



KUZ A.M., PTASHNYK B.YO.

PROBLEM FOR HYPERBOLIC SYSTEM OF EQUATIONS HAVING CONSTANT COEFFICIENTS WITH INTEGRAL CONDITIONS WITH RESPECT TO THE TIME VARIABLE

In a domain specified in the form of a Cartesian product of a segment $[0, T]$ and the space \mathbb{R}^p , we study a problem with integral conditions with respect to the time variable for hyperbolic system with constant coefficients in a class of almost periodic functions in the space variables. A criterion for the unique solvability of this problem and sufficient conditions for the existence of its solution are established. To solve the problem of small denominators arising in the construction of solutions of the posed problem, we use the metric approach.

Key words and phrases: integral conditions, small denominators, Lebesgue measure, almost periodic function, hyperbolic system.

Institute for Applied Problems of Mechanics and Mathematics, 3b Naukova str., 79060, Lviv, Ukraine

E-mail: kuz.anton87@gmail.com (Kuz A.M.), ptashnyk@lms.lviv.ua (Ptashnyk B.Yo.)

INTRODUCTION

Problems with integral conditions with respect to a chosen variable for partial differential equations (PDEs) have become an important area of investigation in recent years (second half of the XX century). Their study driven by a need for constructing a general theory of boundary value problems for such equations, as well as those problems occur in the mathematical simulation of various physical phenomena in the case when it is impossible to directly determine some physical quantities, but the mean values of these quantities are known. Such problems, in general, are ill-posed and their solvability in some cases is related to the problem of small denominators.

Problems with integral conditions for PDEs are studied by many authors (see [1, 7, 9, 13] and the references there), however these problems were investigated insufficient for systems of such equations. Among research works devoted to the study of problems with integral conditions for systems of PDEs are worth noting these [6, 10, 11]. In particular, in [5] author investigate the problem with integral conditions with respect to the time variable in p -dimensional layer for the first order system of PDEs in a class of finite smooth functions with exponential growth for spatial variables. Correct solvability of the problem with integral conditions with respect to the chosen variable and 2π -periodical conditions for other variables to the composite type system of PDEs was established in [6] and [10]. The paper [11] deals with the problem with integral conditions with respect to the time variable (in the form of consecutive moments

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of the required function) for the linear first order system of evolution PDEs with deviating argument.

In the present paper we study a correct solvability of the problem with more general conditions with respect to the time variable, including an integral conditions in the form of moments of arbitrary order of the required functions and the Dirichlet-type conditions as special cases, for the high order hyperbolic system of PDEs with constant coefficients in a class of almost periodic functions in spatial variables in p -dimensional layer.

We use the following notations: $i = \sqrt{-1}$, $x = (x_1, \dots, x_p) \in \mathbb{R}^p$, $dx = dx_1 \cdots dx_p$; $k = (k_1, \dots, k_p) \in \mathbb{Z}^p$, $|k| = |k_1| + \dots + |k_p|$; $\hat{s} = (s_0, s_1, \dots, s_p) \in \mathbb{Z}_+^{p+1}$, $|\hat{s}| = s_0 + s_1 + \dots + s_p$, $|\hat{s}|^* = 2s_0 + s_1 + \dots + s_p$; $\mu_k = (\mu_{k_1}, \dots, \mu_{k_p}) \in \mathbb{R}^p$, $\|\mu_k\|^2 = \mu_{k_1}^2 + \dots + \mu_{k_p}^2$, $|\mu_k| = |\mu_{k_1}| + \dots + |\mu_{k_p}|$, $(\mu_k, x) = \mu_{k_1}x_1 + \dots + \mu_{k_p}x_p$; S_q is the symmetric group of permutations of first q natural numbers; ρ_ω is the number of inversions in the permutation $\omega = (i_1, \dots, i_q) \in S_q$; $D^p = (0, T) \times \mathbb{R}^p$; \mathbf{I}_m is the $m \times m$ identity matrix, C_q^r is the number of all combinations of q elements by r ; C_j , $j = 1, 2, \dots$, are positive values, independent of k and μ_k , $[a]$ is an integer part of a real number a .

1 STATEMENT OF THE PROBLEM

In the domain D^p we consider the problem of finding almost periodic with respect to x solution of the problem

$$L \left(\frac{\partial^2}{\partial t^2}, \frac{\partial}{\partial x} \right) [\vec{u}] := \sum_{|\hat{s}|^*=2n} \mathbf{A}_{\hat{s}} \frac{\partial^{2n} \vec{u}(t, x)}{\partial t^{2s_0} \partial x_1^{s_1} \cdots \partial x_p^{s_p}} = \vec{0}, \quad (t, x) \in D^p, \quad (1)$$

$$\begin{aligned} U_j[\vec{u}] &:= \alpha_j \frac{\partial^{2(j-1)} \vec{u}}{\partial t^{2(j-1)}} \Big|_{t=0} + \beta_j \int_0^T t^j \vec{u}(t, x) dt = \vec{\varphi}_j(x), \quad x \in \mathbb{R}^p, \\ U_{n+j}[\vec{u}] &:= \alpha_{n+j} \frac{\partial^{2(j-1)} \vec{u}}{\partial t^{2(j-1)}} \Big|_{t=T} + \beta_{n+j} \int_0^T t^{n+j} \vec{u}(t, x) dt = \vec{\varphi}_{n+j}(x), \quad x \in \mathbb{R}^p, \end{aligned} \quad (2)$$

where $j \in \{1, \dots, n\}$; $\mathbf{A}_{\hat{s}} = \left\| a_{q,l}^{\hat{s}} \right\|_{l,q=1}^m$, $a_{q,l}^{\hat{s}} \in \mathbb{R}$, $\mathbf{A}_{(n,0,\dots,0)} = \mathbf{I}_m$; $\alpha_l, \beta_l \in \mathbb{R}$, $\alpha_l^2 + \beta_l^2 \neq 0$, $r_l \in \mathbb{Z}_+$, $l \in \{1, \dots, 2n\}$, $0 \leq r_1 < r_2 < \dots < r_{2n}$; $\vec{u}(t, x) = \text{col}(u^1(t, x), \dots, u^m(t, x))$, vector-functions $\vec{\varphi}_l(x) = \text{col}(\varphi_l^1(x), \dots, \varphi_l^m(x))$, $l \in \{1, \dots, 2n\}$, are almost periodic [2] with respect to x with given spectrum

$$M_p := \left\{ \mu_k \in \mathbb{R}^p : \mu_{-k} = -\mu_k, \mu_{\vec{0}} = \vec{0}, d_1 |k|^{\theta_1} \leq |\mu_k| \leq d_2 |k|^{\theta_2}, k \in \mathbb{Z}^p \right\},$$

where $0 < d_1 \leq d_2$, $0 < \theta_1 \leq \theta_2$, and are expanded in Fourier series

$$\vec{\varphi}_l(x) = \sum_{k \in \mathbb{Z}^p} \vec{\varphi}_{lk} \exp(i\mu_k, x), \quad \vec{\varphi}_{lk} = \text{col}(\varphi_{lk}^1, \dots, \varphi_{lk}^m), \quad l \in \{1, \dots, 2n\}, \quad (3)$$

where $\vec{\varphi}_{lk} = \lim_{H \rightarrow \infty} \frac{1}{H^p} \int_{[0,H]^p} \vec{\varphi}_l(x) \exp(-i\mu_k, x) dx$.

We assume that the system (1) is hyperbolic by Petrovsky in narrow sense, that is, for each vector $\eta = (\eta_1, \dots, \eta_p) \in \mathbb{R}^p \setminus \{\vec{0}\}$ the roots $\gamma_j(\eta)$, $j \in \{1, \dots, 2nm\}$, of the characteristic equation

$$\det L(\gamma^2, \eta) := \det \left\| \sum_{|\hat{s}|^* = 2n} \mathbf{A}_{\hat{s}} \gamma^{2s_0} \eta_1^{s_1} \dots \eta_p^{s_p} \right\| = 0, \quad (4)$$

which corresponds to the system (1), are real and different, and therefore (driven by the appearance of the system (1)) are different from zero.

At investigation of the problem (1), (2) we will use the following spaces of almost periodic functions with respect to x with the spectrum M_p :

$H_{M_p}^\alpha := H^\alpha(M_p; \mathbb{R}^p)$, $\alpha \in \mathbb{R}$, is the space obtained by closure of space of finite trigonometric polynomials of the form $v(x) = \sum_{|k| \leq N} v_k \exp(i\mu_k, x)$, $\mu_k \in M_p$, according to the norm function given by [15]

$$\|v; H_{M_p}^\alpha\| = \left(\sum_{k \in \mathbb{Z}^p} |v_k|^2 (1 + |\mu_k|)^{2\alpha} \right)^{1/2}.$$

$\bar{H}_{M_p, m}^\alpha$ is the space of vector functions $\vec{v}(x) = \text{col}(v^1(x), \dots, v^m(x))$ such that $v^q(x) \in H_{M_p}^\alpha$, $q \in \{1, \dots, m\}$, with the following norm

$$\|\vec{v}; \bar{H}_{M_p, m}^\alpha\| = \sum_{q=1}^m \|v^q; H_{M_p}^\alpha\|.$$

$\bar{C}^h([0, T], H_{M_p}^\alpha)$, $h \in \mathbb{Z}_+$, is the space of vector functions $\vec{u}(t, x) = \sum_{k \in \mathbb{Z}^p} \vec{u}_k(t) \exp(i\mu_k, x)$, $\mu_k \in M_p$, $\vec{u}_k(t) = \text{col}(u_k^1(t), \dots, u_k^m(t))$, such, that for any fixed point $t \in [0, T]$ all derivatives $\partial^j \vec{u}(t, \cdot) / \partial t^j = \sum_{k \in \mathbb{Z}^p} \vec{u}_k^{(j)}(t) \exp(i\mu_k, x)$, $j \in \{0, 1, \dots, h\}$, belong to the space $\bar{H}_{M_p, m}^\alpha$ and are continuous with respect to the t according to the norm of this space,

$$\begin{aligned} \|\vec{u}; \bar{C}^h([0, T], H_{M_p}^\alpha)\| &= \sum_{j=0}^h \max_{t \in [0, T]} \left\| \frac{\partial^j \vec{u}}{\partial t^j}; \bar{H}_{M_p, m}^\alpha \right\| \\ &= \sum_{j=0}^h \sum_{q=1}^m \max_{t \in [0, T]} \left(\sum_{k \in \mathbb{Z}^p} \left| \frac{d^j u_k^q(t)}{dt^j} \right|^2 (1 + |\mu_k|)^{2\alpha} \right)^{1/2}. \end{aligned} \quad (5)$$

$\bar{C}_{M_p, m}^h(\bar{D}^p)$ is the space of vector functions $\vec{u}(t, x) = \text{col}(u^1(t, x), \dots, u^m(t, x))$, which are h -times continuously differentiable in \bar{D}^p with respect to all variables and almost periodic for x with the spectrum M_p uniformly by $t \in [0, T]$, with norm given by formula

$$\|\vec{u}; \bar{C}_{M_p, m}^h(\bar{D}^p)\| = \sum_{q=1}^m \sum_{0 \leq |\hat{s}| \leq h} \max_{t \in [0, T]} \sup_{x \in \mathbb{R}^p} \left| \frac{\partial^{|\hat{s}|} u^q(t, x)}{\partial t^{s_0} \partial x_1^{s_1} \dots \partial x_p^{s_p}} \right|;$$

$\bar{C}_{M_p, m}^{h, q}(\mathbb{R}^p)$ is the subspace of vector functions from $\bar{C}_{M_p, m}^h(\bar{D}^p)$, independent of t .

If $\alpha > p/(2\theta_1)$, then such embeddings are valid (see [3] and the references given there):

$$\bar{H}_{M_p, m}^{q+\alpha} \subset \bar{C}_{M_p, m}^q(\mathbb{R}^p), \quad \bar{C}^q([0, T], H_{M_p}^{q+\alpha}) \subset \bar{C}_{M_p, m}^q(\bar{D}^p), \quad q \in \mathbb{Z}_+. \quad (6)$$

2 UNIQUENESS OF THE SOLUTION

Almost periodic with respect to x with the spectrum M_p solution of the problem (1), (2) we seek in the form of the vector series

$$\vec{u}(t, x) = \sum_{k \in \mathbb{Z}^p} \vec{u}_k(t) \exp(i\mu_k x), \quad \mu_k \in M_p. \quad (7)$$

After substituting series (3), (7) into the system (1) and conditions (2), we receive that the each of functions $\vec{u}_k(t)$, $k \in \mathbb{Z}^p$, is a solution of this problem:

$$L\left(\frac{d^2}{dt^2}, i\mu_k\right) \vec{u}_k(t) := \sum_{|\mathbf{s}|^* = 2n} i^{|\mathbf{s}|} \mathbf{A}_{\mathbf{s}} \mu_{k_1}^{s_1} \cdots \mu_{k_p}^{s_p} \frac{d^{2s_0}}{dt^{2s_0}} \vec{u}_k(t) = \vec{0}, \quad (8)$$

$$\begin{aligned} U_j[\vec{u}_k] &:= \alpha_j \frac{d^{2(j-1)} \vec{u}_k(0)}{dt^{2(j-1)}} + \beta_j \int_0^T t^{r_j} \vec{u}_k(t) dt = \vec{\varphi}_{jk}, \\ U_{n+j}[\vec{u}] &:= \alpha_{n+j} \frac{d^{2(j-1)} \vec{u}_k(T)}{dt^{2(j-1)}} + \beta_{n+j} \int_0^T t^{r_{n+j}} \vec{u}_k(t) dt = \vec{\varphi}_{n+j,k}, \quad j \in \{1, \dots, n\}, \end{aligned} \quad (9)$$

If $k = \vec{0}$ ($\mu_{\vec{0}} = \vec{0}$), the system (8) has the form

$$L\left(\frac{d^2}{dt^2}, \vec{0}\right) \vec{u}_{\vec{0}}(t) := \mathbf{I}_m \frac{d^{2n}}{dt^{2n}} \vec{u}_{\vec{0}}(t) = \vec{0},$$

and so, each component $u_{\vec{0}}^q(t)$, $q \in \{1, \dots, m\}$, of the solution $\vec{u}_{\vec{0}}(t) = \text{col}(u_{\vec{0}}^1(t), \dots, u_{\vec{0}}^m(t))$ of the problem (8), (9) is a solution of this problem for scalar differential equation:

$$\frac{d^{2n}}{dt^{2n}} u_{\vec{0}}^q(t) = 0, \quad (10)$$

$$\begin{aligned} U_j[u_{\vec{0}}^q] &:= \alpha_j \frac{d^{2(j-1)} u_{\vec{0}}^q(0)}{dt^{2(j-1)}} + \beta_j \int_0^T t^{r_j} u_{\vec{0}}^q(t) dt = \varphi_{j, \vec{0}}^q, \\ U_{n+j}[u_{\vec{0}}^q] &:= \alpha_{n+j} \frac{d^{2(j-1)} u_{\vec{0}}^q(T)}{dt^{2(j-1)}} + \beta_{n+j} \int_0^T t^{r_{n+j}} u_{\vec{0}}^q(t) dt = \varphi_{n+j, \vec{0}}^q, \quad j \in \{1, \dots, n\}. \end{aligned} \quad (11)$$

The characteristic determinant $\Delta(\vec{0}, T)$ of the problem (10), (11) for each $q \in \{1, \dots, m\}$ has the form

$$\Delta(\vec{0}, T) = \begin{vmatrix} \alpha_1 S_1^0(0) + \beta_1 \frac{T^{r_1+1}}{r_1+1} & \cdots & \alpha_1 S_{2n}^0(0) + \beta_1 \frac{T^{r_1+2n}}{r_1+2n} \\ \vdots & \cdots & \vdots \\ \alpha_n S_1^{2(n-1)}(0) + \beta_n \frac{T^{r_n+1}}{r_n+1} & \cdots & \alpha_n S_{2n}^{2(n-1)}(0) + \beta_n \frac{T^{r_n+2n}}{r_n+2n} \\ \alpha_{n+1} S_1^0(T) + \beta_{n+1} \frac{T^{r_{n+1}+1}}{r_{n+1}+1} & \cdots & \alpha_{n+1} S_{n+1}^0(T) + \beta_{n+1} \frac{T^{r_{n+1}+2n}}{r_{n+1}+2n} \\ \vdots & \cdots & \vdots \\ \alpha_{2n} S_1^{2(n-1)}(T) + \beta_{2n} \frac{T^{r_{2n}+1}}{r_{2n}+1} & \cdots & \alpha_{2n} S_{2n}^{2(n-1)}(T) + \beta_{2n} \frac{T^{r_{2n}+2n}}{r_{2n}+2n} \end{vmatrix},$$

where

$$S_j^{2(l-1)}(z) = \begin{cases} 0, & j < 2l - 1, \\ \frac{(j-1)!}{(j-2l+1)!} z^{j-2l+1}, & j \geq 2l - 1, \end{cases} \quad j \in \{1, \dots, 2n\}, \quad l \in \{1, \dots, n\}.$$

If condition $\Delta(\vec{0}, T) \neq 0$ holds true, the unique solution of the problem (10), (11) always exists for each $q \in \{1, \dots, m\}$. These solutions are expressed by formulas

$$u_0^q(t) = \sum_{l,j=1}^{2n} \frac{\Delta_{lj}(\vec{0}, T)}{\Delta(\vec{0}, T)} \varphi_{l,\vec{0}}^q t^{j-1}, \quad q \in \{1, \dots, m\}, \quad (12)$$

where by $\Delta_{lj}(\vec{0}, T)$ we denote the cofactor of the entry in the l -th row and j -th column in the determinant $\Delta(\vec{0}, T)$.

Remark 1. If $\Delta(\vec{0}, T) = 0$, then the homogeneous problem corresponding to the problem (10), (11), has nontrivial solution $u_0^*(t) = \text{col}(\tilde{u}_0^1(t), \dots, \tilde{u}_0^m(t))$, where $\tilde{u}_0^q(t) = \sum_{j=1}^{2n} C_{jq} t^{j-1}$, $q \in \{1, \dots, m\}$, and coefficients C_{jq} , $j \in \{1, \dots, 2n\}$, are solutions of system of linear algebraic equations

$$\begin{cases} \sum_{j=1}^{2n} C_{jq} \left(\alpha_l S_j^{2(l-1)}(0) + \beta_l \frac{T^{r_l+j}}{r_l+j} \right) = \varphi_{l,\vec{0}}^q \\ \sum_{j=1}^{2n} C_{jq} \left(\alpha_{n+l} S_j^{2(l-1)}(T) + \beta_{n+l} \frac{T^{r_{n+l}+j}}{r_{n+l}+j} \right) = \varphi_{n+l,\vec{0}}^q, \end{cases} \quad l \in \{1, \dots, n\}.$$

Now we consider the problem (8), (9) for all $\mu_k \in M_p \setminus \{\vec{0}\}$. The characteristic equation corresponding to the system of ordinary differential equations (8), may be expressed in the form

$$\det L(\gamma^2, i\mu_k) := \sum_{\omega \in S_m} (-1)^{\rho_\omega} \prod_{q=1}^m \left(\sum_{|s|+2s_0=2n} i^{|s|} a_{i_q,q}^{\hat{s}} \gamma^{2s_0} \mu_{k_1}^{s_1} \dots \mu_{k_p}^{s_p} \right) = 0. \quad (13)$$

Obviously, that roots γ_{jk} of the equation (13) are defined by formulas

$$\gamma_{jk} = i \gamma_j(\mu_k), \quad j \in \{1, \dots, 2nm\}. \quad (14)$$

In (14) by $\gamma_j(\mu_k)$, $j \in \{1, \dots, 2nm\}$, we denote roots of the equation (4) at $\eta = \mu_k$, $\mu_k \in M_p \setminus \{\vec{0}\}$; moreover $\gamma_{nm+q,k} = -\gamma_{qk}$, $q \in \{1, \dots, nm\}$, and following estimates hold [4]:

$$|\gamma_{jk}| \leq C_1(1 + |\mu_k|), \quad j \in \{1, \dots, 2nm\}, \quad \mu_k \in M_p \setminus \{\vec{0}\}, \quad C_1 = (2nm)^p \max_{\substack{|s|^*=2n \\ 1 \leq q, l \leq m}} \{a_{q,l}^{\hat{s}}\}. \quad (15)$$

The fundamental system of solutions of the system of equations (8) is as follows (see [14, p. 116]):

$$\{\vec{u}_{jk}(t) = \vec{h}_{jk} \exp(\gamma_{jk}t), \quad j \in \{1, \dots, 2nm\}\}, \quad k \in \mathbb{Z}^p \setminus \{\vec{0}\}, \quad (16)$$

where by

$$\vec{h}_{jk} = \text{col}(\vec{h}_{jk}^1, \dots, \vec{h}_{jk}^m), \quad j \in \{1, \dots, 2nm\}, \quad (17)$$

we denote some nonzero column of the matrix $L^*(\gamma_{jk}^2, i\mu_k)$ which is adjugate matrix of the matrix $L(\gamma_{jk}^2, i\mu_k)$. Obviously, that $\vec{h}_{nm+j,k} = \vec{h}_{jk}$, $j \in \{1, \dots, nm\}$.

Solution of the problem (8), (9) may be expressed by the formula

$$\vec{u}_k(t) = \sum_{j=1}^{2nm} C_{jk} \vec{h}_{jk} \exp(\gamma_{jk} t), \quad k \in \mathbb{Z}^p \setminus \{\vec{0}\},$$

where constants C_{jk} , $j \in \{1, \dots, 2nm\}$, are defined from this system of linear algebraic equations

$$\sum_{j=1}^{2nm} C_{jk} \left(\alpha_l P_j^l + \beta_l I_l(\gamma_{jk}) \right) \vec{h}_{jk} = \vec{\varphi}_{lk}, \quad l \in \{1, \dots, 2n\}, \quad (18)$$

where for all $l \in \{1, \dots, 2n\}$.

$$P_j^l = \begin{cases} \gamma_{jk}^{2(l-1)}, & 1 \leq l \leq n, \\ \gamma_{jk}^{2(l-n-1)} \exp(\gamma_{jk} T), & n+1 \leq l \leq 2n, \end{cases} \quad j \in \{1, \dots, 2nm\}, \quad (19)$$

$$I_l(z) = \int_0^T t^{r_l} \exp(zt) dt = \frac{(-1)^{r_l} r_l!}{z^{r_l+1}} + \sum_{q=1}^{r_l+1} \frac{(-1)^{q+1} r_l!}{(r_l - q + 1)!} \frac{T^{r_l-q+1}}{z^q} \exp(zT). \quad (20)$$

The determinant of the system of equations (18) matches with the characteristic determinant $\Delta(\mu_k, T)$, $\mu_k \in M_p \setminus \{\vec{0}\}$, of the problem (8), (9) and has the form

$$\begin{aligned} \Delta(\mu_k, T) &= \det \|U_q[\vec{h}_{jk} \exp(\gamma_{jk} t)]\|_{j=1, \dots, 2nm}^{q=1, \dots, 2n} \\ &= \begin{vmatrix} \vec{h}_{1k} (\alpha_1 P_1^1 + \beta_1 I_1(\gamma_{1k})) & \dots & \vec{h}_{2nm,k} (\alpha_1 P_{2nm}^1 + \beta_1 I_1(\gamma_{2nm,k})) \\ \vdots & \ddots & \vdots \\ \vec{h}_{1k} (\alpha_{2n} P_1^{2n} + \beta_{2n} I_{2n}(\gamma_{1k})) & \dots & \vec{h}_{2nm,k} (\alpha_{2n} P_{2nm}^{2n} + \beta_{2n} I_{2n}(\gamma_{2nm,k})) \end{vmatrix}. \end{aligned}$$

The problem (8), (9) can not have (see [16]) two different solutions if and only if $\Delta(\mu_k, T) \neq 0$, $\mu_k \in M_p \setminus \{\vec{0}\}$.

Theorem 1. *For the uniqueness of a solution of the problem (1), (2) in the scale of spaces $\bar{C}^{2n}([0, T], H_{M_p}^\alpha)$ it is necessary and sufficient that the following condition be satisfied*

$$\forall \mu_k \in M_p \quad \Delta(\mu_k, T) \neq 0. \quad (21)$$

Proof. Necessity. Suppose that for some $\mu_{k^0} \in M_p$ $\Delta(\mu_{k^0}, T) = 0$ holds. If $k^0 = \vec{0}$, then homogeneous problem, corresponding to the problem (8), (9) at $k = \vec{0}$, has nontrivial solution $u_0^*(t)$ (see Remark 1). If $k^0 \neq \vec{0}$, then exist nontrivial solutions $\vec{u}_{k^0}(t) = \sum_{j=1}^{2nm} C_{j,k^0} \vec{h}_{j,k^0} \exp(\gamma_{j,k^0} t)$ of the homogeneous problem, corresponding to the problem (8), (9), where C_{j,k^0} , $j \in \{1, \dots, 2nm\}$, are defined from homogeneous system of equation, corresponding to the system (18) at $k = k^0$. Therefore the homogeneous problem, corresponding to the problem (1), (2), has nontrivial solutions $u_0^*(t)$ or $\vec{u}(t, x) = \vec{u}_{k^0} \exp(i\mu_{k^0}, x)$, $k^0 \neq \vec{0}$, and if solution to the problem (1), (2) exists, it won't be unique.

Sufficiency. Let the condition (21) holds true. Suppose to the contrary that there exist two different solutions $\vec{u}_1(t, x)$, $\vec{u}_2(t, x)$ of the problem (1), (2) from the space $\bar{C}^{2n}([0, T], H_{M_p}^\alpha)$.

Then the function $\vec{w}(t, x) = \vec{u}_2(t, x) - \vec{u}_1(t, x)$, which belongs to the space $\bar{C}^{2n}([0, T], H_{M_p}^\alpha)$, is the solution to the homogeneous problem, corresponding to the problem (1), (2). Moreover, functions $\vec{w}(t, x)$, $L[\vec{w}]$, $U_j[\vec{w}]$, $j \in \{1, \dots, 2n\}$, are almost periodic with respect to x with spectrum M_p and expand into Fourier series of the form (7). The Fourier series of functions $L[\vec{w}]$ and $U_j[\vec{w}]$, $j \in \{1, \dots, 2n\}$, match with the series obtained by applying operators L and U_j , $j \in \{1, \dots, 2n\}$, to the Fourier series of the vector function $\vec{w}(t, x)$ respectively. Each of the Fourier coefficients $\vec{w}_k(t)$, $k \in \mathbb{Z}^p$, of the function $\vec{w}(t, x)$ is the solution of homogeneous problem, corresponding to the problem (8), (9). Because $\Delta(\mu_k, T) \neq 0$ for all $\mu_k \in M_p$, then homogeneous problem, corresponding to the problem (8), (9), has only trivial solution for all $\mu_k \in M_p$ and therefore $\vec{w}_k(t) = 0$, $t \in [0, T]$, $k \in \mathbb{Z}^p$. Hence, on the basis of Parseval equality we obtain that $\vec{w}(t, x) = 0$ in the space $\bar{C}^{2n}([0, T], H_{M_p}^\alpha)$, i.e. $\vec{u}_1(t, x) = \vec{u}_2(t, x)$. \square

3 EXISTENCE OF THE SOLUTION

Let condition (21) holds true. Then for each $\mu_k \in M_p$ the unique solution $\vec{u}_k(t) \in C^{2n}([0, T])$ of the problem (8), (9) exists and the formal solution $\vec{u}(t, x)$ of the problem (1), (2) may be expressed in the form

$$\vec{u}(t, x) = u_{\vec{0}}(t) + \sum_{k \in \mathbb{Z}^p \setminus \{\vec{0}\}} \left(\sum_{j=1}^{2nm} C_{jk} \vec{h}_{jk} \exp(\gamma_{jk} t) \right) \exp(i\mu_k x), \quad (22)$$

in which

$$C_{jk} = \sum_{q=1}^m \sum_{l=1}^{2n} \frac{\Delta_{m(l-1)+q,j}(\mu_k, T)}{\Delta(\mu_k, T)} \varphi_{lk}^q, \quad j \in \{1, \dots, 2nm\}, \quad (23)$$

where by $\Delta_{m(l-1)+q,j}(\mu_k, T)$ we denote the cofactor of the entry in the $(m(l-1)+q)$ -th row and j -th column in determinant $\Delta(\mu_k, T)$ and components of vector $u_{\vec{0}}(t)$ are defined by formulas (12).

While proving the existence of a solution of the problem (1), (2) in the scale of spaces $\bar{C}^{2n}([0, T], H_{M_p}^\alpha)$ we will need following lemmas.

We also denote

$$C_2 := C_{2n+p+1}^{2n} C_1 \max_{\substack{|\hat{s}|^* = 2n \\ 1 \leq q, l \leq m}} \{a_{q,l}^{\hat{s}}\}, \quad C_3 = (m-1)!(C_2)^{m-1},$$

$$C_4 = C_3 \max_{1 \leq l \leq 2n} \left\{ |\alpha_l| (C_1)^{2(n-1)}, |\beta_l| T^{r_l+1} / (r_l + 1) \right\}, \quad C_5 = (2nm-1)!(C_4)^{2nm}.$$

Lemma 1. For components of vectors (17) such estimates hold true

$$|h_{jk}^q| \leq C_3(1 + |\mu_k|)^{2n(m-1)}, \quad q \in \{1, \dots, m\}, j \in \{1, \dots, nm\}, \quad \mu_k \in M_p \setminus \{\vec{0}\}.$$

Proof. By $\lambda_{ql}(\gamma_{jk})$, $q, l \in \{1, \dots, m\}$, we denote the element in the q -th row and l -th column in the matrix $L(\gamma_{jk}^2, i\mu_k)$, $j \in \{1, \dots, nm\}$. Note that $\lambda_{ql}(\gamma_{jk}) = \sum_{|\hat{s}|^* = 2n} i^{|\hat{s}|} a_{q,l}^{\hat{s}} \gamma_{jk}^{2s_0} \mu_{k_1}^{s_1} \cdots \mu_{k_p}^{s_p}$ and following estimates hold

$$|\lambda_{ql}(\gamma_{jk})| \leq C_2(1 + |\mu_k|)^{2n}, \quad q, l \in \{1, \dots, m\}, j \in \{1, \dots, nm\}. \quad (24)$$

Now we fix a column with number $l = l^*$ in the matrix $L(\gamma_{jk}^2, i\mu_k)$. Then components h_{jk}^q of vector \vec{h}_{jk} are cofactors of elements $\lambda_{q,l^*}(\gamma_{jk})$, $q \in \{1, \dots, m\}$, in matrix $L(\gamma_{jk}^2, i\mu_k)$ respectively. They may be expressed in form

$$h_{jk}^q = \sum_{\omega \in S_{m-1}} (-1)^{\rho_\omega} \prod_{\substack{l=1 \\ l \neq l^*, i_l \neq q}}^m \lambda_{i_l, l}(\gamma_{jk}), \quad q \in \{1, \dots, m\}, \quad j \in \{1, \dots, nm\}. \quad (25)$$

Based on (24) and (25) we obtain that

$$|h_{jk}^q| \leq (m-1)! \prod_{l=1, l \neq l^*}^m |\lambda_{ll}(\gamma_{jk})| \leq C_3(1 + |\mu_k|)^{2n(m-1)}, \quad j \in \{1, \dots, nm\}, \quad q \in \{1, \dots, m\}.$$

The lemma is proved. \square

By $\psi(\alpha)$ we denote the function of discrete argument, defined on the set $\{\alpha_1, \dots, \alpha_{2n}\}$ as follows:

$$\psi(\alpha_j) := 0, \quad \alpha_j = 0, \quad j \in \{1, \dots, 2n\}; \quad \psi(\alpha_l) = \psi(\alpha_{n+l}) = 2(l-1), \quad \alpha_l \neq 0, \quad l \in \{1, \dots, n\}.$$

Lemma 2. For cofactors $\Delta_{m(l-1)+q,j}(\mu_k, T)$, $q \in \{1, \dots, m\}$, $l \in \{1, \dots, 2n\}$, $j \in \{1, \dots, 2nm\}$, of the determinant $\Delta(\mu_k, T)$, $\mu_k \in M_p \setminus \{\vec{0}\}$, such estimates hold true

$$|\Delta_{m(l-1)+q,j}(\mu_k, T)| \leq C_5(1 + |\mu_k|)^{2n(m-1)(2nm-1)+\Psi_l}, \quad \Psi_l = m \sum_{j=1}^{2n} \psi(\alpha_j) - \psi(\alpha_l).$$

Proof. At first we hold some auxiliary estimates. On basis of formulas (19) and (20) we receive inequalities

$$|P_j^{n+l}| < |P_j^l| \leq (1 + |\mu_k|)^{2(l-1)}, \quad l \in \{1, \dots, n\}, \quad j \in \{1, \dots, 2nm\}, \quad t \in [0, T], \quad (26)$$

$$|I_l(\gamma_{jk})| \leq \int_0^T |t^{r_l} \exp(\gamma_{jk} t)| dt \leq \frac{T^{r_l+1}}{r_l+1}, \quad l \in \{1, \dots, 2n\}, \quad j \in \{1, \dots, nm\}. \quad (27)$$

By $\delta_{rj}(\mu_k) := h_{jk}^q(\alpha_l P_j^l + \beta_l I_l(\gamma_{jk}))$ we denote the element on the entry of r -th row, $r = m(l-1) + q$, $l \in \{1, \dots, 2n\}$, $q \in \{1, \dots, m\}$ and j -th column, $j \in \{1, \dots, 2nm\}$ in the determinant $\Delta(\mu_k, T)$. On basis of formulas (26), (27) and Lemma 1 we obtain that

$$|\delta_{rj}(\mu_k)| < |h_{jk}^q|(|\alpha_l| |P_j^l| + |\beta_l| |I_l(\gamma_{jk})|) < C_4(1 + |\mu_k|)^{2n(m-1)+\psi(\alpha_l)}. \quad (28)$$

Cofactors $\Delta_{m(l-1)+q,j}(\mu_k, T)$, $l \in \{1, \dots, 2n\}$, $q \in \{1, \dots, m\}$, $j \in \{1, \dots, 2nm\}$, may be expressed by formulas

$$\Delta_{m(l-1)+q,j}(\mu_k, T) = \sum_{\omega \in S_{2nm-1}} (-1)^{\rho_\omega} \prod_{\substack{r=1 \\ r \neq m(l-1)+q \\ i_r \neq j}}^{2nm} \delta_{r,i_r}(\mu_k). \quad (29)$$

On basis of (28), (29) we receive that

$$|\Delta_{m(l-1)+q,j}(\mu_k, T)| \leq (2nm-1)! \prod_{\substack{r=1 \\ r \neq m(l-1)+q}}^{2nm} |\delta_{r,r}(\mu_k)| \leq C_5(1 + |\mu_k|)^{2n(m-1)(2nm-1)+\Psi_l},$$

where $l \in \{1, \dots, 2n\}$, $s \in \{1, \dots, m\}$, $j \in \{1, \dots, 2nm\}$. The lemma is proved. \square

The series (22), in general, is divergent because of the expression $|\Delta(\mu_k, T)|$, being different from zero, can take arbitrarily small values for an infinite number (for some subsequence) of vectors $\mu_k \in M_p$.

Theorem 2. *Let condition (21) holds true and there exists a constant $\eta > 0$ such that for all (except for finite number of) vectors $\mu_k \in M_p$ such inequality holds*

$$|\Delta(\mu_k, T)| > (1 + |\mu_k|)^{-\eta}. \quad (30)$$

If $\vec{\varphi}_l(x) \in \bar{H}_{M_p, m}^{\xi_l}$, $\xi_l = \alpha + 2n(2nm(m-1) + 1) + \eta + \Psi_l$, $l \in \{1, \dots, 2n\}$, then there exists a solution of the problem (1), (2) from the space $\bar{C}^{2n}([0, T], H_{M_p}^\alpha)$ which depends continuously on the functions $\vec{\varphi}_l(x)$, $l \in \{1, \dots, 2n\}$. This solution is given by formula (22).

Proof. On basis of formulas (5) and (22) we obtain estimate

$$\begin{aligned} \|\vec{u}; \bar{C}^{2n}([0, T], H_{M_p}^\alpha)\| &= \sum_{q=1}^m \sum_{r=0}^{2n} \max_{t \in [0, T]} \left(\left| \frac{d^r u_0^q(t)}{dt^r} \right|^2 + \sum_{k \in \mathbb{Z}^p \setminus \{\vec{0}\}} \left| \frac{d^r u_k^q(t)}{dt^r} \right|^2 (1 + |\mu_k|)^{2\alpha} \right)^{1/2} \\ &\leq \sum_{q=1}^m \sum_{r=0}^{2n} \left(\max_{t \in [0, T]} \left| \frac{d^r u_0^q(t)}{dt^r} \right|^2 + \sum_{k \in \mathbb{Z}^p \setminus \{\vec{0}\}} \max_{t \in [0, T]} \left| \frac{d^r u_k^q(t)}{dt^r} \right|^2 (1 + |\mu_k|)^{2\alpha} \right)^{1/2}, \end{aligned} \quad (31)$$

in which $u_0^q(t)$, $q \in \{1, \dots, m\}$, are defined by formulas (12), and

$$u_k^q(t) = \sum_{j=1}^{2nm} C_{jk} h_{jk}^q \exp(\gamma_{jk} t), \quad k \in \mathbb{Z}^p \setminus \{\vec{0}\}, \quad (32)$$

where h_{jk}^q , $s \in \{1, \dots, m\}$, are components of the corresponding vector (17). Constants C_{jk} , $j \in \{1, \dots, 2nm\}$, are defined by formulas (23).

From formulas (12) it follows that

$$\max_{t \in [0, T]} \left| \frac{d^r}{dt^r} u_0^q(t) \right| \leq C_6 \sum_{j=1}^{2n} |\varphi_{j, \vec{0}}^q|^2, \quad s \in \{1, \dots, m\}, \quad (33)$$

where constant C_6 depends on T and α_l, β_l, r_l , $l \in \{1, \dots, 2n\}$.

On basis of (15), (23), (32) and Lemma 1 we obtain that

$$\max_{t \in [0, T]} \left| \frac{d^r u_k^s(t)}{dt^r} \right| \leq C_7 \sum_{j=1}^{2nm} \sum_{q=1}^m \sum_{l=1}^{2n} \frac{|\Delta_{m(l-1)+q,j}(\mu_k, T)|}{|\Delta(\mu_k, T)|} |\varphi_{lk}^q| (1 + |\mu_k|)^{2n(m-1)+r}, \quad (34)$$

where $r \in \{0, 1, \dots, 2n\}$ and $C_7 = C_3(C_1)^{2n}$.

Taking into account (30), (34) and Lemma 2, we obtain following estimates:

$$\max_{t \in [0, T]} \left| \frac{d^r u_k^s(t)}{dt^r} \right| \leq 2nm C_5 C_7 \sum_{q=1}^m \sum_{l=1}^{2n} |\varphi_{lk}^q| (1 + |\mu_k|)^{4mn^2(m-1)+\theta_l+\eta+r}, \quad r = 0, 1, \dots, 2n. \quad (35)$$

From estimates (31), (33) and (35) follows that

$$\|\vec{u}; \bar{C}^{2n}([0, T], H_{M_p}^\alpha)\| \leq C_8 \sum_{l=1}^{2n} \sum_{q=1}^m \left(\sum_{k \in \mathbb{Z}^p} |\varphi_{lk}^q|^2 (1 + |\mu_k|)^{2\xi_l} \right)^{1/2} = C_8 \sum_{l=1}^{2n} \|\vec{\varphi}_l; \bar{H}_{M_p, m}^{\xi_l}\|.$$

where $C_8 = 2nm \max\{C_6, 2nm C_5 C_7\}$. From the obtained inequality follows the proof of the theorem. \square

Remark 2. If in Theorem 2 $\alpha > 2n + p/(2\theta_1)$ then, according to (6), such embedding is valid $\bar{C}^{2n}([0, T], H_{M_p}^\alpha) \subset C_{M_p, m}^{2n}(\bar{D}^p)$ and the solution of the problem (1), (2), defined by the formula (22), is a solution in the classical sense.

4 ESTIMATES OF SMALL DENOMINATORS

Let's find when the inequality (30) holds true. To do this, we show that $\Delta(\mu_k, T)$, as function of variable T , is a quasi-polynomial and apply Theorem 2.1 from [9]. We denote by $\mathcal{J}_z, z \in \mathbb{N}$, the set of all vectors of the form $J = (j_1, \dots, j_z), j_l \in \{0, 1\}, l \in \{1, \dots, z\}$;

$$\begin{aligned} A &= (\underbrace{\alpha_1, \dots, \alpha_1}_m, \dots, \underbrace{\alpha_{2n}, \dots, \alpha_{2n}}_m), \quad B = (\underbrace{\beta_1, \dots, \beta_1}_m, \dots, \underbrace{\beta_{2n}, \dots, \beta_{2n}}_m), \\ R &= (\underbrace{r_1, \dots, r_1}_m, \dots, \underbrace{r_{2n}, \dots, r_{2n}}_m), \quad \Gamma_k = (\gamma_{1k}, \dots, \gamma_{nm,k}, -\gamma_{1k}, \dots, -\gamma_{nm,k}), \end{aligned}$$

by $A_q, B_q, R_q, \Gamma_{qk}, q \in \{1, \dots, 2nm\}$, we denote coordinates of vectors A, B, R and Γ_k respectively; $\Gamma_{\omega,k} = (\Gamma_{i_1,k}, \dots, \Gamma_{i_{2nm},k}), \omega = (i_1, \dots, i_{2nm}) \in S_{2nm}$,

$$V_j = (\underbrace{p_j^1, \dots, p_j^1}_m, \dots, \underbrace{p_j^{2n}, \dots, p_j^{2n}}_m), \quad j \in \{1, \dots, 2nm\},$$

by $H_{sj}, s, j \in \{1, \dots, 2nm\}$, we denote values defined as follows:

$$H_{ml+q,j} = H_{qj} = h_{jk}^q, \quad q \in \{1, \dots, m\}, l \in \{1, \dots, 2n-1\}, j \in \{1, \dots, 2nm\},$$

where h_{jk}^q are components of vectors (17).

Further, we will need the following proposition which is proved in the paper [7].

Lemma 3. For arbitrary $x_q, y_q \in \mathbb{C}, q \in \{1, \dots, z\}$, following equality holds true

$$\prod_{q=1}^z (x_q + y_q) = \sum_{j_1=0}^1 \dots \sum_{j_z=0}^1 \prod_{q=1}^z x_q^{j_q} \prod_{l=1}^z y_l^{1-j_l}.$$

For each $\mu_k \in M_p \setminus \{\vec{0}\}$ the determinant $\Delta(\mu_k, T)$ can be expressed by the formula [8]

$$\Delta(\mu_k, T) = \sum_{\omega \in S_{2nm}} (-1)^{\rho_\omega} \prod_{q=1}^{2nm} H_{i_q,q} \left(A_q V_{i_q,q} + B_q I \left(R_q, \Gamma_{i_q,k} \right) \right), \quad (36)$$

where $V_{i_q,q}$ is the element at number q of the vector V_{i_q} , and

$$I(R_q, \Gamma_{i_q,k}) = \int_0^T t^{R_q} \exp(\Gamma_{i_q,k} t) dt = Q_{R_q}(\Gamma_{i_q,k}, T) \exp(T \Gamma_{i_q,k}) - Q_{R_q}(\Gamma_{i_q,k}, 0), \quad (37)$$

$$Q_{R_q}(\Gamma_{i_q,k}, t) = \sum_{l=1}^{R_q+1} \frac{(-1)^{l+1} R_q!}{(R_q - l + 1)!} \frac{t^{R_q-l+1}}{(\Gamma_{i_q,k})^n}, \quad q \in \{1, \dots, 2nm\}. \quad (38)$$

On basis of formulas (36), (37) we obtain that

$$\begin{aligned} \Delta(\mu_k, T) = \sum_{\omega \in S_{2nm}} (-1)^{\rho\omega} \prod_{q=1}^{2nm} H_{i_q, q} \left([A_q V_{i_q, q} - B_q Q_{R_q}(\Gamma_{i_q, k}, 0)] \right. \\ \left. + B_q Q_{R_q}(\Gamma_{i_q, k}, T) \exp(T\Gamma_{i_q, k}) \right). \end{aligned} \quad (39)$$

Formula (39) on basis of Lemma 3 may be expressed in the form

$$\Delta(\mu_k, T) = \sum_{\omega \in S_{2nm}} (-1)^{\rho\omega} \sum_{J \in \mathcal{J}_{2nm}} \Delta_{1k}(\omega, J, T) \Delta_{2k}(\omega, J, T),$$

where

$$\Delta_{1k}(\omega, J, T) = \prod_{q=1}^{2nm} \left(B_q Q_{R_q}(\Gamma_{i_q, k}, T) \exp(T\Gamma_{i_q, k}) \right)^{j_q} = B(J) Q_J(\Gamma_{\omega, k}, T) \exp(T(J, \Gamma_{\omega, k})), \quad (40)$$

$$B(J) = \prod_{q=1}^{2nm} (B_q)^{j_q}, \quad Q_J(\Gamma_{\omega, k}, T) = \prod_{q=1}^{2nm} \left(Q_{R_q}(\Gamma_{i_q, k}, T) \right)^{j_q}, \quad (41)$$

$$(J, \Gamma_{\omega, k}) = \sum_{q=1}^{2nm} j_q \Gamma_{i_q, k}, \quad J \in \mathcal{J}_{2nm}, \quad \omega \in S_{2nm}; \quad (42)$$

$$\Delta_{2k}(\omega, J, T) = \prod_{l=1}^{2nm} (A_l V_{i_l, l} - B_l Q_{R_l}(\Gamma_{i_l, k}, 0))^{1-j_l}. \quad (43)$$

The formula (43) by opening brackets, in view of (38), can be expressed in the following form

$$\Delta_{2k}(\omega, J, T) = P_{1k}(\omega, J) \exp \left(T \sum_{l=nm+1}^{2nm} (1-j_l) \Gamma_{i_l, k} \right) + P_{2k}(\omega, J), \quad (44)$$

where values $P_{1k}(\omega, J)$, $P_{2k}(\omega, J)$ don't depend on T .

On basis of (39), (40), (44), we obtain the following expression for $\Delta(\mu_k, T)$

$$\Delta(\mu_k, T) = \sum_{\omega \in S_{2nm}} (-1)^{\rho\omega} \sum_{J \in \mathcal{J}_{2nm}} \bar{Q}_J(\Gamma_{\omega, k}, T) \exp(T(J, \Gamma_{\omega, k})), \quad (45)$$

where $\bar{Q}_J(\Gamma_{\omega, k}, T)$, $\omega \in S_{2nm}$, $J \in \mathcal{J}_{2nm}$, are some polynomials of variable T with complex coefficients, such that

$$\begin{aligned} \deg \bar{Q}_J(\Gamma_{\omega, k}, T) &\leq \max_{J \in \mathcal{J}_{2nm}} \{\deg Q_J(\Gamma_{\omega, k}, T)\} \\ &= \sum_{q=1}^{2nm} \deg Q_{R_q}(\Gamma_{i_q, k}, T) = \sum_{q=1}^{2nm} R_q = m(r_1 + \dots + r_{2n}). \end{aligned} \quad (46)$$

Estimates (46) we obtained by using (38) and (41). From (45) follows that $\Delta(\mu_k, T)$ is a quasi-polynomial of variable T .

For each $\mu_k \in M_p \setminus \{\vec{0}\}$ we consider the function $\Delta(\mu_k, \tau)$ defined of interval $(0, \infty)$ by formula (45), where T is replaced by τ . On basis of formula (45) and inequalities (46) $\Delta(\mu_k, \tau)$ can be expressed in the form

$$\Delta(\mu_k, \tau) = \sum_{J \in \mathcal{J}_{2nm}} F_J(\tau) \exp(\tau(J, \Gamma_k)), \quad (47)$$

where $F_j(\tau)$ is the polynomial with constant coefficients of degree N_j , $N_j \leq m(r_1 + \dots + r_{2n})$, and the number of terms with different exponents does not exceed $1 + 2^{nm+1}$. From the formula (47) follows that the function $\Delta(\mu_k, \tau)$ is analytic on interval $(0, \infty)$. We analytically continue it on \mathbb{R} and obtained function we denote by $D := D(\mu_k, \tau)$.

By $E(D, \varepsilon, [0, b])$ we denote a set of $\tau \in [0, b]$, $b \in \mathbb{R}_+$, for which the inequality $|D(\mu_k, \tau)| \leq \varepsilon$ holds. On basis of Theorem 2.1 from [9], given that $\text{Re}(J, \Gamma_k) = 0$ (it's follows from (14)), for each $\mu_k \in M_p \setminus \{\vec{0}\}$ following estimate holds

$$\text{mes}_{\mathbb{R}} E(D, \varepsilon, [0, b]) \leq C_9 B(\mu_k) \left(\frac{4\varepsilon}{G(\mu_k)} \right)^{\frac{1}{N-1}}, \quad C_9 = C_9(N, b), \quad (48)$$

where

$$N := \sum_{J \in \mathcal{J}_{2nm}} (1 + N_J) \leq (1 + 2^{nm+1}) (1 + m(r_1 + \dots + r_2)), \quad (49)$$

$$B(\mu_k) := 1 + \max_{J \in \mathcal{J}_{2nm}} |(J, \Gamma_k)|, \quad \mu_k \in M_p \setminus \{\vec{0}\}, \quad (50)$$

$$G(\mu_k) = \max_{1 \leq j \leq N} \left\{ |(d/d\tau)^{j-1} D(\mu_k, \tau)|_{\tau=0} (B(\mu_k))^{-j} \right\}, \quad \mu_k \in M_p \setminus \{\vec{0}\}. \quad (51)$$

Taking into account (15), (42) and (50) we obtain

$$B(\mu_k) \leq C_{10} (1 + |\mu_k|), \quad C_{10} = 2nmC_1.$$

Lemma 4. *There exists a number $\delta(\vec{\alpha}, \vec{\beta}) \in \mathbb{N}$, $\vec{\alpha} = (\alpha_1, \dots, \alpha_{2n})$, $\vec{\beta} = (\beta_1, \dots, \beta_{2n})$, such that*

$$\left. \frac{d^q}{d\tau^q} D(\mu_k, \tau) \right|_{\tau=0} = \begin{cases} 0, & q < \delta(\vec{\alpha}, \vec{\beta}), \\ \delta(\vec{\alpha}, \vec{\beta})! C_{11}(\vec{\alpha}, \vec{\beta}, \vec{r}) W(\mu_k), & q = \delta(\vec{\alpha}, \vec{\beta}), \end{cases} \quad (52)$$

where $\vec{r} = (r_1, \dots, r_{2n})$ and by $W(\mu_k) = \det \|\vec{h}_{jk} \gamma_{jk}^{l-1}\|_{l=1, \dots, 2n}^{j=1, \dots, 2nm}$ we denote the value of Wronskian of the system of functions (16) at point $t = 0$.

Proof. We denote $g_{lj}(\mu_k, \tau) := \alpha_l P_j^l + \beta_l I_l(\gamma_{jk})$, $l, j \in \{1, \dots, 2n\}$, where P_j^l , $I_l(\gamma_{jk})$ are defined by formulas (19), (20) respectively. We have following extensions:

$$\begin{aligned} \exp(\gamma_{jk}\tau) &= \sum_{q=0}^{2n-1} \frac{\gamma_{jk}^q}{q!} \tau^q + \tau^{2n} \nu_{jk}(\tau), \quad j \in \{1, \dots, 2nm\}, \\ I_l(\gamma_{jk}) &= \int_0^\tau t^{r_l} \exp(\gamma_{jk}t) dt = \sum_{q=0}^{2n-1} \frac{\gamma_{jk}^q}{q!(r_l + q + 1)} \tau^{r_l + q + 1} + \tau^{r_l + 2n + 1} \nu_{jl}(\tau), \end{aligned} \quad (53)$$

where $l \in \{1, \dots, 2n\}$, $j \in \{1, \dots, 2nm\}$;

$$P_j^l = \begin{cases} \gamma_{jk}^{2(l-1)}, & 1 \leq l \leq n, \\ \sum_{q=0}^{2n-1} \frac{\gamma_{jk}^{q+2(l-n-1)}}{q!} \tau^q + \tau^{2n} \nu_{jk}(\tau), & n+1 \leq l \leq 2n, \end{cases} \quad j \in \{1, \dots, 2nm\}, \quad (54)$$

where $v_{jk}(\tau)$, $V_{jlk}(\tau) = (r_l + 2n + 2)^{-1} \int_0^\tau v_{jk}(t)dt$ are some analytic in a neighborhood of the point $\tau = 0$ functions. We rewrite the extension (54) in the form

$$P_j^l = \begin{cases} \gamma_{jk}^{2(l-1)}, & 1 \leq l \leq n, \\ \sum_{q=2(l-n-1)}^{2n-1} \frac{\tau^{q-2(l-n-1)}}{(q-2(l-n-1))!} \gamma_{jk}^q + \tau^{4n-2l+2} \bar{v}_{jlk}(\tau), & n+1 \leq l \leq 2n, \end{cases} \quad (55)$$

where

$$\bar{v}_{jlk}(\tau) = \begin{cases} v_{jk}(\tau), & l = n+1, \\ \sum_{q=2n}^{2l-3} \frac{\gamma_{jk}^q \tau^{q-2n}}{(q-2(l-n-1))!} + \tau^{2(l-n-1)} v_{jk}(\tau), & n+2 \leq l \leq 2n, \end{cases} \quad j \in \{1, \dots, 2nm\}.$$

By substituting extensions (53) and (55) in the expression for $g_{lj}(\mu_k, \tau)$ for each $j \in \{1, \dots, 2nm\}$ we obtain following extensions:

$$\begin{aligned} g_{lj}(\mu_k, \tau) &= \alpha_l \gamma_{jk}^{2(l-1)} + \beta_l \sum_{q=0}^{2n-1} \frac{\gamma_{jk}^q}{q!(r_l + q + 1)} \tau^{r_l + q + 1} + \beta_l \tau^{r_l + 2n + 1} V_{jlk}(\tau), \quad 1 \leq l \leq n, \\ g_{lj}(\mu_k, \tau) &= \alpha_l \sum_{q=2(l-n-1)}^{2n-1} \frac{\tau^{q-2(l-n-1)}}{(q-2(l-n-1))!} \gamma_{jk}^q + \beta_l \sum_{q=0}^{2n-1} \frac{\gamma_{jk}^q}{q!(r_l + q + 1)} \tau^{r_l + q + 1} \\ &\quad + \alpha_l \tau^{4n-2l+2} \bar{v}_{jlk}(\tau) + \beta_l \tau^{r_l + 2n + 1} V_{jlk}(\tau), \quad n+1 \leq l \leq 2n. \end{aligned} \quad (56)$$

In formulas (56) we group terms on degrees of γ_{jk} . We obtain that

$$g_{lj}(\mu_k, \tau) = \sum_{q=0}^{2n-1} \gamma_{jk}^q \tilde{g}_{lq}(\alpha_l, \beta_l, \tau) + \tilde{V}_{jlk}(\tau), \quad l \in \{1, \dots, 2n\}, \quad j \in \{1, \dots, 2nm\}, \quad (57)$$

where

$$\tilde{g}_{lq}(\alpha_l, \beta_l, \tau) = \begin{cases} \beta_l \frac{\tau^{r_l + q + 1}}{q!(r_l + q + 1)}, & q \neq 2(l-1), \\ \alpha_l + \beta_l \frac{\tau^{r_l + q + 1}}{q!(r_l + q + 1)}, & q = 2(l-1), \end{cases} \quad (58)$$

if $l \in \{1, \dots, n\}$ and

$$\tilde{g}_{lq}(\alpha_l, \beta_l, \tau) = \begin{cases} \beta_l \frac{\tau^{r_l + q + 1}}{q!(r_l + q + 1)}, & q \leq 2(l-n) - 3, \\ \alpha_l \frac{\tau^{q-2(l-n-1)}}{(q-2(l-n-1))!} + \beta_l \frac{\tau^{r_l + q + 1}}{q!(r_l + q + 1)}, & 2(l-n-1) \leq q \leq 2n, \end{cases} \quad (59)$$

if $l \in \{n+1, \dots, 2n\}$,

$$\tilde{V}_{jlk}(\tau) = \begin{cases} \beta_l \tau^{r_l + 2n + 1} V_{jlk}(\tau), & 1 \leq l \leq n, \\ \tau^{4n-2l+2} (\alpha_l \bar{v}_{jlk}(\tau) + \beta_l \tau^{r_l - 2n + 2l - 1} V_{jlk}(\tau)), & n+1 \leq l \leq 2n. \end{cases} \quad (60)$$

Due to the definition of the function $D(\mu_k, \tau)$ it can be expressed by the formula

$$D(\mu_k, \tau) = \det \|\vec{h}_{jk} g_{lj}(\mu_k, \tau)\|_{l=1, \dots, 2n}^{j=1, \dots, 2nm}. \quad (61)$$

We substitute obtained extensions (57) in the formula (61) and by using elementary properties of determinants receive that

$$\begin{aligned} D(\mu_k, \tau) &= \det \left\| \vec{h}_{jk} \left(\sum_{q=0}^{2n-1} \gamma_{jk}^q \tilde{g}_{lq}(\alpha_l, \beta_l, \tau) + V_{lj}(\alpha_l, \beta_l, \mu_k, \tau) \right) \right\|_{l=1, \dots, 2n}^{j=1, \dots, 2nm} \\ &= \det \left\| \vec{h}_{jk} \sum_{q=0}^{2n-1} \gamma_{jk}^q \tilde{g}_{lq}(\alpha_l, \beta_l, \tau) \right\|_{l=1, \dots, 2n}^{j=1, \dots, 2nm} + \tilde{D}_k(\vec{\alpha}, \vec{\beta}, \tau), \end{aligned} \quad (62)$$

where by $\tilde{D}_k(\vec{\alpha}, \vec{\beta}, \tau) := \tilde{D}(\mu_k, \vec{\alpha}, \vec{\beta}, \tau)$ we denote some analytic at the point $\tau = 0$ function, which have at this point zero of higher order than

$$\det \left\| \vec{h}_{jk} \sum_{q=0}^{2n-1} \gamma_{jk}^q \tilde{g}_{lq}(\alpha_l, \beta_l, \tau) \right\|_{l=1, \dots, 2n}^{j=1, \dots, 2nm}.$$

It's follows from formulas (58)–(60).

Let us consider the matrix

$$\mathbf{F} = \left\| \vec{h}_{jk} \sum_{q=0}^{2n-1} \gamma_{jk}^q \tilde{g}_{lq}(\alpha_l, \beta_l, \tau) \right\|_{l=1, \dots, 2n}^{j=1, \dots, 2nm}$$

and split it into m blocks, each is of size $2n \times 2nm$

$$\mathbf{F} = \begin{pmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \mathbf{F}_m \end{pmatrix}, \quad \mathbf{F}_s = \left\| h_{jk}^s \sum_{q=0}^{2n-1} \gamma_{jk}^q \tilde{g}_{lq}(\alpha_l, \beta_l, \tau) \right\|_{l=1, \dots, 2n}^{j=1, \dots, 2nm},$$

where h_{jk}^s , $s \in \{1, \dots, m\}$, are components of vectors (17). It is easy to see that each of the blocks \mathbf{F}_s , $s \in \{1, \dots, m\}$, is a product of two matrices:

$$\mathbf{F}_s = \mathbf{G} \cdot \mathbf{W}_s, \quad \mathbf{G} = \left\| \tilde{g}_{l, q-1}(\alpha_l, \beta_l, \tau) \right\|_{l, q=1}^{2n}, \quad \mathbf{W}_s = \left\| h_{jk}^s \gamma_{jk}^{q-1} \right\|_{j=1, \dots, 2nm}^{q=1, \dots, 2n}.$$

Size of the matrix \mathbf{G} is $2n \times 2n$, and of the matrix \mathbf{W}_s is $2n \times 2nm$. Therefore

$$\mathbf{F} = \begin{pmatrix} \mathbf{G} \cdot \mathbf{W}_1 \\ \mathbf{G} \cdot \mathbf{W}_2 \\ \vdots \\ \mathbf{G} \cdot \mathbf{W}_m \end{pmatrix}.$$

Note that the determinant of the matrix $\text{col} \|\mathbf{W}_1, \mathbf{W}_2, \dots, \mathbf{W}_m\|$ is accurate to a sign equal to $W(\mu_k)$. We assume that $\det \mathbf{G} \neq 0$. Let us consider the block matrix of size $2nm \times 2nm$ of the form

$$\mathbf{G}_m = \begin{pmatrix} \mathbf{G}^{-1} & \mathbf{O}_{2n} & \cdots & \mathbf{O}_{2n} \\ \mathbf{O}_{2n} & \mathbf{G}^{-1} & \cdots & \mathbf{O}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_{2n} & \mathbf{O}_{2n} & \cdots & \mathbf{G}^{-1} \end{pmatrix},$$

where by \mathbf{O}_{2n} we denote zero matrix of size $2n \times 2n$, and \mathbf{G}^{-1} is an inverse matrix to \mathbf{G} . It's obviously that $\det \mathbf{G}_m = (\det \mathbf{G})^{-m}$. Then, according to the rule of multiplication of block matrices, we obtain that

$$\mathbf{G}_m \cdot \mathbf{F} = \left\| \begin{array}{cccc} \mathbf{G}^{-1} & \mathbf{O}_{2n} & \cdots & \mathbf{O}_{2n} \\ \mathbf{O}_{2n} & \mathbf{G}^{-1} & \cdots & \mathbf{O}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_{2n} & \mathbf{O}_{2n} & \cdots & \mathbf{G}^{-1} \end{array} \right\| \cdot \left\| \begin{array}{c} \mathbf{G} \cdot \mathbf{W}_1 \\ \mathbf{G} \cdot \mathbf{W}_2 \\ \vdots \\ \mathbf{G} \cdot \mathbf{W}_m \end{array} \right\| = \left\| \begin{array}{c} \mathbf{W}_1 \\ \mathbf{W}_2 \\ \vdots \\ \mathbf{W}_m \end{array} \right\|.$$

Wherefrom

$$\det(\mathbf{G}_m \cdot \mathbf{F}) = \det \mathbf{G}_m \cdot \det \mathbf{F} = \det \mathbf{F} (\det \mathbf{G})^{-m} = \pm W(\mu_k). \quad (63)$$

On the basis of the formula (63) we obtain equality

$$\det \mathbf{F} = \pm W(\mu_k) (\det \mathbf{G})^m = \pm W(\mu_k) (\det \|\tilde{g}_{l,q-1}(\alpha_l, \beta_l, \tau)\|_{l,q=1}^{2n})^m. \quad (64)$$

Taking into account the formula (64), the equality (62) can be written as

$$D(\mu_k, \tau) = \pm W(\mu_k) (\det \|\tilde{g}_{l,q-1}(\alpha_l, \beta_l, \tau)\|_{l,q=1}^{2n})^m + \tilde{D}_k(\vec{\alpha}, \vec{\beta}, \tau). \quad (65)$$

From (58), (59) follows, that $\det \|\tilde{g}_{l,q-1}(\alpha_l, \beta_l, \tau)\|_{l,q=1}^{2n}$ is a polynomial with respect to τ (and therefore is different from zero for all except a finite number of points τ) and don't depends on μ_k . From the resulting expansion (65) it follows that the smallest degree of τ , in the polynomial $(\det \|\tilde{g}_{l,q-1}(\alpha_l, \beta_l, \tau)\|_{l,q=1}^{2n})^m$ is equal to the number $\delta(\vec{\alpha}, \vec{\beta})$, and coefficient beside it is we denote as $C_{11}(\vec{\alpha}, \vec{\beta}, \vec{r})$. In other words, equalities (52) hold true. The lemma is proved. \square

For some values of parameters $\vec{\alpha}$ i $\vec{\beta}$ values $\delta(\vec{\alpha}, \vec{\beta})$ and $C_{11}(\vec{\alpha}, \vec{\beta}, \vec{r})$ can be easily calculated.

Example 1. Let in the conditions (2) $\alpha_l = 0, l \in \{1, \dots, 2n\}$. Then from formulas (58), (59) we obtain that

$$\begin{aligned} \tilde{g}_{l,q-1}(0, \beta_l, \tau) &= \beta_l \frac{\tau^{r_l+q}}{(q-1)!(r_l+q)}, \quad q \in \{1, \dots, 2n\}, \\ \det \|\tilde{g}_{l,q-1}(0, \beta_l, \tau)\|_{l,q=1}^{2n} &= \det \left\| \beta_l \frac{\tau^{r_l+q}}{(q-1)!(r_l+q)} \right\|_{l,q=1}^{2n} \\ &= \prod_{l=1}^{2n} \frac{\beta_l}{(l-1)!} \det \left\| \frac{1}{r_l+q} \right\|_{l,q=1}^{2n} \tau^{r+n(2n+1)}, \end{aligned} \quad (66)$$

where we denote $r = r_1 + \dots + r_{2n}$. According to [12, p.110] this equality is valid

$$\det \left\| \frac{1}{r_l+q} \right\|_{l,q=1}^{2n} = \prod_{2n \geq j > l \geq 1} (r_j - r_l)(j-l) \prod_{j,l=1}^{2n} (r_j + l)^{-1}. \quad (67)$$

On basis of (66), (67) we obtain that

$$\begin{aligned} \delta(\vec{0}, \vec{\beta}) &= m(r + n(2n+1)), \\ C_{11}(\vec{0}, \vec{\beta}, \vec{r}) &= \left(\prod_{j=1}^{2n} \frac{\beta_l}{(l-1)!} \prod_{2n \geq j > l \geq 1} (r_j - r_l)(j-l) \prod_{j,l=1}^{2n} (r_j + l)^{-1} \right)^m. \end{aligned}$$

Now we estimate the below value of $G(\mu_k)$, defined by the formula (51). Taking into account formulas (51)–(52) we obtain that

$$G(\mu_k) = \left| (\partial/\partial\tau)^{\delta(\vec{\alpha}, \vec{\beta})} D(\mu_k, \tau) \right|_{\tau=0} (B(\mu_k))^{-\delta(\vec{\alpha}, \vec{\beta})-1} \geq C_{12} |W(\mu_k)| (1 + |\mu_k|)^{-\delta(\vec{\alpha}, \vec{\beta})-1}, \quad (68)$$

where $C_{12} = \delta(\vec{\alpha}, \vec{\beta})! C_{11}(\vec{\alpha}, \vec{\beta}, \vec{r})(C_{10})^{-\delta(\vec{\alpha}, \vec{\beta})-1}$.

Theorem 3. *Let there exists a constant $\eta_0 \geq 0$ such that for all (except for finite number of) vectors $\mu_k \in M_p$ the inequality*

$$|W(\mu_k)| > C_{13}(1 + |\mu_k|)^{\eta_0} \quad (69)$$

holds. Then for almost all (with respect to Lebesgue measure on \mathbb{R}) numbers $T > 0$ the inequality (30) holds true for all (except for finite number of) vectors $\mu_k \in M_p$, if

$$\eta > \delta(\vec{\alpha}, \vec{\beta}) - \eta_0 + 1 + \left(1 + 2^{nm+1}\right) \left(\frac{p}{\theta_1} + 1\right) (1 + m(r_1 + \dots + r_{2n})).$$

Proof. Let $\varepsilon_k = (1 + |\mu_k|)^{-\eta}$, $k \in \mathbb{Z}^p \setminus \{\vec{0}\}$. Taking into account (48), (49), (51) and (68) for the measure of those $\tau \in [0, b]$ for which the inequality $|D(\mu_k, \tau)| \leq \varepsilon_k$ holds we obtain estimate

$$\begin{aligned} \text{mes}_{\mathbb{R}} E(D, \varepsilon_k, [0, b]) &\leq C_9 C_{10} (1 + |\mu_k|) \left(\frac{4(1 + |\mu_k|)^{-\eta}}{C_{12} C_{13} (1 + |\mu_k|)^{-\delta(\vec{\alpha}, \vec{\beta}) + \eta_0 - 1}} \right)^{1/\chi} \\ &= C_{14} (1 + |\mu_k|)^{-\frac{\eta - \delta(\vec{\alpha}, \vec{\beta}) + \eta_0 - 1}{\chi} + 1} \leq C_{14} d_1 |k|^{-\left(\frac{\eta - \delta(\vec{\alpha}, \vec{\beta}) + \eta_0 - 1}{\chi} - 1\right)\theta_1}, \end{aligned} \quad (70)$$

where $\chi = (1 + 2^{nm+1})(1 + m(r_1 + \dots + r_{2n}))$. Because of $\left(\frac{\eta - \delta(\vec{\alpha}, \vec{\beta}) + \eta_0 - 1}{\chi} - 1\right)\theta_1 > p$, the series $\sum_{k \in \mathbb{Z}^p \setminus \{\vec{0}\}} \text{mes}_{\mathbb{R}} E(D, \varepsilon_k, [0, b])$ is convergent. Then by Borel-Kantelli Lemma [14] the measure of those $\tau \in (0, b]$, which belongs to an infinite number of sets $E(D, \varepsilon_k, [0, b])$, is equal to zero. Thus, for almost all (with respect to Lebesgue measure on \mathbb{R}) numbers $\tau \in (0, b]$ the inequality $|D(\mu_k, \tau)| \geq \varepsilon_k$ holds for all (except a finite number) of vectors $\mu_k \in M_p$. Since from the inequality (70) follows, that the measures of sets $E(D, \varepsilon_k, [0, b])$ don't depend on b (this fact is the consequence of that the system (1) is of hyperbolic type), then, sending b to infinity, we obtain that for almost all (with respect to Lebesgue measure on \mathbb{R}) numbers $\tau \in (0, \infty)$ the inequality $|D(\mu_k, \tau)| \geq \varepsilon_k$ holds for all (excepting a finite number of) vectors $\mu_k \in M_p$. Since $\Delta(\mu_k, T) \equiv D(\mu_k, T)$ for all $T \in (0, \infty)$, then from the above follows the proof of the theorem. \square

Proposition 1. *If in the problem (1), (2) $p = 1$ then the inequality (69) holds true at $\eta_0 > 4n^2m(m-1) + nm(2n-1)$.*

Proof. Under the condition of the proposition roots of the equation (13) at $p = 1$ have the form $\gamma_{jk} = \gamma_j \mu_k$, $j \in \{1, \dots, 2nm\}$, where by γ_j we denote roots of the equation

$$\det \left\| \sum_{|\hat{s}|^* = 2n} i^{|\hat{s}|} \mathbf{A}_{\hat{s}} \gamma^{2s_0} \right\| = 0.$$

Vectors \vec{h}_{jk} at $p = 1$ have the form $\vec{h}_{jk} = \vec{h}_j \mu_k^{2n(m-1)}$, $j \in \{1, \dots, 2nm\}$, respectively, where by \vec{h}_j we denote some nonzero column of the matrix $L^*(\gamma_j^2, i)$, which is adjugate matrix of the matrix $L(\gamma_j^2, i)$, $j \in \{1, \dots, 2nm\}$. Hence

$$\begin{aligned} W(\mu_k) &= \det \|\vec{h}_{jk} \gamma_{jk}^{l-1}\|_{l=1, \dots, 2n}^{j=1, \dots, 2nm} = \det \|\vec{h}_j \gamma_j^{l-1} \mu_k^{2n(m-1)+l-1}\|_{l=1, \dots, 2n}^{j=1, \dots, 2nm} \\ &= \mu_k^{4n^2 m(m-1) + nm(2n-1)} \det \|\vec{h}_j \gamma_j^{l-1}\|_{l=1, \dots, 2n}^{j=1, \dots, 2nm}. \end{aligned}$$

From the above equality follows aforesaid statement. \square

Proposition 2. *If $m = 1$, i.e. the system (1) consists of a single equation, then the inequality (69) holds at $\eta_0 = 0$.*

Proof. Under the condition of the proposition we have that $W(\mu_k) = \prod_{1 \leq l < j \leq 2n} (\gamma_{jk} - \gamma_{lk})$, where by γ_{jk} , $j \in \{1, \dots, 2n\}$, we denote roots of the equation (13) at $m = 1$. Hence, at $m = 1$ the equation (1) is strictly hyperbolic, then from inequalities 2.21 in [14, p. 100], follows that $|\gamma_{jk} - \gamma_{lk}| \geq C_{15} > 0$, where $1 \leq l < j \leq 2n$. From these inequalities follows that $|W(\mu_k)| \geq (C_{15})^{n(2n+1)}$. \square

5 COROLLARY

In the present paper we investigated the correctness of the problem with integral conditions with respect to the time for hyperbolic in the narrow sense system of PDE's with constant coefficients in a class of almost periodic by spatial variables functions. We established the criterion of unique solvability of this problem and the sufficient conditions for the existence of its solutions. To solve the problem, small denominators (which are the quasi-polynomials with respect to the upper limit of integration) arising in the construction of solutions of the posed problem, we used the metric approach.

Our results can be extended to the Gårding hyperbolic systems of equations of the form

$$L\left(\frac{\partial^2}{\partial t^2}, \frac{\partial}{\partial x}\right)[\vec{u}] := \sum_{|\vec{s}| \leq 2n} \mathbf{A}_{\vec{s}} \frac{\partial^{2n} \vec{u}(t, x)}{\partial t^{2s_0} \partial x_1^{s_1} \dots \partial x_p^{s_p}} = \vec{0}, \quad (t, x) \in D^p.$$

REFERENCES

- [1] Avalishvili G., Avalishvili M., Gordeziani D. *On integral nonlocal boundary value problems for some partial differential equations*. Bull. Georg. Natl. Acad. Sci. 2011, **5** (1), 31–37.
- [2] Besicovitch A.S. *Almost periodic functions*. Dover, Cambridge, 1954.
- [3] Dell'acqua G., Santucci P. *Embedding theorems of Sobolev-Besicovitch spaces $W_{ap}^{k,1}(\mathbb{R}^s)$* . Rend. Mat. Appl. 1996, **16** (7), 525–536.
- [4] Faddeev D.K., Sominskii I.S. *Collection of Problems in Higher Algebra*. Vyshcha shkola, Kyiv, 1971. (in Ukrainian)
- [5] Fardigola L.V. *Influence of parameters on the properties of solutions of integral boundary value problems in a layer*. Izv. Vuzov. Matematika 1993, **7**, 51–58. (in Russian)
- [6] Il'kiv V.S. *A problem with integral conditions for system of partial differential equations with variable coefficients*. Bull. National Univ. "Lviv Polytechnic". Appl. Math. Series 1999, **364**, 318–323. (in Ukrainian)

- [7] Kuz' A.M., Ptashnyk B.I. *A problem with integral conditions with respect to time for Gårding hyperbolic equations*. Ukr. Math. J. 2013, **65**(2), 277–293. doi:10.1007/s11253-013-0777-7
- [8] Lancaster P. *Theory of matrices*. Academic Press, New-York, 1969.
- [9] Medvid O.M., Symotyuk M.M. *Diophantine approximation of characteristic determinant of an integral problem for linear partial differential equation*. Sci. Bull. Chernivtsi Univ. Ser. Matematika 2004, **228**, 74–85. (in Ukrainian)
- [10] Medvid O.M., Symotyuk M.M. *A problem with integral conditions for linear system of partial differential equations*. Math. Methods Phys. Mech. Fields 2007, **50** (1), 32–39. (in Ukrainian)
- [11] Medvid O.M., Symotyuk M.M. *A problem with integral conditions for system of partial differential equations with deviating argument*. Math. Bull. Shevchenko Sci. Soc. 2007, **4**, 414–427. (in Ukrainian)
- [12] Pólya G., Szegő G. *Problems and Theorems in Analysis II*. Springer, Berlin, 1998.
- [13] Pul'kina L.S. *A Nonlocal Problem with Integral Conditions for a Hyperbolic Equation*. Differ. Equ. 2004, **40** (7), 947–953. doi:10.1023/B:DIEQ.0000047025.64101.16
- [14] Ptashnyk B.Yo. *Ill-posed Boundary Value Problems for Partial Differential Equations*. Naukova dumka, Kyiv, 1984. (in Russian)
- [15] Shubin M.A. *Almost periodic functions and partial differential operators*. Russ. Math. Surv. 1978, **33** (2), 1–47. doi:10.1070/RM1978v033n02ABEH002303
- [16] Tamarkin J.D. *On some general problems in the theory of ordinary linear differential equations and on the expansion in series of arbitrary functions*. Petrograd, 1917. (in Russian)

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Кузь А.М., Пташник Б.І. *Задача з інтегральними умовами за часовою змінною для системи гіперболічних рівнянь зі сталими коефіцієнтами* // Карпатські матем. публ. — 2014. — Т.6, №2. — С. 282–299.

В області, що є декартовим добутком відрізка $[0, T]$ і простору \mathbb{R}^p , досліджено задачу з інтегральними умовами за часовою координатою для системи гіперболічних рівнянь зі сталими коефіцієнтами у класі майже періодичних за просторовими змінними функцій. Знайдено критерій єдиності та достатні умови існування розв'язку задачі. Для розв'язання проблеми малих знаменників, які виникли при побудові розв'язку задачі, використано метричний підхід

Ключові слова і фрази: інтегральні умови, малі знаменники, міра Лебега, майже періодичні функції, гіперболічна система.

Кузь А.М., Пташник Б.И. *Задача с интегральными условиями по времени для системы гиперболических уравнений с постоянными коэффициентами* // Карпатские матем. публ. — 2014. — Т.6, №2. — С. 282–299.

В области, являющейся декартовым произведением отрезка $[0, T]$ и пространства \mathbb{R}^p , исследована задача с интегральными условиями по временной координате для системы гиперболических уравнений с постоянными коэффициентами в классе почти периодических по пространственным переменным функций. Найдены критерий единственности и достаточные условия существования решения задачи. Для решения проблемы малых знаменателей, которые возникли при построении решения задачи, использовано метрический подход.

Ключевые слова и фразы: интегральные условия, малые знаменатели, мера Лебега, почти периодические функции, гиперболическая система.