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THE FIBONACCI Q-MATRIX CODING METHOD

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This paper presents the results of research, formalization and mathematical justification of the Fibonacci Q-matrix coding method. This method allows finding errors in the encoded message with high probability and correcting them in certain cases. The notes on algorithm implementation are given. The developed "block Q-matrix" method based on the standard method is described. The comparative analysis of the algorithms is presented.

Keywords: coding methods, Fibonacci numbers, error detection and correction, Q-matrix.

Introduction

The question of effective encoding and protection of the data in communication channels is rather important in the modern IT sphere.

Most of the known error detection and correction codes make it possible to restore single bits or combinations of bits [1-4], which is surely useful for many fields of application. However, the presented "Fibonacci Q-matix" coding method uses an entirely different approach: it allows restoring one of the predefined parts of the message – no matter how big is – given the condition that the damage affected only that part. The flaw is that errors, even small ones, in other parts of the message make the whole message unreadable.

However, there is an opportunity to develop new methods based on the standard Q-matrix method. The developed "block Q-matrix method" presented in the paper divides the message into fixed-length segments and applies the standard algorithm to them. That allows correcting errors scattered throughout the whole message. Also, in case method fails to restore some damaged segments, only that segments becomes unreadable.

The aim of the research is to study and evolve the coding methods based on the Fibonacci numbers.

The task of the research is to formalize, justify mathematically and analyze the Fibonacci Q-matrix coding method, analyze the ways to improve the algorithm, develop a program library which implements the method and make the characteristic of its work.

«Fibonacci Q-matrix» properties

Fibonacci Q-matrix is a following square 2×2 matrix [5]:

$$Q = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}. \qquad (1)$$

Property 1. There is a property which connects the Q-matrix with the Fibonacci numbers:

$$Q^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}, \tag{2}$$

where F_i is the Fibonacci number *i*.

This can be proved by the induction method.

$$n = 2: \; Q^2 = \begin{bmatrix} 1+1 & 1+0 \\ 1+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

The elements of the Q^2 matrix are the corresponding Fibonacci numbers: $F_1 = F_2 = 1$, $F_3 = 2$.

Assume that $Q^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$ and calculate Q^{n+1}.

$$Q^{n+1} = Q^n \times Q = \begin{bmatrix} F_{n+1} + F_n & F_{n+1} + 0 \\ F_{n+1} + 0 & F_n + 0 \end{bmatrix} = \begin{bmatrix} F_{n+2} & F_{n+1} \\ F_{n+1} & F_n \end{bmatrix}.$$

The property is proved.

Property 2. $\det(Q^n) = (-1)$.

To prove it a determinant of Q-matrix can be calculated:

$$\det(Q) = 1 \times 0 - 1 \times 1 = -1.$$

Using a property stating that a determinant of the product of two square matrixes of the same size equals the product of their determinants [8]:

$$\det(Q^n) = (-1)^n. \tag{3}$$

Consequence. As $\det(Q^n) = F_{n+1}F_{n-1} - F_n^2 = (-1)^n$, then for each three consecutive Fibonacci numbers the following is true:

$$F_{n+1}F_{n-1} - F_n^2 = (-1)^n. \tag{4}$$

Property 3. With the help of the cofactor method [8] the following presentation of the Q^{-n} matrix can be acquired:

$$Q^{-n} = (-1)^n \begin{bmatrix} F_{n-1} & -F_n \\ -F_n & F_{n+1} \end{bmatrix} = \begin{bmatrix} (-1)^n F_{n-1} & (-1)^{n+1} F_n \\ (-1)^{n+1} F_n & (-1)^n F_{n+1} \end{bmatrix}. \tag{5}$$

The standard Fibonacci Q-matrix coding method

Let's look at the standard Fibonacci Q-matrix coding method [5].

Assume there is a message M, which can be divided into 4 parts, represented in numeric values as a 2×2 matrix. Each element m_i is a nonnegative integer number

$$M = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix}. \tag{6}$$

Encoding. An encoded message M' can be fetched in the following way.

$$M' = M \times Q^n = \begin{bmatrix} F_{n+1}m_1 + F_n m_2 & F_n m_1 + F_{n-1} m_2 \\ F_{n+1}m_3 + F_n m_4 & F_n m_3 + F_{n-1} m_4 \end{bmatrix} = \begin{bmatrix} m_1' & m_2' \\ m_3' & m_4' \end{bmatrix}. \tag{7}$$

As F_i and m_i are nonnegative integer numbers, then the elements m_i' of the encoded matrix M' are nonnegative as well.

The power of the Q-matrix can be a random positive number and it serves as an encryption key of this method.

Decoding. The primal message M'' can be fetched from the decoded message M' in the following way.

$$M'' = M' \times Q^{-n} = \begin{bmatrix} m_1'' & m_2'' \\ m_3'' & m_4'' \end{bmatrix} = \begin{bmatrix} (-1)^n F_{n-1} m_1' + (-1)^{n+1} F_n m_2' & (-1)^{n+1} F_n m_1' + (-1)^n F_{n+1} m_2' \\ (-1)^n F_{n-1} m_3' + (-1)^{n+1} F_n m_4' & (-1)^{n+1} F_n m_3' + (-1)^n F_{n+1} m_4' \end{bmatrix}. \tag{8}$$

We need proof that $M'' = M$. For this we must prove that each m_i'' equals m_i :

$$m_1'' = (-1)^n F_{n-1} m_1' + (-1)^{n+1} F_n m_2' =$$

$$= (-1)^n F_{n-1} F_{n+1} m_1 + (-1)^n F_{n-1} F_n m_2 + (-1)^{n+1} F_n^2 m_1 + (-1)^{n+1} F_n F_{n-1} m_2 =$$

$$= (-1)^n F_{n-1} F_{n+1} m_1 - (-1)^n F_n^2 m_1 + (-1)^n F_n F_{n-1} m_2 - (-1)^n F_n F_{n-1} m_2 =$$

$$= (-1)^n m_1 (F_{n-1} F_{n+1} - F_n^2) = (-1)^n \times (-1)^n m_1 = m_1;$$

$$m_2'' = (-1)^{n+1} F_n m_1' + (-1)^n F_{n+1} m_2' =$$

$$= (-1)^{n+1} F_n F_{n+1} m_1 + (-1)^{n+1} F_n^2 m_2 + (-1)^n F_{n+1} F_n m_1 + (-1)^n F_{n+1} F_{n-1} m_2 =$$

$$= (-1)^n F_{n+1} F_n m_1 - (-1)^n F_{n+1} F_n m_1 + (-1)^n F_{n+1} F_{n-1} m_2 - (-1)^n F_n^2 m_2 =$$

$$= (-1)^n m_2 (F_{n+1} F_{n-1} - F_n^2) = (-1)^n \times (-1)^n m_2 = m_2;$$

$$m_3'' = (-1)^n F_{n-1} m_3' + (-1)^{n+1} F_n m_4' =$$

$$= (-1)^n F_{n-1} F_{n+1} m_3 + (-1)^n F_{n-1} F_n m_4 + (-1)^{n+1} F_n^2 m_3 + (-1)^{n+1} F_n F_{n-1} m_4 =$$

$$= (-1)^n F_{n-1} F_{n+1} m_3 - (-1)^n F_n^2 m_4 + (-1)^n F_n F_{n-1} m_3 - (-1)^n F_n F_{n-1} m_4 =$$

$$= (-1)^n m_3 (F_{n-1} F_{n+1} - F_n^2) = (-1)^n \times (-1)^n m_3 = m_3;$$

$$m_4'' = (-1)^{n+1} F_n m_3' + (-1)^n F_{n+1} m_4' =$$

$$= (-1)^{n+1} F_n F_{n+1} m_3 + (-1)^{n+1} F_n^2 m_4 + (-1)^n F_{n+1} F_n m_3 + (-1)^n F_{n+1} F_{n-1} m_4 =$$

$$= (-1)^n F_{n+1} F_n m_3 - (-1)^n F_{n+1} F_n m_4 + (-1)^n F_{n+1} F_{n-1} m_3 - (-1)^n F_n^2 m_4 =$$

$$= (-1)^n m_4 (F_{n+1} F_{n-1} - F_n^2) = (-1)^n \times (-1)^n m_4 = m_4.$$

Error detection and correction

One of the features of this coding method is the possibility of detecting and correcting the errors. From (3) it can be known that:

$$\det(M') = (-1)^n \det(M).$$

By passing the value of the determinant with the message, we can allow the receiver to check whether it matches the determinant of the received matrix before starting to decode. If the message is corrupted, one or more of the elements of the matrix will differ, and the determinants won't match.

As $\det(M') = m_1' m_4' - m_2' m_3'$, we can restore the corrupted part of the message, if the determinant and the other three parts are unharmed.

$$m_1' = \frac{m_2' m_3' + \det(M')}{m_4'},\ m_2' = \frac{m_1' m_4' + \det(M')}{m_3'},$$
$$m_3' = \frac{m_1' m_4' + \det(M')}{m_2'},\ m_4' = \frac{m_2' m_3' + \det(M')}{m_1'}. \quad (9)$$

If it is not obvious which part of the message was corrupted, the correction for each part can be calculated. The corrected part must be integer, so with high probability there will be a single matching result.

If no correction provides integer results, then two or more parts of the message were damaged, and the restoration is likely impossible.

Example:

$$M = \begin{bmatrix} 65 & 115 \\ 104 & 97 \end{bmatrix},\ Q^6 = \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix},$$

$$M' = M \times Q^6 = \begin{bmatrix} 1765 & 1095 \\ 2128 & 1317 \end{bmatrix},$$

$$\det(M') = -5655.$$

Let's add a «corruption» into the second part of the message, changing it to 1112:

$$M^{err} = \begin{bmatrix} 1765 & 1112 \\ 2128 & 1317 \end{bmatrix},$$

$$\det(M^{err}) = -41831 \neq -5655.$$

Let's assume that the first part is damaged, and try to correct it:

$$m_1' = \frac{m_2' m_4' + \det(M')}{m_4'} = \frac{1112 \times 2128 - 5655}{1317} = 1792.47\,.$$

The result is non-integer. Let’s assume the second part was damaged:

$$m_2' = \frac{m_1' m_4' + \det(M')}{m_3'} = \frac{1765 \times 1317 - 5655}{2128} = 1095\,.$$

Let’s assume the correction is true:

$$M'' = M' \times Q^{-n} = \begin{bmatrix} 1765 & 1095 \\ 2128 & 1317 \end{bmatrix} \times \begin{bmatrix} 5 & -8 \\ -8 & 13 \end{bmatrix} = \begin{bmatrix} 65 & 115 \\ 104 & 97 \end{bmatrix}.$$

So, we have restored the message, where one of the parts was corrupted.

Notes on algorithm implementation

The following notes are given for the implementation of the Fibonacci Q-matrix coding method.

1. The encrypted parts of the message along with the determinant are translated into the Fibonacci code [7]. Because each Fibonacci code contains only two consecutive 1-digits, which are located at the end of the code, it can help to determine which parts of the message were corrupted. Also, if a binary data transmission is used, we won’t have to translate the encrypted values into the binary numeral system, because the Fibonacci code consists only of 0- and 1-digits. In this way we avoid unnecessary calculations while transforming numbers from one numeral system into another.
2. The encrypted parts are brought to the common length by adding 0-bits to the lesser parts. The message is increased by 1-2 bytes in most cases this way. In computer implementation developed for this method the common length of each part is increased till it becomes divisible by 8, so that each byte has only one corresponding part of the message.
3. The writing of encoded data is done in the following way. The size of the determinant in bytes and its sign are written. The determinant is written. The decoded parts are written.
4. The reading of encoded data is done in the following way. The size of the determinant and its sign are read. The determinant is acquired by reading the number of bytes equal to its size. The rest of the message is divided into 4 equal-sized parts, each one has its 0-bits deleted from the end until the first 1-bit is met.
5. The data read is checked for the correct Fibonacci codes. If the determinant is damaged, the correcting of other errors will be useless. If the Fibonacci code of the determinant is correct, and one part of the message is damaged, that part is restored. If the Fibonacci codes of the determinant and the encoded parts are correct, but the received determinant doesn’t match the calculated determinant, an attempt to restore each part is made, and an integer result is treated as a correct one. If no attempt gave the integer result, the message is decoded with errors.

Block Q-matrix algorithm

The following algorithm has been developed on the basis of studied properties of Fibonacci Q-matrix and the Fibonacci code.

Assume there is a message M , which can be divided into segments 4 bytes each:

$$M = \{b_{11}b_{12}b_{13}b_{14}, b_{21}b_{22}b_{23}b_{24}, ..., b_{n1}b_{n2}b_{n3}b_{n4}\}, 0 \le b_{ij} \le 255 . \quad (10)$$

Let's encode the message M into M' by applying the standard Fibonacci Q-matrix method for each quad of bytes

$$M' = \{B'_1, B'_2, ..., B'_n\}, \quad (11)$$

where B'_i is a group of bytes $b_{i1}b_{i2}b_{i3}b_{i4}$ encoded with the help of the standard Fibonacci Q-matrix method.

The power of Q-matrix $n = 5$ is chosen for the implementation of this method. The implementation of the standard Fibonacci Q-matrix method is done as described previously.

To decode the message, the standard Q-matrix method needs to be applied for each encoded segments B'_i. However, in order to separate B'_i correctly in the encoded message, the length of these segments must be a fixed length of k bytes. To find it, we'll calculate the largest possible values of determinant and the elements of the encoded matrix.

The biggest value of determinant in case $n = 5$ is achieved when $\det(M') = m'_1m'_4 - m'_2m'_3 = 255 \times 255 - 0 \times 0 = 65025$. Translated into Fibonacci code it will be written as 100001000101010000001011; the length of this presentation is 24 bits, i.e. 3 bytes.

The biggest value of the element of the encoded matrix is achieved when $m'_i = F_{n+1}m_i + F_n m_{i+1} = 8 \times 255 + 5 \times 255 = 3315$. Translated into Fibonacci code it will be written as 001010100100010011; the length of this presentation is 18 bits. All four elements of the matrix will have their collective length no more than 9 bytes.

Therefore, considering that at least 1 more byte is needed for the sign and the size of determinant, we can say that each encoded segment B'_i won't exceed $k = 13$ bytes. If the segment is smaller, the missing bits are filled with the values which can be ignored by the decoding algorithm (for example, 0-bits).

So, the optimal block Q-matrix method produces 104 bits (13 bytes) of encoded message from each 32 bits (4 bytes) on the initial message.

It is worth noting that the computer implementation, the work of which is described later in the paper, uses the implementation of the standard Q-matrix method with more massive amount of output data (providing divisibility by 8 for encoded parts), and each 4 bytes are encoded into 18 bytes.

Interleaving modification

The described algorithm can handle the errors in different parts of the message, restoring each 4-byte segment if the damage affected only one byte of this segment. However, this means that the errors must affect only single bytes, not consecutive ones. In any case, the undamaged segments of the message will remain readable.

To increase the effectiveness, the algorithm is modified with the help of interleaving [9]: the encoded bytes are shuffled in certain order, and the consecutive damaged bytes will affect different 4-byte segments in the end.

In the computer implementation, the first bytes of each segment are placed at the beginning, then the second ones, and so on, till the last bytes. This process allocates bytes of each segment equally, and the errors will need to "guess" several bytes of each segment to prevent it from restoration. Even if it happens, only the 4 bytes of this segment will become unreadable in the decoded message.

Results of the numerical experiments

A C++ library has been developed, which performs encoding and decoding with the help of the standard Fibonacci Q-matrix method and the block method, applying the notes listed above.

The ability to correct errors has been tested for each algorithm. The given number of byte damages has been generated for the message: in one case the single damaged bytes were randomly scattered throughout the message, in another case the consecutive bytes were damaged in the random position. For each number of errors 100 tests were handled, which calculated the number of completely restored message cases.

The message was 576 bytes long. For the standard Fibonacci Q-matrix method ($n = 5$) the encoded message was 1250 bytes. For the block Q-matrix method it was 2592 bytes. The test results are shown on the diagrams on fig. 1 and fig. 2. The comparative analysis of the work of the algorithms is presented in table 1.

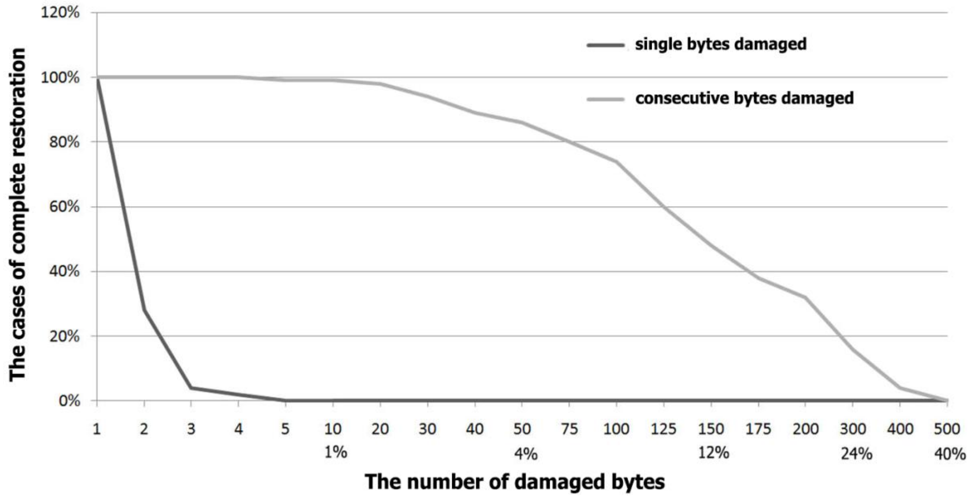

Fig. 1. The cases of complete restoration of the initial message in the standard Fibonacci Q-matrix method

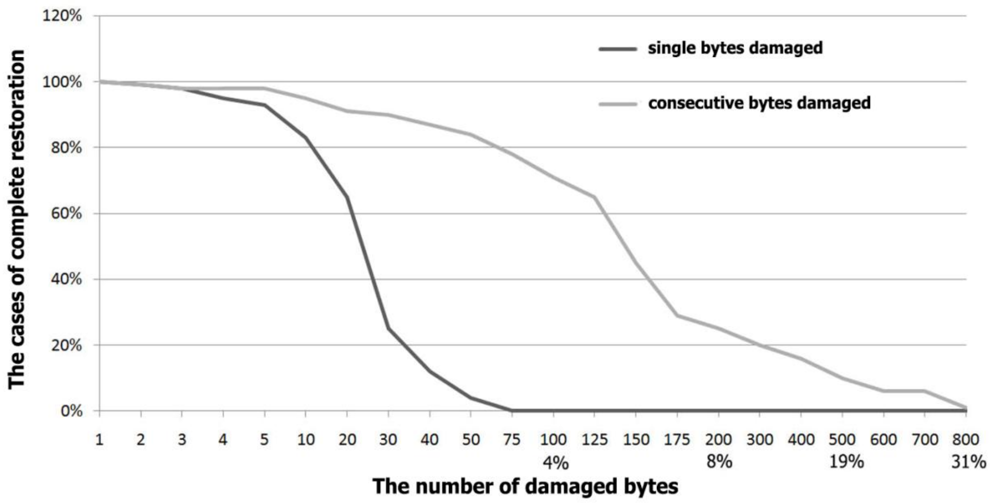

Fig. 2. The cases of complete restoration of the initial message in the block Q-matrix method

The standard Fibonacci Q-matrix method survives the considerable amount of consecutive damaged bytes (being completely ineffective in case more than 40% of the message is damaged), although it is useless in case of single damaged bytes in different parts of the message. In that case the half or the whole message becomes unreadable.

The block Q-matrix method handles the consecutive damaged bytes a little worse (being completely ineffective in case more than 31% of the message is damaged), and allows the restoration of the scattered single error bytes (up to 4% if the message). Also, even if it fails to correct, only the failed damaged groups remain unreadable.

Table 1.

Comparative analysis of the methods

	Standard Ficonacci Q-matrix ($n=5$)	Block Q-matrix method
Ratio of the encoded message size to the initial message size	2.1622	4.5
Code rate	0.4625	0.22
Complete restoration of consecutive error bytes	Up to 40% damage of the message	Up to 31% damage of the message
Complete restoration of single error bytes	Less than 1% damage of the message	Up to 4% damage of the message
If failed to correct all errors	Half or all of the message is unreadable	Only the failed 4-byte segments remain unreadable
Time to encode 4000 bytes	1.13 sec	1.1 sec
Time to decode 4000 bytes	4.65 sec	0.75 sec

The standard Fibonacci Q-matrix method doesn't belong to linear block or convolutional codes [2], which are widespread in coding theory for error detection and correction, or any other category known to the authors. In terms of this it is difficult to compare it with the work of other codes.

While the widespread codes make it possible to correct single bits and their combinations, the Fibonacci Q-matrix allows restoring the elements of the matrix, the size of which is theoretically unlimited. The implementation of this method, however, may require arbitrary-precision arithmetic.

This code also allows encrypting the data, using the power of the Q-matrix as an encryption key. This gives the code an additional advantage in cryptography.

The use of this method might prove useful for the **digital signature** technology. If both the power of the Q-matrix and the determinant are present, the validation of the document can be verified by comparing the calculated determinant with the known one.

The results of numerical experiments made it possible to calculate the rate of the code. The **code rate** [3] is a relation of bits of «useful» information (primal message) to the number of bits of redundant information (encoded message).

Using the developed library it was experimentally proved that for $n=5$, while the size of the message tends to infinity, the rate of the standard Fibonacci Q-matrix code approaches approximately 0.4625. Increasing the power n for Q-matrix will decrease the code rate, therefore increasing the size of the encoded message compared to the primal.

For comparison, the rate of Hamming (3.1) - code is 0.333, and Hamming (7.4) - code is 0.571 [3].

The standard Fibonacci Q-matrix coding method is quite useful if the damage took place in only one part of the message, even if the whole part was corrupted. However, the damage in more parts will make half of the message, or the whole message, unreadable and uncorrectable. Interleaving isn't effective for this method, unless we can predict which bytes will be damaged beforehand.

Nevertheless, given that $\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \varphi$, where F_n is a Fibonacci number n, and φ is the Golden ratio, there are ways to correct errors even in two or three elements in the matrix [5,6]. However, these methods require additional study.

The developed block Q-matrix method relates to the block codes [10]. The code rate is $\frac{4}{18} = 0.22$ in computer implementation, and the optimized algorithm's code rate can reach $\frac{4}{13} = 0.307$. In both cases the number of redundant bits exceeds the standard method.

With the help of interleaving this code can correct the consecutive damaged bytes a little worse than the standard Fibonacci Q-matrix method. However, at first, it leaves the unharmed code readable, and, at second, is able to restore single damaged bytes in different parts of the message even without interleaving.

Both methods are inferior to most of the modern codes in speed and amount of calculations, but their correcting ability can be high for specific types of damage.

Conclusion

This paper presents the Fibonacci Q-matrix coding method, which allows detecting and correcting data errors. The authors have formalized, systematized and justified the existing researches in this field. The "block Q-matrix" method has been developed on the basis of the standard algorithm. The comparative analysis of these methods has been made.

The advantage of these methods is that they allow correcting considerably large information units, the size of which is theoretically unlimited, instead of single bits and their combinations. A matrix element that can be an integer of unlimited value is a minimal information unit for the Fibonacci Q-matrix coding method.

However, both methods are inferior to most of the modern codes in speed and amount of calculations. The standard method also cannot restore the message, if the damage is out of limits of predefined area. This disadvantage has been amended in the block Q-matrix method.

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МЕТОД КОДУВАННЯ НА ОСНОВІ ФІБОНАЧЧІЄВОЇ Q-МАТРИЦІ

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В статті викладаються результати дослідження, формалізації та математичного обґрунтування метода кодування на основі «фібоначчієвої Q-матриці». Даний метод кодування дозволяє з високою ймовірністю виявляти помилки в закодованому повідомленні і виправляти їх у певних випадках. Наведені зауваження щодо реалізації алгоритму. Розроблений «блоковий» алгоритм Q-матриці на основі базового. Наведений порівняльний аналіз роботи алгоритмів.

Ключові слова: методи кодування, числа Фібоначчі, виявлення та виправлення помилок, Q-матриця.

МЕТОД КОДИРОВАНИЯ НА ОСНОВЕ ФИБОНАЧЧИЕВОЙ Q-МАТРИЦЫ

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В статье излагаются результаты исследования, формализации и математического обоснования метода кодирования на основе «фибоначчиевой Q-матрицы». Данный метод кодирования позволяет с высокой вероятностью обнаруживать ошибки в закодированном сообщении и исправлять их в определённых случаях. Приведены замечания к реализации алгоритма. Разработан «блочный» алгоритм Q-матрицы на основе базового метода. Приведен сравнительный анализ работы алгоритмов.

Ключевые слова: методы кодирования, числа Фибоначчи, обнаружение и исправление ошибок, Q-матрица.