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THE HEAT EQUATION ON LINE WITH RANDOM RIGHT PART FROM ORLICZ SPACE

In this paper the heat equation with random right part is examined. In particular, we give conditions for existence with probability one of the solutions in the case when the right part is a random field, sample continuous with probability one from the Orlicz space. Estimation for the distribution of the supremum of solutions of such equations is found.

Key words and phrases: the heat equation, Orlicz space.

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INTRODUCTION

The Cauchy problem for the heat equation with random factors is a classical problem of mathematical physics. Several researchers have investigated solutions of the heat equation depending on various types with random conditions [1, 7, 10–12]. In this paper we consider the Cauchy problem for the heat equation on line with random right part. We consider right part as a class of random fields from the Orlicz spaces.

Similar problems for the parabolic type equations are considered in [2], for the hyperbolic type equations are considered in [3, 4, 8, 14, 15].

The paper is organized as follows. Setion 1 contains necessary definitions and results of the theory of the Orlicz space. In section 2 we consider heat equations with random right part. For such problem conditions of existence with probability one of classical solution with random right part from the Orlicz space are found. The estimation for distribution of supremum of solution of this problem has been got in Section 3.

Using this results one can construct modeless, which approximate solutions of such equations with given accuracy and reliability in the uniform metric (see [9, 13]).

1 STOCHASTIC PROCESSES OF THE ORLICZ SPACE

Definition 1 ([3]). *An even, continuous, convex function $U(x)$, $x \in \mathbb{R}$ such that $U(x) > 0$ for $x \neq 0$ is called a C-function.*

Definition 2 ([5]). *We say that a C-function U satisfies g-condition if there exit constants $z_0 > 0$, $k > 0$ and $A > 0$ such that the inequality*

$$U(x)U(y) \leq AU(kxy)$$

holds for all $x > z_0$ and $y > z_0$.

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Definition 3 ([5]). *Suppose that* (T, ρ) *is a nonempty metric space and* $\varepsilon > 0$. *Denote by* $N_\rho(t, \varepsilon)$ *the smallest number of points in* ε*-net for the set* T *with respect to the metric* ρ. *The function* $(N_\rho(t, \varepsilon), \ \varepsilon > 0)$ *is called the massiveness of the set* T *with respect to the metric* ρ.

Let $\{\Omega, \mathcal{B}, P\}$ be a probability space.

Definition 4 ([3]). *The space* $L_U(\Omega)$ *of a random variables* $\xi(\omega) = \xi$, $\omega \in \Omega$, *is called the Orlicz space generated by a C-function* $U(x)$ *if, for any* $\xi \in L_U(\Omega)$ *there exists a constant* r_ξ *such that* $EU\left(\frac{\xi}{r_\xi}\right) \leq \infty$.

The Orlicz space $L_U(\Omega)$ is a Banach space with the norm

$$\|\xi\|_{L_U} = inf\left\{r > 0 : Eu\left(\frac{\xi}{r_\xi}\right) \leq 1\right\}.$$

Definition 5 ([3]). *A stochastic process* $X = \{X(t), t \in T\}$ *is said to be from the Orlicz space* $L_U(\Omega)$ *if for all* $t \in T$ *the random variable* $X(t)$ *belongs to* $L_U(\Omega)$.

Definition 6 ([3]). *Let* $U(x)$ *be a C-function such that* $U(x)$ *is stronger than* $V(x) = x^2$ *that is* $V(x) > cx^2$ *as* $x > x_0, c > 0$. *The set of random variables* ξ $(E\xi = 0)$ *from the space* $L_U(\Omega)$ *is called strongly Orlicz family of random variables if there exists a constant* C_Δ *such that for* $\xi_i \in \Delta, i \in I$ *and for all* $\lambda_i \in \mathbb{R}^1$ *the following inequality holds (*I *is any finite set)*

$$\left\|\sum_{i\in I}\lambda_i\xi_i\right\|_{L_U} \leq C_\Delta\left(E\left(\sum_{i\in I}\lambda_i\xi_i\right)^2\right)^{1/2}.$$

Definition 7 ([3]). *A stochastic process* $X = \{X(t), t \in T\}$, $(X \in L_U(\Omega))$ *is called a strongly Orlicz process if the family of random variable* $X = \{X(t), t \in T\}$ *is a strongly Orlicz family.*

Theorem 1 ([3]). *Let* Δ *be a strongly Orlicz family of random variables. Then the linear closure* $\overline{\Delta}$ *of the family* Δ *in the space* $L_2(\Omega)$ *is a strongly Orlicz family.*

Theorem 2 ([3]). *Let* $X_i = \{X_i(t), t \in T, i \in I\}$ *be a family of strongly Orlicz stochastic processes. Let* (T, Θ, μ) *is a measurable space. If*

$$\varphi_{k_i}(t), \ i \in I, \ k = 1, \ldots, \infty$$

is a family of measurable function in (T, Θ, μ) *and the integral*

$$\xi_{ki} = \int_T \varphi_k(t)\, x_i(t)\, d\mu(t)$$

is well defined in the mean square sense, then the family of random variables

$$\Delta_\xi = \left\{\xi_{ki}, i \in I, k = \overline{1, \infty}\right\}$$

is a strongly Orlicz family.

Theorem 3 ([15]). *Let $\mathbf{R}^k$ be the k-dimensional space,*

$$d(t,s) = \max_{1\le i\le k} |t_i - s_i|,$$

$T = \{0 \le t_i \le T_i, i = 1,2,\dots,k\}$, $X_n = \{X_n(t), t \in T\}$, $n = 1,2,\dots$ be a sequence of stochastic processes belonging to the Orlicz space $L_U(\Omega)$, and let the function u satisfy the g-condition. Assume that the process $X_n(t)$ is separable and

$$\sup_{d(t,s)\le h} \sup_{n=\overline{1,\infty}} \|X_n(t) - X_n(s)\| \le \sigma(h),$$

where $\sigma = \{\sigma(h), h > 0\}$ is a monotone increasing continuous function such that $\sigma(h) \to 0$ as $h \to 0$. We also assume that

$$\int_0^{\varepsilon} U^{(-1)}\left(\prod_{i=1}^{k}\left(\frac{T_i}{2\sigma^{(-1)}(u)}+1\right)\right) du < \infty,$$

where $\sigma^{(-1)}(u)$ is the inverse function to $\sigma(u)$. If the processes $X_n(t)$ converge in probability to the process $X(t)$ for all $t \in T$, then $X_n(t)$ converge in probability in the space $C(T)$.

The following result contains for the existence of partial derivatives for stochastic processes of Orlicz space.

Theorem 4 ([14]). *Let $T = \{a_i \le x_i \le b_i,\ i = 1,\dots,m\}$. $\xi(X)$, $X \in T$, be a separable random field such that $\xi(X)$ is a strongly Orlicz stochastic processes. Let $B_{0000}(X,Y) = E\xi(X)\xi(Y)$ and assume that the partial derivatives $B_{i0i0}(X,Y) = \frac{\partial^2 B(X,Y)}{\partial x_i \partial y_i}$, $i = 1,\dots,m$, and*

$$B_{ikik}(X,Y) = \frac{\partial^4 B(X,Y)}{\partial x_i \partial y_i \partial x_k \partial y_k}, \quad i = 1,\dots,m,\ k = 1,\dots,m$$

exist. Let there exist a monotone increasing continuous increasing functions $\sigma_z(h) > 0$, $h > 0$, that $\sigma_z(h) \to 0$ as $h \to 0$ for $z = (0,0,0,0)$, $z = (i,0,i,0)$, $i = 1,\dots,m$ and $z = (i,k,i,k)$, $i = 1,\dots,m$, $k = 1,\dots,m$. Assume that

$$\sup_{\substack{|x_i-y_i|\le h\\ i=1,\dots,m}} \left(B_z(X,X) + B_z(Y,Y) - 2B_z(X,Y)\right)^{\frac{1}{2}} \le \sigma_z(h). \tag{1}$$

If

$$\int_0^{\varepsilon} U^{-1}\left(\left(\frac{\pi}{2\sigma_z^{(-1)}(u)}+1\right)\left(\frac{T}{2\sigma_z^{(-1)}(u)}+1\right)\right) du < \infty \tag{2}$$

for all z and for sufficiently small ε, then with probability one the partial derivatives

$$\frac{\partial \xi(X)}{\partial x_i}, \quad \frac{\partial^2 \xi(X)}{\partial x_i \partial x_j}, \quad i,j = 1,\dots,m.$$

2 THE CAUCHY PROBLEM FOR THE HEAT EQUATION WITH A RANDOM RIGHT PART FROM ORLICZ SPACE

We consider the Cauchy problem for the heat equation

$$\frac{\partial u(x,t)}{\partial t} = a^2 \frac{\partial^2 u(x,t)}{\partial x^2} + \xi(x,t), \tag{3}$$
$$-\infty < x < +\infty, \quad t > 0,$$

subject to the initial condition

$$u(x,0) = 0, \quad -\infty < x < +\infty. \tag{4}$$

Let the function $\xi(x,t) = \{\xi(x,t), \ \ x \in \mathbb{R}, \ t > 0\}$ is a random field sample continuity with probability one from the Orlicz space such that $E\xi(x,t) = 0$, $E(\xi(x,t))^2 < +\infty$. Let us denote

$$B(x,t,y,s) = E\xi(x,t)\xi(y,s).$$

Let $B(x,t,z,s)$ be a continuous function.

Problem with the nonrandom function $\xi(x,t)$ is seen in [6].

Theorem 5 ([10]). *Let the conditions* $\int\limits_R \sqrt{E(\xi^2(x,t))}dx < \infty$ *be satisfied and let*

$$G(y,t) = \frac{1}{\sqrt{2\pi}} \int\limits_0^t e^{-a^2y^2(t-\tau)}\tilde{\xi}(y,\tau)d\tau, \qquad \tilde{\xi}(y,\tau) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} \cos(yx)\, \xi(x,\tau)dx,$$

and

$$u(x,t) = \int\limits_{-\infty}^{+\infty} \cos(yx)\, G(y,t)dy. \tag{5}$$

If the integrals

$$\int\limits_{-\infty}^{+\infty} y \sin(yx)\, G(y,t)dy, \ \int\limits_{-\infty}^{+\infty} y^s \cos(yx)\, G(y,t)dy, \quad s = 0, \ 2,$$

exist and for all $A > 0$ *and* $T > 0$ *there exists a sequence* a_n *with* $a_n \to \infty$ *for* $n \to \infty$ *such that the sequence of integrals*

$$\int\limits_{-a_n}^{+a_n} y \sin(yx)\, G(y,t)dy, \quad \int\limits_{-a_n}^{+a_n} y^s \cos(yx)\, G(y,t)dy, \quad s = 0, \ 2, \tag{6}$$

converges in probability, uniformly for $|x| \le A$, $0 \le t \le T$, *then the function* $u(x,t)$ *is the classical solution to the problem* (3)–(4).

Indeed,

$$\frac{\partial u(x,t)}{\partial t} = -a^2 \int\limits_{-\infty}^{+\infty} y^2 \cos(yx)\, G(y,t)dy + \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{+\infty} \cos(yx)\, \tilde{\xi}(y,t)dy = a^2 \frac{\partial^2 u(x,t)}{\partial x^2} + \xi(x,t).$$

Lemma 1 ([11]). *Let a function $X(\lambda, u)$, $\lambda > 0$ and $u > 0$ be such that:*

1) $\sup\limits_{0\le u<\infty, 0\le\lambda<\infty} |X(\lambda, u)| \le B$;

2) $|X(\lambda, u) - X(\lambda, v)| \le C\lambda|u - v|$ *for all* $u > 0$, $v > 0$.

Let $\varphi(\lambda)$, $\lambda > 0$ be a continuous increasing function such that $\varphi(\lambda) > 0$ for all $\lambda > 0$, and the function $\frac{\lambda}{\varphi(\lambda)}$ is increasing for $\lambda > v_0$, and for some constant $v_0 \ge 0$. Then

$$|X(\lambda, u) - X(\lambda, v)| \le \max(C, 2B)\frac{\varphi(\lambda + v_0)}{\varphi\left(\frac{1}{|u-v|} + v_0\right)}$$

for all $\lambda \ge 0$ and $v > 0$.

Let

$$u_{a_n}^{(0)}(x,t) = \int\limits_{-a_n}^{a_n} \cos(yx)\, G(y,t)dy, \quad u_{a_n}^{(1)}(x,t) = \int\limits_{-a_n}^{a_n} y\sin(yx)\, G(y,t)dy,$$

$$u_{a_n}^{(2)}(x,t) = \int\limits_{-a_n}^{a_n} y^2\cos(yx)\, G(y,t)dy.$$

Theorem 6. *Let $\xi(x,t)$ be a random field, sample continuous with probability one from the Orlicz space. Let*

1) $\int\limits_R \sqrt{E(\xi^2(x,t))}dx < \infty$;

2) *the derivatives* $\frac{\partial^k B(x,t,v,s)}{\partial x^l \partial v^m}$, $k = 0, \ldots 4$, $l + m = k$ *exist;*

3) $\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \left|\frac{\partial^k B(x,t,v,s)}{\partial x^l \partial v^m}\right| dxdx \le B(k,l,m) < \infty$, $k = 0, \ldots 4$, $l + m = k$;

4) $\frac{\partial^k B(x,t,v,s)}{\partial x^l \partial v^m} \to 0, k = 0, \ldots 4$, $l + m = k$, *as* $x \to \infty$ *or* $v \to \infty$;

5) $\sup\limits_{\substack{|x-x_i|\le h\\ |t-t_1|\le h}} \tau_\varphi\left(u_{a_n}^{(k)}(x,t) - u_{a_n}^{(k)}(x_1,t_1)\right) \le \sigma_k(h)$, *for* $k = 0, 1, 2$, *where* $\sigma_k(h)$ *is a monotone increasing continuous function such that* $\sigma_k(h) \to 0$ *as* $h \to 0$, *moreover*

$$\int_{0+}^{\varepsilon} U^{(-1)}\left(\left(\frac{A}{\sigma_k^(-1)(u)} + 1\right)\left(\frac{T}{2\sigma_k^(-1)(u)} + 1\right)\right) du < \infty, \tag{7}$$

where $\sigma_k^{-1}(\varepsilon)$ *is the inverse function to* $\sigma_k(\varepsilon)$

Then the function $u(x,t)$, which is represented in the form (5), *is a classical solution to the problem* (3)–(4).

Proof. This theorem follows from Theorem 3 and Theorem 5. □

Example 1. *Assume that $\xi(x)$, $\eta(x)$ are strongly Orlicz processes $L_U(\Omega)$. Let $u(x)$ be a function such that $u(x) = |x|^p$ for some $p > 2$ and all $|x| > 1$. Then condition (7) of Theorem 6 holds the function $\sigma_k(h) = C_k|h|^\delta$ for $0 < \delta \le 1$. Indeed for $\varepsilon > 0$*

$$I = \int_0^\varepsilon U^{(-1)} \left(\left(\frac{A}{\sigma_k^{(-1)}(u)} + 1 \right) \left(\frac{T}{2\sigma_k^{(-1)}(u)} + 1 \right) \right) du < \infty,$$

$$I \le \int_0^\varepsilon \left(\frac{AC_k^{\frac{1}{\delta}}}{u^{\frac{1}{\delta}}} \cdot \frac{TC_k^{\frac{1}{\delta}}}{2u^{\frac{1}{\delta}}} \right)^{\frac{1}{2}} du \le D \int_0^\varepsilon \frac{1}{u^{\frac{2}{p\delta}}} du.$$

The latter integral converges under $\delta > \frac{2}{p}$.

Theorem 7. *Let $\xi(x,t)$ be strongly Orlicz processes $L_U(\Omega)$ where $u(x)$ is a function such that $u(x) = |x|^p$ for some $p > 2$ and all $|x| > 1$, sample continuous with probability one. Let*

1) $\int\limits_R \sqrt{E(\xi^2(x,t))} dx < \infty$;

2) *the derivatives* $\frac{\partial^k B(x,t,v,s)}{\partial x^l \partial v^m}$, $\quad k = 0,\dots 4,\ l + m = k$ *exist;*

3) $\int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \left| \frac{\partial^k B(x,t,v,s)}{\partial x^l \partial v^m} \right| dxdx \le B(k,l,m) < \infty,\ k = 0,\dots 4,\ l + m = k;$

4) $\frac{\partial^k B(x,t,v,s)}{\partial x^l \partial v^m} \to 0, k = 0,\dots 4,\ l + m = k,$ *as* $x \to \infty$ *or* $v \to \infty$*;*

5) $\int\limits_{-\infty}^{+\infty} \left(E\left|\xi(x,\tau)\right|^2 \right)^{\frac{1}{2}} dx < \Theta;\ \int\limits_{-\infty}^{+\infty} \left(E\left| \frac{\partial \xi(x,\tau)}{dx} \right|^2 \right)^{\frac{1}{2}} \le \Theta_1;\ \int\limits_{-\infty}^{+\infty} \left(E\left| \frac{\partial^2 \xi(x,\tau)}{dx^2} \right|^2 \right)^{\frac{1}{2}} \le \Theta_2$ *for some* $\Theta > 0,\ \Theta_1 > 0,\ \Theta_2 > 0.$

Then the function $u(x,t)$, which is represented in the form (5)*, is classical solution to the problem* (3)–(4).

Proof. It follows from the condition 1) (see Lemma 1 of [10]) that the integral Fourier transform

$$\tilde{\xi}(y,\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \cos(yx)\, \xi(x,\tau) dx$$

exists and

$$\xi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \cos(yx)\, \tilde{\xi}(y,\ t) dy.$$

It follows conditions 2)–4) (see Lemma 2 of [10]) that there exist with probability one integrals

$$\int_{-\infty}^{+\infty} y \sin(yx)\, G(y,t) dy,\ \int_{-\infty}^{+\infty} y^s \cos(yx)\, G(y,t) dy, \quad s = 0,\ 2.$$

Then by Theorem 5 the integrals in (6) converge in probability to integrals

$$\int_{-\infty}^{+\infty} y \sin(yx)\, G(y,t) dy,\ \int_{-\infty}^{+\infty} y^s \cos(yx)\, G(y,t) dy, \quad s = 0,\ 2,$$

for $|x| \le A, 0 \le t \le T$.

According to Theorem 3 and Example 1, when the conditions in the probability space $C(\widetilde{T})$ required that the conditions

$$\left(E\left|u_{a_n}(x,t)-u_{a_n}(x_1,t_1)\right|^2\right)^{\frac{1}{2}} \le Ch^{\alpha}, \qquad \left(E\left|u^{(1)}_{a_n}(x,t)-u^{(1)}_{a_n}(x_1,t_1)\right|^2\right)^{\frac{1}{2}} \le C_1h^{\alpha},$$

$$\left(E\left|u^{(2)}_{a_n}(x,t)-u^{(2)}_{a_n}(x_1,t_1)\right|^2\right)^{\frac{1}{2}} \le C_2h^{\alpha}$$

hold, where

$$u_{a_n}(x,t)=\int_{-a_n}^{a_n}\cos(yx)\,G(y,t)dy, \qquad u^{(1)}_{a_n}(x,t)=\int_{-a_n}^{a_n}y\sin(yx)\,G(y,t)dy,$$

$$u^{(2)}_{a_n}(x,t)=\int_{-a_n}^{a_n}y^2\cos(yx)\,G(y,t)dy,$$

then the integrals in integrals (6) converge in the probability space $C(T)$.

Using the generalized Minkovski's inequality we obtain

$$\begin{aligned}
&\left(E\left|u_{a_n}(x,t)-u_{a_n}(x_1,t_1)\right|^2\right)^{\frac{1}{2}}\\
&=\left(E\left|\int_{-a_n}^{a_n}\cos(yx)\,G(y,t)dy-\int_{-a_n}^{a_n}\cos(yx_1)\,G(y,t_1)dy\right|^2\right)^{\frac{1}{2}}\\
&=\left(E\left|\int_{-a_n}^{a_n}\left[\cos(yx)\,G(y,t)-\cos(yx_1)\,G(y,t_1)\right]dy\right|^2\right)^{\frac{1}{2}}\\
&=\left(E\left|\int_{-a_n}^{a_n}\left[(\cos(yx)-\cos(yx_1))G(y,t_1)+(G(y,t)-G(y,t_1))\cos(yx)\right]dy\right|^2\right)^{\frac{1}{2}}\\
&\le\int_{-\infty}^{\infty}\left[\left|\cos(yx)-\cos(yx_1)\right|\left(|G(y,t_1)|^2\right)^{\frac{1}{2}}+\left(E|G(y,t)-G(y,t_1)|^2\right)^{\frac{1}{2}}\right]dy.
\end{aligned}$$

Using the inequality $|\sin x| \le |x|^{\alpha}$ for $0<\alpha\le 1$ and arbitrary h, $|x-x_1|\le h$ we have

$$\left|\cos(yx)-\cos(yx_1)\right| \le 2\left|\sin\frac{y(x-x_1)}{2}\right| \le 2^{1-\alpha}|y|^{\alpha}h^{\alpha}.$$

Consider

$$\begin{aligned}
\left(E|G(y,t_1)|^2\right)^{\frac{1}{2}} &=\frac{1}{\sqrt{2\pi}}\left(E\left|\int_0^{t_1}e^{-a^2y^2(t_1-\tau)}\tilde{\xi}(y,\tau)d\tau\right|^2\right)^{\frac{1}{2}}\\
&\le\frac{1}{\sqrt{2\pi}}\int_0^{t_1}e^{-a^2y^2(t_1-\tau)}\left(E|\tilde{\xi}(y,\tau)|^2\right)^{\frac{1}{2}}d\tau.
\end{aligned}$$

$$\left(E|\tilde{\xi}(y,\tau)|^2\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2\pi}}\left(E\left|\int_{-\infty}^{+\infty}\cos(yx)\,\xi(x,\tau)dx\right|^2\right)^{\frac{1}{2}}$$
$$\leq \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left(E\,|\xi(x,\tau)|^2\right)^{\frac{1}{2}}dx < \frac{1}{\sqrt{2\pi}}\Theta.$$

Therefore

$$\left(E|G(y,t_1)|^2\right)^{\frac{1}{2}} \leq \frac{1}{2\pi}\int_0^{t_1}\Theta e^{-a^2y^2(t_1-\tau)}d\tau \leq \frac{1}{2\pi}\Theta^{\frac{1}{2}}\frac{1}{a^2y^2}\left|1-e^{-a^2y^2t_1}\right|.$$

Let $t_1 < t$, then

$$\left(E|G(y,t)-G(y,t_1)|^2\right)^{\frac{1}{2}}$$
$$= \frac{1}{\sqrt{2\pi}}\left(E\left|\int_0^t e^{-a^2y^2(t-\tau)}\tilde{\xi}(y,\tau)d\tau - \int_0^{t_1} e^{-a^2y^2(t_1-\tau)}\tilde{\xi}(y,\tau)d\tau\right|^2\right)^{\frac{1}{2}}$$
$$= \frac{1}{\sqrt{2\pi}}\left(E\left|\int_0^{t_1}\left[e^{-a^2y^2(t-\tau)} - e^{-a^2y^2(t_1-\tau)}\right]\tilde{\xi}(y,\tau)d\tau + \int_{t_1}^{t} e^{-a^2y^2(t-\tau)}\tilde{\xi}(y,\tau)d\tau\right|^2\right)^{\frac{1}{2}}$$
$$= \frac{1}{\sqrt{2\pi}}\left(\int_0^{t_1}\left[\left|e^{-a^2y^2(t-\tau)} - e^{-a^2y^2(t_1-\tau)}\right|\left(E|\tilde{\xi}(y,\tau)|^2\right)^{\frac{1}{2}}\right]d\tau + \int_{t_1}^{t} e^{-a^2y^2(t-\tau)}\left(E|\tilde{\xi}(y,\tau)|^2\right)^{\frac{1}{2}}d\tau\right).$$

Using Lemma 1, we obtain the estimate

$$\left|e^{-a^2y^2(t-\tau)} - e^{-a^2y^2(t_1-\tau)}\right| = \left|e^{-a^2y^2(t_1-\tau)}\right|\left|e^{-a^2y^2(t-t_1)}-1\right|$$
$$\leq e^{-a^2y^2(t_1-\tau)}\max(1,a^2)y^{2\alpha}|t-t_1|^{\alpha} \leq e^{-a^2y^2(t_1-\tau)}\max(1,a^2)y^{2\alpha}h^{\alpha}.$$

Therefore

$$\left(E|G(y,t)-G(y,t_1)|^2\right)^{\frac{1}{2}}$$
$$\leq \frac{1}{2\pi}\left(\int_0^t e^{-a^2y^2(t_1-\tau)}\max(1,a^2)y^{2\alpha}h^{\alpha}\Theta^{\frac{1}{2}}d\tau + \int_{t_1}^{t} e^{-a^2y^2(t-\tau)}\Theta^{\frac{1}{2}}d\tau\right)$$
$$= \frac{\Theta}{2\pi}\left(\max(1,a^2)y^{2\alpha}h^{\alpha}\frac{1}{a^2y^2}\left|1-e^{-a^2y^2t_1}\right| + \int_{t_1}^{t} e^{-a^2y^2(t-\tau)}d\tau\right)$$
$$= \frac{\Theta}{2\pi}\left(\max(1,a^2)\frac{h^{\alpha}}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right| + \int_{t_1}^{t} e^{-a^2y^2(t-\tau)}d\tau\right).$$

Then

$$
\begin{aligned}
&\left(E\left|u_{a_n}(x,t)-u_{a_n}(x_1,t_1)\right|^2\right)^{\frac{1}{2}}\\
&\le \frac{\Theta}{2\pi}\int\limits_{-\infty}^{+\infty}\left[2^{1-\alpha}|y^\alpha h^\alpha|\frac{1}{a^2y^2}\left|1-e^{-a^2y^2t_1}\right|+h^\alpha\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right|\right.\\
&\left.+\int\limits_{t_1}^{t}e^{-a^2y^2(t-\tau)}d\tau\right]dy=\frac{\Theta}{\pi}\int\limits_0^{+\infty}\left[2^{1-\alpha}\frac{h^\alpha}{a^2y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right|\right.\\
&\left.+h^\alpha\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right|+\int\limits_{t_1}^{t}e^{-a^2y^2(t-\tau)}d\tau\right]dy\\
&=\frac{\Theta}{\pi}\left\{\int\limits_0^1\left[2^{1-\alpha}\frac{h^\alpha}{a^2y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right|+h^\alpha\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha}\left|1-e^{-a^2y^2t_1}\right|\right.\right.\\
&\left.+\int\limits_{t_1}^{t}e^{-a^2y^2(t-\tau)}d\tau\right]dy+\int\limits_1^{+\infty}\left[2^{1-\alpha}\frac{h^\alpha}{a^2y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right|\right.\\
&\left.\left.+h^\alpha\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right|+\int\limits_{t_1}^{t}e^{-a^2y^2(t-\tau)}d\tau\right]dy\right\}=\frac{\Theta}{\pi}\left(I_1+I_2\right).
\end{aligned}
$$

Consider

$$
\begin{aligned}
I_1&=\int\limits_0^1\left[2^{1-\alpha}\frac{h^\alpha}{a^2y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right|+h^\alpha\max(1,a^2)\frac{h^\alpha}{a^2y^2(1-\alpha)}\left|1-e^{-a^2y^2t_1}\right|\right.\\
&\left.+\int\limits_{t_1}^{t}e^{-a^2y^2(t-\tau)}d\tau\right]dy=\frac{2^{1-\alpha}h^\alpha}{a^2}\int\limits_0^1\frac{1}{y^{2-\alpha}}\left|1-e^{-a^2y^2t_1}\right|dy\\
&+\frac{h^\alpha}{a^2}\max(1,a^2)\int\limits_0^1\frac{1}{y^{2(1-\alpha)}}\left|1-e^{-a^2y^2t_1}\right|dy+\int\limits_0^1\left(\int\limits_{t_1}^{t}e^{-a^2y^2(t-\tau)}d\tau\right)dy\\
&=\frac{2^{1-\alpha}h^\alpha}{a^2}I_{11}+\frac{h^\alpha}{a^2}\max(1,a^2)I_{12}+I_{13}.
\end{aligned}
$$

Since $\left|1-e^{-a^2y^2t_1}\right|\le a^2y^2t_1\le a^2y^2T$,

$$
I_{11}=\int\limits_0^1\frac{1}{y^{2(1-\alpha)}}\left|1-e^{-a^2y^2t_1}\right|dy\le\frac{a^2T}{\alpha+1},\qquad I_{12}=\int\limits_0^1\frac{1}{y^{2(1-\alpha)}}\left|1-e^{-a^2y^2t_1}\right|dy\le\frac{a^2T}{2\alpha+1}.
$$

Using that $e^{-a^2y^2(t-\tau)}\le 1$, we have

$$
I_{13}=\int\limits_0^1\left(\int\limits_{t_1}^{t}e^{-a^2y^2(t-\tau)}d\tau\right)dy\le\int\limits_0^1(t-t_1)dy\le h\le h^\alpha T^{1-\alpha}.
$$

So we have

$$I_1 \le h^\alpha \left(\frac{2^{1-\alpha}T}{\alpha+1} + \frac{\max(1,a^2)T}{2\alpha+1} + T^{1-\alpha} \right).$$

$$I_2 = \int_1^{+\infty} \Bigg[2^{1-\alpha} \frac{h^\alpha}{a^2 y^{2-\alpha}} \left| 1 - e^{-a^2y^2t_1} \right| dy$$
$$+ h^\alpha \max(1,a^2) \frac{h^\alpha}{a^2y^2(1-\alpha)} \left| 1 - e^{-a^2y^2t_1} \right| + \int_{t_1}^{t} e^{-a^2y^2(t-\tau)} d\tau \Bigg] dy$$
$$= \frac{2^{1-\alpha}h^\alpha}{a^2} \int_1^{+\infty} \frac{1}{y^{2-\alpha}} \left| 1 - e^{-a^2y^2t_1} \right| dy + \frac{h^\alpha}{a^2} \max(1,a^2) \int_1^{+\infty} \frac{1}{y^{2(1-\alpha)}} \left| 1 - e^{-a^2y^2t_1} \right| dy$$
$$+ \int_1^{+\infty} \left(\int_{t_1}^{t} e^{-a^2y^2(t-\tau)} d\tau \right) dy = \frac{2^{1-\alpha}h^\alpha}{a^2} I_{21} + \frac{h^\alpha}{a^2} \max(1,a^2) I_{22} + I_{23}.$$

$$I_{21} = \int_1^{+\infty} \frac{1}{y^{2-\alpha}} \left| 1 - e^{-a^2y^2t_1} \right| dy \le \int_1^{+\infty} \frac{1}{y^{2-\alpha}} dy = \frac{1}{1-\alpha}.$$

$$I_{22} = \int_1^{+\infty} \frac{1}{y^{2(1-\alpha)}} \left| 1 - e^{-a^2y^2t_1} \right| dy \le \int_1^{+\infty} \frac{1}{y^{2(1-\alpha)}} dy = \frac{1}{1-2\alpha}.$$

$$I_{23} = \int_1^{+\infty} \left(\int_{t_1}^{t} e^{-a^2y^2(t-\tau)} d\tau \right) dy = \frac{1}{a^2} \int_1^{+\infty} \frac{1}{y^2} \left(1 - e^{-a^2y^2(t-t_1)} \right) dy$$
$$\le \frac{h^\alpha}{a^2} \max 1, a^2 \int_1^{+\infty} \frac{dy}{y^{2(1-\alpha)}} = \frac{h^\alpha}{a^2} \max(1,a^2) \frac{1}{1-2\alpha}.$$

Therefore

$$I_2 = \left(\frac{2^{1-\alpha}}{a^2} \cdot \frac{1}{1-\alpha} + \frac{2\max(1,a^2)}{a^2} \right) h^\alpha.$$

Then for $0 < \alpha < \frac{1}{2}$, we have

$$\left(E \left| u_{a_n}(x,t) - u_{a_n}(x_1,t_1) \right|^2 \right)^{\frac{1}{2}} \le Ch^\alpha,$$

where

$$C = \frac{\Theta}{\pi} \left(\frac{2^{1-\alpha}T}{\alpha+1} + \frac{\max(1,a^2)T}{2\alpha+1} + T^{1-\alpha} + \frac{2^{1-\alpha}}{a^2} \cdot \frac{1}{1-\alpha} + \frac{2\max(1,a^2)}{a^2} \right).$$

Consider

$$\left(E\left|u_{a_n}^{(1)}(x,t)-u_{a_n}^{(1)}(x_1,t_1)\right|^2\right)^{\frac{1}{2}}$$
$$=\left(E\left|\int_{-a_n}^{a_n} y\sin(yx)\,G(y,t)dy-\int_{-a_n}^{a_n} y\sin(yx_1)G(y,t_1)dy\right)^2\right|^{\frac{1}{2}}$$
$$=\left(E\left|\int_{-a_n}^{a_n} y\left[\sin(yx)\,G(y,t)-\sin(yx_1)G(y,t_1)\right]dy\right|^2\right)^{\frac{1}{2}}$$
$$=\left(E\left|\int_{-a_n}^{a_n} y\left[(\sin(yx)-\sin(yx_1))G(y,t_1)+(G(y,t)-G(y,t_1))\sin(yx)\right]dy\right|^2\right)^{\frac{1}{2}}$$
$$\leq\int_{-\infty}^{\infty} y\left[|\sin(yx)-\sin(yx_1)|\left(E\left|G(y,t_1)\right|^2\right)^{\frac{1}{2}}+\left(E|G(y,t)-G(y,t_1)|^2\right)^{\frac{1}{2}}\right]dy$$
$$=\int_{-\infty}^{\infty}\left[|\sin(yx)-\sin(yx_1)|\left(y^2E|G(y,t_1)|^2\right)^{\frac{1}{2}}+\left(y^2E|G(y,t)-G(y,t_1)|^2\right)^{\frac{1}{2}}\right]dy.$$

Since

$$|\sin(yx)-\sin(yx_1)|\leq 2\left|\sin\frac{y(x-x_1)}{2}\right|\leq 2^{1-\alpha}|y|^{\alpha}h^{\alpha}.$$

$$\left(y^2EG|(y,t_1)|^2\right)^{\frac{1}{2}}\leq\frac{1}{\sqrt{2\pi}}\int_0^{t_1}e^{-a^2y^2(t_1-\tau)}\left(y^2E|\tilde{\xi}(y,\tau)|^2\right)^{\frac{1}{2}}d\tau.$$

$$\left(y^2E|\tilde{\xi}(y,\tau)|^2\right)^{\frac{1}{2}}=\frac{1}{\sqrt{2\pi}}\left(E\left|\int_{-\infty}^{+\infty}y\cos(yx)\,\xi(x,\tau)dx\right)^2\right|^{\frac{1}{2}}$$
$$\leq\frac{1}{\sqrt{2\pi}}\left(E\left|\int_{-\infty}^{+\infty}\sin(yx)\frac{\partial\xi(x,\tau)}{\partial x}dx\right|^2\right)^{\frac{1}{2}}\leq\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left(E\left|\frac{\partial\xi(x,\tau)}{dx}\right|^2\right)^{\frac{1}{2}}\leq\frac{1}{\sqrt{2\pi}}\Theta_1.$$

Similarly,

$$\left(E\left|u_{a_n}^{(1)}(x,t)-u_{a_n}^{(1)}(x_1,t_1)\right|^2\right)^{\frac{1}{2}}\leq C_1h^{\alpha},$$

where

$$C_1=\frac{\Theta_1}{\pi}\left(\frac{2^{1-\alpha}T}{\alpha+1}+\frac{\max(1,a^2)T}{2\alpha+1}+T^{1-\alpha}+\frac{2^{1-\alpha}}{a^2}\cdot\frac{1}{1-\alpha}+\frac{2\max(1,a^2)}{a^2}\right).$$

Consider

$$\left(E\left|u^{(2)}_{a_n}(x,t)-u^{(2)}_{a_n}(x_1,t_1)\right|^2\right)^{\frac{1}{2}}$$

$$=\left(E\left|\int_{-a_n}^{a_n} y^2\cos(yx)\,G(y,t)dy-\int_{-a_n}^{a_n} y^2\cos(yx_1)\,G(y,t_1)dy\right|^2\right)^{\frac{1}{2}}$$

$$=\left(E\left|\int_{-a_n}^{a_n} y^2\left[\cos(yx)\,G(y,t)-\cos(yx_1)\,G(y,t_1)\right]dy\right|^2\right)^{\frac{1}{2}}$$

$$=\left(E\left|\int_{-a_n}^{a_n} y^2\left[(\cos(yx)-\cos(yx_1))G(y,t_1)+(G(y,t)-G(y,t_1))\cos(yx)\right]dy\right|^2\right)^{\frac{1}{2}}$$

$$\le\int_{-\infty}^{\infty}\left[|\cos(yx)-\cos(yx_1)|\left(y^4E|G(y,t_1)^2\right)^{\frac{1}{2}}+\left(y^4E|G(y,t)-G(y,t_1)|^2\right)^{\frac{1}{2}}\right]dy.$$

$$\left(y^4E|G(y,t_1)|^2\right)^{\frac{1}{2}}\le\frac{1}{\sqrt{2\pi}}\int_0^{t_1}e^{-a^2y^2(t_1-\tau)}\left(y^4E|\tilde{\xi}(y,\tau)|^2\right)^{\frac{1}{2}}d\tau.$$

$$\left(y^4E|\tilde{\xi}(y,\tau)|^2\right)^{\frac{1}{2}}=\frac{1}{\sqrt{2\pi}}\left(E\left|\int_{-\infty}^{+\infty}y^2\cos(yx)\,\xi(x,\tau)dx\right|^2\right)^{\frac{1}{2}}$$

$$\le\frac{1}{\sqrt{2\pi}}\left(E\left|\int_{-\infty}^{+\infty}\cos(yx)\frac{\partial^2\xi(x,\tau)}{\partial x^2}dx\right|^2\right)^{\frac{1}{2}}\le\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left(E\left|\frac{\partial^2\xi(x,\tau)}{dx^2}\right|^2\right)^{\frac{1}{2}}\le\frac{1}{\sqrt{2\pi}}\Theta_2.$$

Then

$$\left(E\left|u^{(2)}_{a_n}(x,t)-u^{(2)}_{a_n}(x_1,t_1)\right|^2\right)^{\frac{1}{2}}\le C_2h^{\alpha},$$

where

$$C_2=\frac{\Theta_2}{\pi}\left(\frac{2^{1-\alpha}T}{\alpha+1}+\frac{\max(1,a^2)T}{2\alpha+1}+T^{1-\alpha}+\frac{2^{1-\alpha}}{a^2}\cdot\frac{1}{1-\alpha}+\frac{2\max(1,a^2)}{a^2}\right),$$

for $0<\alpha<\frac{1}{2}$.

According to Theorem 3, the conditions of Theorem 7 hold for $0<\alpha<\frac{1}{2}$.

□

3 ESTIMATES OF THE DISTRIBUTION OF THE SUPREMUM OF A SOLUTION OF THE HEAT EQUATION

Theorem 8 ([11]). *Let $(\widetilde{T},\widetilde{\rho})$ be a compact metric space and $N(u)$ the metric massiveness of the space $(\widetilde{T},\widetilde{\rho})$, that is, the minimum number of closed balls of radius U that cover $(\widetilde{T},\widetilde{\rho})$. Let $X=$*

$\{X(t),\ t\in\widetilde{T}\}$ be a separable stochastic process from the space $L_U(\Omega)$, and let the function U satisfies the g-condition. Assume that there exists a monotone increasing continuous function $\sigma=\sigma(h),\ 0\le h\le \sup\limits_{t,s\in T}\widetilde{\rho}(t,s)$ such that

$$\sup_{\widetilde{\rho}(t,s)\le h}\|X(t)-X(s)\|_U\le\sigma(h).$$

If for some ε

$$\int\limits_0^{\varepsilon}\chi_U\left(N\left(\sigma^{(-1)}(u)\right)\right)du<\infty, \tag{8}$$

where

$$\chi_U(n)=\begin{cases} n, & n<U(z_0);\\ C_UU^{(-1)}(n), & n\ge U(z_0),\end{cases}$$

$C_U=k(1+U(z_0))\max(1,A), z_0,\ k,\ A$ are constants from definition of C-function and $\sigma^{(-1)}(h)$ is the inverse of $\sigma(h)$, then the random variable $\sup\limits_{t\in\widetilde{T}}|X(t)|$ belongs to the space $L_U(\Omega)$ with probability one and

$$\left\|\sup_{t\in T}|X(t)|\right\|_U\le\|X(t_0)\|_U+\frac{1}{\theta(1-\theta)}\int\limits_0^{\omega_0\theta}\chi_U\left(N\left(\sigma^{(-1)}(u)\right)\right)du=B(t_0,\ \theta),$$

where $t_0\in\widetilde{T}$, $\omega_0=\sigma(\sup\limits_{t\in T}\rho(t_0,\ t)),\quad 0<\theta<1$. In addition, for all $\varepsilon>0$ the following inequality holds:

$$P\left\{\sup_{t\in\widetilde{T}}|X(t)|>\varepsilon\right\}\le\left(U\left(\frac{\varepsilon}{B(t_0,\theta)}\right)\right)^{-1}.$$

Theorem 9. *Let in conditions of Theorem 8*

$$\widetilde{T}=\{(x,t):\ x\in[A,A],\ t\in[0,T],\},\ \ \widetilde{\rho}(x,x_1,t,t_1)=\max\{|x-x_1|,|y-y_1|\}.$$

Let

$$u(x,t)=\int\limits_{-\infty}^{+\infty}\cos(yx)\,G(y,t)dy,$$

where

$$G(y,t)=\frac{1}{\sqrt{2\pi}}\int\limits_0^t e^{-a^2y^2(t-\tau}\tilde{\xi}(y,\tau)d\tau,\quad \tilde{\xi}(y,\tau)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}\cos(yx)\,\xi(x,\tau)dx,$$

be a separable stochastic process from the space $L_U(\Omega)$, and let the function U satisfies the g-condition. Assume that there exists a monotone increasing continuous function $\sigma=\sigma(h)$, $0\le h\le\sup\limits_{t,s\in T}\widetilde{\rho}(x,x_1,t,t_1)$ such that

$$\sup_{\substack{|x-x_i|\le h\\|t-t_1|\le h}}\tau_\varphi\left(u(x,t)-u(x_1,t_1)\right)\le\sigma_(h).$$

If for some ε

$$\int_{0+}^{\varepsilon} U^{(-1)}\left(\left(\frac{A}{\sigma^{(-1)}(u)}+1\right)\left(\frac{T}{2\sigma^{(-1)}(u)}+1\right)\right) du < \infty,$$

where $\sigma^{(-1)}(h)$ *is the inverse of* $\sigma(h)$*, then the random variable* $\sup\limits_{(x,t)\in\widetilde{T}} |u(x,t)|$ *belongs to the space* $L_U(\Omega)$ *with the probability one and*

$$\left\|\sup_{(x,t)\in\widetilde{T}} |u(x,t)|\right\|_U \leq \|u(x_0,t_0)\|_U$$

$$+\frac{1}{\theta(1-\theta)}\int\limits_0^{\omega_0\theta}\int_{0+}^{\varepsilon} U^{(-1)}\left(\left(\frac{A}{\sigma^{(-1)}(u)}+1\right)\left(\frac{T}{2\sigma^{(-1)}(u)}+1\right)\right) du = B(x_0,t_0,\ \theta),$$

where $(x_0,t_0) \in \widetilde{T}$*,* $\omega_0 = \sigma(\sup\limits_{t\in T}\rho(x_0,x,t_0,t))$*,* $0<\theta<1$*. In addition, for all* $\varepsilon>0$ *the following inequality holds:*

$$P\left\{\sup_{(x,t)\in\widetilde{T}} |u(x,t)| > \varepsilon\right\} \leq \left(U\left(\frac{\varepsilon}{B(x_0,t_0,\theta)}\right)\right)^{-1}.$$

Proof. This Theorem follows from Theorem 8 since

$$N(\sigma^{(-1)}(u)) \leq \left(\frac{A}{\sigma^{(-1)}(u)}+1\right)\left(\frac{T}{2\sigma^{(-1)}(u)}+1\right).$$

□

REFERENCES

[1] Angulo J.M., Ruiz-Medina M.D., Ang V.V., Grecksch W. *Fractional diffusion and fractional heat equation.* Adv. in Appl. Probab. 2000, **32** (4), 1077–1099. doi:10.1239/aap/1013540349

[2] Beghin L., Kozachenko Yu., Orsingher E., Sakhno L. *On the solution of linear odd-order heat-type equations with random initial.* J. Stat. Phys. 2007, **127** (4), 721–739. doi:10.1007/s10955-007-9309-x

[3] Barrasa de La Krus E., Kozachenko Yu.V. *Boundary-value problems for equations of mathematical physics with strictly Orlics random initial conditions.* Random Oper. and Stoch. Equ. 1995, **3** (3), 201–220.

[4] Bejsenbaev E., Kozachenko Yu.V. *Uniform convergence in probability of random series, and solutions of boundary value problems with random initial conditions.* Teor. Imovir. Mat. Stat. 1979, **21**, 9–23. (in Russian)

[5] Buldygin V.V., Kozachenko Yu.V. Metric Characterization of Random Variables and Random processes. AMS, Providence, Rhode Island, 2000.

[6] Markovich B.M. Equations of Mathematical Physics. Lviv Polytechnic Publishing House, Lviv, 2010. (in Ukrainian)

[7] Kozachenko Yu.V., Leonenko G.M. *Extremal behavior of the heat random field.* Extremes 2006, **8** (3), 191–205. doi:10.1007/s10687-006-7967-8

[8] Kozachenko Yu.V., Slyvka G.I. *Justification of the Fourier method for hyperbolic equations with random initial conditions.* Theory Probab. and Math. Statist. 2004, **69**, 67–83. doi:10.1090/S0094-9000-05-00615-0 (translation of Teor. Imovir. Mat. Stat. 2003, **69**, 63–78. (in Russian))

[9] Kozachenko Yu.V., Slyvka G.I. *Modelling a solution of a hyperbolic equation with random initial conditions.* Theory Probab. Math. Statist. 2007, **74**, 59–75. doi:10.1090/S0094-9000-07-00698-9 (translation of Teor. Imovir. Mat. Stat. 2006, **74**, 52–67 (in Russian))

[10] Kozachenko Yu.V., Slyvka-Tylyshchak A.I. *The Cauchy problem for the heat equation with a random right side.* Random Oper. Stoch. Equ. 2014, **22** (1), 53–64. doi:10.1515/rose-2014-0006

[11] Kozachenko Yu.V., Veresh K.J. *The heat equation with random initial conditions from Orlicz space.* Theory Probab. Math. Statist. 2010, (80), 71–84. doi:10.1090/S0094-9000-2010-00795-2 (translation of Teor. Imovir. Mat. Stat. 2009, **80**, 63–75 (in Ukrainian))

[12] Kozachenko Yu.V., Veresh K.J. *Boundary-value problems for a nonhomogeneous parabolic equation with Orlicz right side*. Random Oper. Stoch. Equ. 2010, **18** (2), 97–119. doi:10.1515/rose.2010.005

[13] Slyvka-Tylyshchak A.I. *Simulation of vibrations of a rectangular membrane with random initial conditions*. Annales Mathematicae et Informaticae 2012, **39**, 325–338.

[14] Slyvka-Tylyshchak A.I. *Justification of the Fourier method for equations of homogeneous string vibration with random initial conditions.* Annales Univ. Sci. Budapest., Sect. Comp. 2012, **38**, 211–232.

[15] Slyvka G.I., Veresh K.J. *Justifications of the Fourier method for hyperbolic equations with random initials conditions from Orlicz Spaces.* Bull. Uzhgorod Univ. Ser. Math. Inform. 2008, **16**, 174–183. (in Ukrainian)

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Сливка-Тилищак Г.І. *Рівняння теплопровідності на прямій з випадковою правою частиною з простору Орліча* // Карпатські матем. публ. — 2014. — Т.6, №1. — С. 134–148.

В роботі розглядається рівняння теплопровідності на прямій з випадковою правою частиною з простору Орліча. Знайдено умови існування з ймовірністю одиниця розв'язку задачі Коші такого рівняння. Отримано оцінку для розподілу супремуму розв'язку даної задачі.

Ключові слова і фрази: рівняння теплопровідності, простір Орліча.

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В работе рассматривается уравнение теплопроводности на прямой со случайной правой частью с пространства Орлича. Найдены условия существования с вероятностью единица решения задачи Коши для такого уравнения. Получены оценки для распределения супремума решения данной задачи.

Ключевые слова и фразы: уравнение теплопроводности, пространство Орлича.