows of PBA;

$j = \overline{0, N-1}$ — numbers of columns of PBA;

which has an ideal two-dimensional periodic autocorrelation function (2D PACF)

$$R(N) = \left\| r_{m,n} \right\| = \begin{bmatrix} N^2 & 0 \cdots 0 \\ 0 & 0 \cdots 0 \\ \vdots & \vdots \ddots \vdots \\ 0 & 0 \cdots 0 \end{bmatrix}, \tag{2}$$

where $m = \overline{0, N-1}$ — numbers of 2D PACF rows;

$n = \overline{0, N-1}$ — numbers of 2D PACF columns.

Elements of 2D PACF (2) of P0BA are calculated using an equation

$$r_{m,n} = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} p_{i,j} p_{i+m,j+n} = \begin{cases} N^2, \text{ when } m = n = 0; \\ 0, \text{ others } m \text{ and } n, \end{cases} \qquad (3)$$

while values *i*+*m* and *j*+*n* are calculated modulo *N*.

An example of the PBA $P(4)$ size of 4×4 and its 2D PACF $R(4)$ respectively given below

$$P(4) = \begin{bmatrix} ++-- \\ ++++ \\ -++- \\ -+-+ \end{bmatrix}, \ R(4) = \begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (4)$$

Here and further the elements of PBA «+1» and «–1» denoted as «+» and «–» respectively.

It is known the PBAs of order $N = 2^k$ and $N = 3 \cdot 2^k$, where $k = 1, 2, 3, 4, \ldots, k$ — non-negative integer [3].

Research, given in this paper was performed within the Research work "Methods of informational security in broadband telecommunication systems", which is carried out by Information Security Department, Odessa National Polytechnic University.

The aim of the research

The research is devoted to the consideration of the properties of the equivalence class perfect binary arrays thinned matrices while the cyclic shifts of its rows and/or columns are performed. Such properties are able to simplify design of the equivalence class PBA generator.

The following problems must be solved to achieve this:

1. Evaluate the mutual properties of the PBAs and its thinned matrices while the cyclic shifts of their rows and/or columns are performed.
2. Determine the relationship between the cyclic shift parameters of generator PBA and the cyclic shift parameters of thinned matrices.
3. Determine the relationship between the cyclic shift parameters of generator PBA and the location of thinned matrices when interleaving is performed.

Main Body

It is known [1], that each PBA of arbitrary order *N* can be a generator (base) to build the equivalence class of PBAs — *E*(*N*)-class. Such class can be obtained from a generator matrix by means of cyclic shifts of its rows and/or columns. Thus, if any arbitrary PBA is chosen as a generator, on its basis an *E*(*N*)-class can be constructed and the power of it is given by

$$S_{E(N)} = N^2. \qquad (5)$$

The union of all equivalence classes of PBA enables to construct a complete *U*(*N*)-class of PBAs. The random selection of any PBA from each *E*(*N*)-class as a base allows to construct the generator class of PBAs — П(*N*)-class, the power of which when $N = 2^k > 4$ is given by [3]

$$S_{\Pi(N)} = \begin{cases} 7^{(k-1)/2} \cdot 3^{(k-1)/2} \cdot 2^{2^{k+1}-2(k+1)}, & \text{если } k \text{ нечетное;} \\ 7^{(k-2)/2} \cdot 3^{k/2} \cdot 2^{2^{k+1}-2(k+1)}, & \text{если } k \text{ четное.} \end{cases} \qquad (6)$$

The power of generator Π(2)-class of PBAs is $S_{\Pi(2)} = 2$, the power of Π(4)-class — $S_{\Pi(4)} = 12$.

Consider the next PBA as a generator

$$P_0(4) = \begin{bmatrix} - & + & - & - \\ + & + & + & - \\ - & + & - & - \\ - & - & - & + \end{bmatrix}, \tag{7}$$

where the subscript denotes its belonging to one of equivalence classes.

On the basis of given PBA (7) by means of cyclic shift of rows and/or columns operations the corresponding *E*(4)-class (Table 1) was constructed.

Table 1.

Equivalence E(4)-class of PBAs

	$k2 = 0$	$k2 = 1$	$k2 = 2$	$k2 = 3$
$k1 = 0$	$P_0^{[0,0]}(4) = \begin{bmatrix} - & + & - & - \\ + & + & + & - \\ - & + & - & - \\ - & - & - & + \end{bmatrix}$	$P_0^{[0,1]}(4) = \begin{bmatrix} + & - & - & - \\ + & + & - & + \\ + & - & - & - \\ - & - & + & - \end{bmatrix}$	$P_0^{[0,2]}(4) = \begin{bmatrix} - & - & - & + \\ + & - & + & + \\ - & - & - & + \\ - & + & - & - \end{bmatrix}$	$P_0^{[0,3]}(4) = \begin{bmatrix} - & - & + & - \\ - & + & + & + \\ - & - & + & - \\ + & - & - & - \end{bmatrix}$
$k1 = 1$	$P_0^{[1,0]}(4) = \begin{bmatrix} - & - & - & + \\ - & + & - & - \\ + & + & + & - \\ - & + & - & - \end{bmatrix}$	$P_0^{[1,1]}(4) = \begin{bmatrix} - & - & + & - \\ + & - & - & - \\ + & + & - & + \\ + & - & - & - \end{bmatrix}$	$P_0^{[1,2]}(4) = \begin{bmatrix} - & + & - & - \\ - & - & - & + \\ + & - & + & + \\ - & - & - & + \end{bmatrix}$	$P_0^{[1,3]}(4) = \begin{bmatrix} + & - & - & - \\ - & - & + & - \\ - & + & + & + \\ - & - & + & - \end{bmatrix}$
$k1 = 2$	$P_0^{[2,0]}(4) = \begin{bmatrix} - & + & - & - \\ - & - & - & + \\ - & + & - & - \\ + & + & + & - \end{bmatrix}$	$P_0^{[2,1]}(4) = \begin{bmatrix} + & - & - & - \\ - & - & + & - \\ + & - & - & - \\ + & + & - & + \end{bmatrix}$	$P_0^{[2,2]}(4) = \begin{bmatrix} - & - & - & + \\ - & + & - & - \\ - & - & - & + \\ + & - & + & + \end{bmatrix}$	$P_0^{[2,3]}(4) = \begin{bmatrix} - & - & + & - \\ + & - & - & - \\ - & - & + & - \\ - & + & + & + \end{bmatrix}$
$k1 = 3$	$P_0^{[3,0]}(4) = \begin{bmatrix} + & + & + & - \\ - & + & - & - \\ - & - & - & + \\ - & + & - & - \end{bmatrix}$	$P_0^{[3,1]}(4) = \begin{bmatrix} + & + & - & + \\ + & - & - & - \\ - & - & + & - \\ + & - & - & - \end{bmatrix}$	$P_0^{[3,2]}(4) = \begin{bmatrix} + & - & + & + \\ - & - & - & + \\ - & + & - & - \\ - & - & - & + \end{bmatrix}$	$P_0^{[3,3]}(4) = \begin{bmatrix} - & + & + & + \\ - & - & + & - \\ + & - & - & - \\ - & - & + & - \end{bmatrix}$

In Table 1 parameters *k*1 and *k*2 denote number of cyclic shifts of base PBA's rows down and number of cyclic shifts of columns to the left respectively. Let denote by $P_0^{[k1,k2]}(4)$ any PBA belonging to an equivalence *E*(4)-class.

An arbitrary square PBA $P(N) = \left\| p_{i,j} \right\|$ of order *N* can be thinned in the spatial coordinates [4] in order to obtain four matrices

$$\left.\begin{aligned} A(N/2) &= \left\| a_{m,n} \right\|, \\ B(N/2) &= \left\| b_{m,n} \right\|, \\ C(N/2) &= \left\| c_{m,n} \right\|, \\ D(N/2) &= \left\| d_{m,n} \right\|, \end{aligned}\right\}, \; m = \overline{0, N/2-1}, \; n = \overline{0, N/2-1}. \tag{8}$$

Let call these structures the thinned matrices. The dimension of such matrices is $N/2 \times N/2$.

The elements of thinned matrices are calculated using the next equation

$$\left.\begin{aligned} a_{m,n} &= p_{2i,2j}, \\ b_{m,n} &= p_{2i,2j+1}, \\ c_{m,n} &= p_{2i+1,2j}, \\ d_{m,n} &= p_{2i+1,2j+1} \end{aligned}\right\}, \; \left.\begin{aligned} i = \overline{0, N/2-1}, \, m = \overline{0, N/2-1}, \\ j = 0, N/2-1, \, n = 0, N/2-1 \end{aligned}\right\}, \tag{9}$$

where i — number of PBA's row;

j — number of PBA's column;

m — number of thinned matrix row;

n — number of thinned matrix column.

Consider an operation of thinning in spatial coordinates and obtaining the thinned matrices $A(N/2)$, $B(N/2)$, $C(N/2)$, $D(N/2)$ on the example of PBA of order N=4. The elements of PBA are located as follows

$$P(4)=\begin{bmatrix} p_{0,0} & p_{0,1} & p_{0,2} & p_{0,3} \\ p_{1,0} & p_{1,1} & p_{1,2} & p_{1,3} \\ p_{2,0} & p_{2,1} & p_{2,2} & p_{2,3} \\ p_{3,0} & p_{3,1} & p_{3,2} & p_{3,3} \end{bmatrix}. \tag{10}$$

Operation of thinning PBA (10) in spatial coordinates leads to the construction of four thinned matrices, which constitute the PBA

$$A(N/2)=A(2)=\begin{bmatrix} p_{0,0} & p_{0,2} \\ p_{2,0} & p_{2,2} \end{bmatrix}=\begin{bmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & a_{1,1} \end{bmatrix}, \tag{11}$$

$$B(N/2)=B(2)=\begin{bmatrix} p_{0,0} & p_{0,2} \\ p_{2,0} & p_{2,2} \end{bmatrix}=\begin{bmatrix} b_{0,0} & b_{0,1} \\ b_{1,0} & b_{1,1} \end{bmatrix}, \tag{12}$$

$$C(N/2)=C(2)=\begin{bmatrix} p_{0,0} & p_{0,2} \\ p_{2,0} & p_{2,2} \end{bmatrix}=\begin{bmatrix} c_{0,0} & c_{0,1} \\ c_{1,0} & c_{1,1} \end{bmatrix}, \tag{13}$$

$$D(N/2)=D(2)=\begin{bmatrix} p_{0,0} & p_{0,2} \\ p_{2,0} & p_{2,2} \end{bmatrix}=\begin{bmatrix} d_{0,0} & d_{0,1} \\ d_{1,0} & d_{1,1} \end{bmatrix}. \tag{14}$$

Opposite is also true. The square PBA $H(N)=\left\|h_{i,j}\right\|$ of order N can be constructed from four thinned matrices (11)–(14) of order N/2 by means of spatial interleaving of thinned matrices elements, which is an inverse of the thinning (9).

Equation to calculate elements of PBA using the thinned matrices $A(N/2)$, $B(N/2)$, $C(N/2)$, $D(N/2)$ is given below

$$\left.\begin{array}{l} h_{2i,2j}=a_{m,n}, \\ h_{2i,2j+1}=b_{m,n}, \\ h_{2i+1,2j}=c_{m,n}, \\ h_{2i+1,2j+1}=d_{m,n} \end{array}\right\},\quad \left.\begin{array}{l} i=\overline{0,N/2-1},\ m=\overline{0,N/2-1}, \\ j=0,N/2-1,\ n=0,N/2-1 \end{array}\right\}. \tag{15}$$

While performing the operation of interleaving (15), the thinned matrices can be permuted. The number of these permutations is $4!=24$. Thus, by means of permutations of four thinned matrices $A(N/2)$, $B(N/2)$, $C(N/2)$, $D(N/2)$ and the operation of interleaving, 24 PBAs can be constructed.

In this way, if for construction of PBA the permutation of thinned matrixes $A(N/2)B(N/2)C(N/2)D(N/2)$ is used, the elements of resulting PBA will be located as shown below

$$P(4)=\begin{bmatrix} a_{0,0} & b_{0,0} & a_{0,1} & b_{0,1} \\ c_{0,0} & d_{0,0} & c_{0,1} & d_{0,1} \\ a_{1,0} & b_{1,0} & a_{1,1} & b_{1,1} \\ c_{1,0} & d_{1,0} & c_{1,1} & d_{1,1} \end{bmatrix}.$$

If another permutation is used, e.g. $A(N/2)C(N/2)B(N/2)D(N/2)$, the elements of resulting PBA will be located as follows

$$P(4)=\begin{bmatrix} a_{0,0} & c_{0,0} & a_{0,1} & c_{0,1} \\ b_{0,0} & d_{0,0} & b_{0,1} & d_{0,1} \\ a_{1,0} & c_{1,0} & a_{1,1} & c_{1,1} \\ b_{1,0} & d_{1,0} & b_{1,1} & d_{1,1} \end{bmatrix}.$$

For example, for the PBA (7) the thinned matrices look as follows

$$A(N/2)=A(2)=\begin{bmatrix} - & - \\ - & - \end{bmatrix}, \tag{16}$$

$$B(N/2)=B(2)=\begin{bmatrix} + & - \\ + & - \end{bmatrix}, \tag{17}$$

$$C(N/2)=C(2)=\begin{bmatrix} + & + \\ - & - \end{bmatrix}, \tag{18}$$

$$D(N/2)=D(2)=\begin{bmatrix} + & - \\ - & + \end{bmatrix}. \tag{19}$$

Consider the relationship between the cyclic shift parameters $k1$ and $k2$ of generator PBA and the cyclic shift parameters of its thinned matrices. Then consider location of thinned matrices after the cyclic shifts of generator PBA will be performed.

Let parameters $a1$, $b1$, $c1$, $d1$ and $a2$, $b2$, $c2$, $d2$ denote the number of cyclic shifts of thinned matrices' $A(N/2)$, $B(N/2)$, $C(N/2)$, $D(N/2)$ rows down and columns to the left respectively.

For brevity the thinned matrices will be denoted by A, B, C, and D.

As follows from Table 1 and equation (9) and taking into account (16)–(19), the following combinations of thinned matrices can be obtained (Table 2).

Similar studies have been conducted to PBA of larger orders, in particular for $N=6$ and $N=8$ that can more clearly illustrate the results presented below.

Table 2.

E(4)-class of PBAs presented as combinations of thinned matrices

	$k2=0$	$k2=1$	$k2=2$	$k2=3$
$k1=0$	$A^{[0,0]}B^{[0,0]}C^{[0,0]}D^{[0,0]}$	$B^{[0,0]}A^{[0,1]}D^{[0,0]}C^{[0,1]}$	$A^{[0,1]}B^{[0,1]}C^{[0,1]}D^{[0,1]}$	$B^{[0,1]}A^{[0,0]}D^{[0,1]}C^{[0,0]}$
$k1=1$	$C^{[1,0]}D^{[1,0]}A^{[0,0]}B^{[0,0]}$	$D^{[1,0]}C^{[1,1]}B^{[0,0]}A^{[0,1]}$	$C^{[1,1]}D^{[1,1]}A^{[0,1]}B^{[0,1]}$	$D^{[1,1]}C^{[1,0]}B^{[0,1]}A^{[0,0]}$
$k1=2$	$A^{[1,0]}B^{[1,0]}C^{[1,0]}D^{[1,0]}$	$B^{[1,0]}A^{[1,1]}D^{[1,0]}C^{[1,1]}$	$A^{[1,1]}B^{[1,1]}C^{[1,1]}D^{[1,1]}$	$B^{[1,0]}A^{[1,1]}D^{[1,0]}C^{[1,1]}$
$k1=3$	$C^{[0,0]}D^{[0,0]}A^{[1,0]}B^{[1,0]}$	$D^{[0,0]}C^{[0,1]}B^{[1,0]}A^{[1,1]}$	$C^{[0,1]}D^{[0,1]}A^{[1,1]}B^{[1,1]}$	$D^{[0,1]}C^{[0,0]}B^{[1,1]}A^{[1,0]}$

As follows from Table 2, there exist only four possible combinations of thinned matrices A, B, C, and D with different number of their cyclic shifts of rows or/and columns, in particular

$$A^{[a1,a2]}B^{[b1,b2]}C^{[c1,c2]}D^{[d1,d2]},\ B^{[b1,b1]}A^{[a1,a2]}D^{[d1,d2]}C^{[c1,c2]},$$

$$C^{[c1,c2]}D^{[d1,d2]}A^{[a1,a2]}B^{[b1,b2]},\ D^{[d1,d2]}C^{[c1,c2]}B^{[b1,b2]}A^{[a1,a2]}.$$

Combinations of thinned matrices permutations depend on whether the cyclic shift parameters $k1$ and $k2$ of generator PBA are odd or even. The Table 3 illustrates such relationship.

Table 3.

Combinations of thinned matrices depending on the parameters k1 and k2

$k1$	$k2$	Combinations of thinned matrices
even	even	$A^{[a1,a2]}B^{[b1,b2]}C^{[c1,c2]}D^{[d1,d2]}$
even	odd	$B^{[b1,b1]}A^{[a1,a2]}D^{[d1,d2]}C^{[c1,c2]}$
odd	even	$C^{[c1,c2]}D^{[d1,d2]}A^{[a1,a2]}B^{[b1,b2]}$
odd	odd	$D^{[d1,d2]}C^{[c1,c2]}B^{[b1,b2]}A^{[a1,a2]}$

In addition, there is a dependence of the cyclic shifts parameters of thinned matrices *A*, *B*, *C*, and *D* on parity of the cyclic shifts parameters *k*1 and *k*2 of generator PBA. Let consider such relationship in detail (Table 4)

Table 4.

The cyclic shifts parameters of thinned matrices

$k1,\ k2$	$a1,\ a2$	$b1,\ b2$	$c1,\ c2$	$d1,\ d2$
even	$k1/2$	$k1/2$	$k1/2$	$k1/2$
odd	$(k1+1)/2$	$(k1-1)/2$	$(k1+1)/2$	$(k1-1)/2$

For example, if following cyclic shifts parameters of PBA are given: *k*1=2, *k*2=1, then according to the Table 3 the cyclic shifts of rows and columns of generator PBA by *k*1 and *k*2 elements respectively lead to the structure $B^{[b1,b1]}A^{[a1,a2]}D^{[d1,d2]}C^{[c1,c2]}$. The cyclic shifts parameters of thinned matrices (*a*1, *a*2), (*b*1, *b*2), (*c*1, *c*2), (*d*1, *d*2) can be determined from Table 4. In this way, the following PBA is obtained

$$B^{[1,0]}(N/2)A^{[1,1]}(N/2)D^{[1,0]}(N/2)C^{[1,1]}(N/2),$$

that corresponds to the Table 2.

Conclusions

The properties of PBA and their thinned matrices while the cyclic shifts of rows and/or columns are performed are discussed. The relationship between the cyclic shifts parameters of generator PBA and the cyclic shifts parameters of thinned matrices discovered. Moreover, the relation between the cyclic shifts parameters of generator PBA and location of thinned matrices when performing operation of interleaving revealed.

The results obtained in this study can be utilized in the design of the equivalence class PBA generator which can be used for cryptographic transmission of information, for broadband signals construction and other purposes.

The equivalence class PBA generator can be constructed in various ways:

- store in memory all the PBA of equivalence class and use the switch to select the desired PBA, in this case it is possible to reach a minimum hardware complexity, but it requires a large amount of expensive memory elements;
- store generator PBA in memory and using shift registers obtain the necessary number of rows and then columns shifts; this method has a very low performance;
- use other known methods, each of which has a high hardware complexity or poor performance.

The results presented in this paper will allow the construction of equivalence class PBA generator with stored in memory thinned matrices. Such approach will drastically reduce the complexity of the cyclic shift device since the number of elements in matrices decreases four times. Furthermore, such a device can be implemented on the basis of switches. It allows performing necessary number of cyclic shifts of rows and columns in one clock cycle. Thus, the discovered properties of thinned matrices of equivalence class will allow creating cost-effective schemes of generators for PBAs of different orders.

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СТРУКТУРНІ ВЛАСТИВОСТІ ПРОРІДЖЕНИХ МАТРИЦЬ ЕКВІВАЛЕНТНОГО КЛАСУ ДОСКОНАЛИХ ДВІЙКОВИХ РЕШІТОК

Н.І. Кушніренко, В.Я. Чечельницький

Одеський національний політехнічний університет,
просп. Шевченка, 1, Одеса, 65044, Україна; e-mail: natalka_kni@ukr.net

У роботі розглянуті властивості досконалих двійкових решіток та їх проріджених матриць при виконанні операції циклічного зсуву рядків і/або стовпців. Знайдено взаємозв'язок між параметрами циклічних зсувів породжувальної решітки та її проріджених матриць, встановлено їх розташування при виконанні операції перемежування. Виявлені властивості проріджених матриць еквівалентного класу дозволять створити економічні схеми генераторів досконалих двійкових решіток різних порядків, які можна використовувати для криптографічної передачі інформації, для побудови різних широкосмугових сигналів та інших цілей.

Ключові слова: досконала двійкова решітка, проріджена матриця, породжувальна ДДР, параметри циклічного зсуву, генератор еквівалентного класу ДДР

СТРУКТУРНЫЕ СВОЙСТВА ПРОРЕЖЕННЫХ МАТРИЦ ЭКВИВАЛЕНТНОГО КЛАССА СОВЕРШЕННЫХ ДВОИЧНЫХ РЕШЕТОК

Н.И. Кушниренко, В.Я. Чечельницкий

Одесский национальный политехнический университет,
просп. Шевченко, 1, Одесса, 65044, Украина; e-mail: natalka_kni@ukr.net

В работе рассмотрены свойства совершенных двоичных решеток и их прореженных матриц при выполнении операции циклического сдвига строк и/или столбцов. Найдена взаимосвязь между параметрами циклических сдвигов порождающей решетки и ее прореженных матриц, установлено их расположение при выполнении операции перемежения. Обнаруженные свойства прореженных матриц эквивалентного класса позволят создать экономичные схемы генераторов совершенных двоичных решеток различных порядков, которые можно использовать для криптографической передачи информации, для построения различных широкополосных сигналов и других целей.

Ключевые слова: совершенная двоичная решетка, прореженная матрица, порождающая СДР, параметры циклического сдвига, генератор эквивалентного класса СДР