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CONTINUOUS APPROXIMATIONS OF CAPACITIES ON METRIC COMPACTA

A method of "almost optimal" continuous approximation of capacities on a metric compactum with possibility measures, necessity measures, or with capacities on a closed subspace, is presented.

Key words and phrases: capacity, metric compactum, approximation.

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INTRODUCTION

Capacities were introduced by Choquet [1] and found numerous applications in different theories. Spaces of upper semicontinuous capacities on compacta were systematically studied in [2]. In particular, in the latter paper functoriality of the construction of a space of capacities was proved and Prokhorov-style and Kantorovich-Rubinstein-style metrics on the set of capacities on a metric compactum were introduced. Needs of practice require that a capacity can be approximated with capacities of simpler structure or with some convenient properties. It was shown in[3] that each normalized capacity on a compactum is the value of a so-called $\cup$-capacity (or possibility measure) on the space of $\cap$-capacities (necessity measures) under the multiplication mapping of the capacity monad. Nevertheless it is impossible to represent every capacity in this manner using only capacities of one of the two mentioned classes. We can discuss only approximation of an arbitrary capacity with $\cup$- or $\cap$-capacities. A construction of the capacity from the class of $\cup$- or $\cap$-capacities that is the closest to the given one w.r.t. the Prokhorov metric was described in [4]. A method of optimal approximation of a capacity with a capacity on a closed subspace was also presented there. Although the proposed approximations are optimal (belong to the optimal ones), they does not depend continuously on the original capacity. In this paper we consider the problem of *continuous* approximation. It is proved that the space $\underline{M}X$ of subnormalized capacities on a metric compactum X is an I-convex compactum, hence all elements of $\underline{M}X$ can be approximated with "almost optimal" precision with elements of an arbitrary closed I-convex subset $X_0 \subset \underline{M}X$, in particular, with $\cup$-capacities, $\cap$-capacities, or capacities on a fixed closed subspace $X_0 \subset X$, so that the approximation is continuous w.r.t. the original capacity and the chosen "tolerance".

1 BASIC FACTS AND DEFINITIONS

We follow the terminology and notation of [2] and denote by $\exp X$ the set of all non-empty closed subsets of a compactum X. The set $\exp X$ is considered with the Vietoris topology. If

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a metric d on X is admissible, then the Hausdorff metric $\hat{d}$ is admissible on $\exp X$. For a point x in (X, d) and a non-empty subset $S \subset X$ we denote $d(x, S) = \inf\{d(x, x') \mid x' \in S\}$, and I is the unit segment $[0, 1]$.

We call a function $c : \exp X \cup \{\varnothing\} \to I$ a *capacity* on a compactum X if the three following properties hold for all subsets $F, G \underset{\text{cl}}{\subset} X$:

1. $c(\varnothing) = 0$;
2. if $F \subset G$, then $c(F) \leq c(G)$ (monotonicity);
3. if $c(F) < a$, then there is an open subset $U \supset F$ such that for all $G \subset U$ the inequality $c(G) < a$ is valid (upper semicontinuity).

If, additionally, $c(X) = 1$ (or $c(X) \leq 1$) holds, then the capacity is called *normalized* (resp. *subnormalized*).

We denote by MX and $\underline{M}X$ the sets of all normalized and of all subnormalized capacities respectively. It was shown in [2] that MX carries a compact Hausdorff topology with the subbase of all sets of the form

$$O_-(F, a) = \{c \in MX \mid c(F) < a\}, \text{ where } F \underset{\text{cl}}{\subset} X, a \in I,$$

and

$$\begin{aligned} O_+(U, a) &= \{c \in MX \mid c(U) > a\} \\ &= \{c \in MX \mid \text{there is a compactum } F \subset U, c(F) > a\}, \text{where } U \underset{\text{op}}{\subset} X, a \in I. \end{aligned}$$

The same formulae determine a subbase of a compact Hausdorff topology on $\underline{M}X$ and therefore $MX \subset \underline{M}X$ is a subspace.

We consider the following subclasses of MX.

1. $M_\cap X$ is the set of the so-called $\cap$-*capacities* (or necessity measures) with the property: $c(A \cap B) = \min\{c(A), c(B)\}$ for all $A, B \underset{\text{cl}}{\subset} X$.
2. $M_\cup X$ is the set of the so-called $\cup$-*capacities* (or possibility measures) with the property: $c(A \cup B) = \max\{c(A), c(B)\}$ for all $A, B \underset{\text{cl}}{\subset} X$.
3. Class MX_0 of capacities defined *on a closed subspace* $X_0 \subset X$. We regard each capacity c_0 on X_0 as a capacity on X extended with the formula $c(F) = c_0(F \cap X_0)$, $F \underset{\text{cl}}{\subset} X$.

Analogous subclasses are defined in $\underline{M}X$, with the obvious denotations. It was proved in [3] that the subsets $M_\cap X$, $M_\cup X$, and MX_0 are closed in MX, hence for a compactum X they are compacta as well.

From now on we restrict to $\underline{M}X$, results for MX are quite analogous. We consider the metric on the set $\underline{M}X$ of subnormalized capacities on a metric compactum (X, d) :

$$\hat{d}(c, c') = \inf\{\varepsilon > 0 \mid c(\bar{O}_\varepsilon(F)) + \varepsilon \geq c'(F), c'(\bar{O}_\varepsilon(F)) + \varepsilon \geq c(F), \forall F \underset{\text{cl}}{\subset} X\}.$$

Here $\bar{O}_\varepsilon(F)$ is the closed ε-neighborhood of a subset $F \subset X$. This metric is admissible [2]. Recall some definitions and well-known facts on compact topological semilattices and compact idempotent semimodules.

A poset $(X, \leq)$ is called an *upper semilattice* is pairwise suprema $x \vee y$ exist for all $x, y \in X$. A subset Y of an upper semilattice Y is called an *upper subsemilattice* if the supremum of each two elements of Y is in Y. Then Y is an upper semilattice as well, and suprema of all finite non-empty subsets of Y in X and in Y exist and are equal.

An upper semilattice $(X, \leq)$ is called *topological* if a topology is fixed on X such that the pairwise supremum $x \vee y$ depends on $x, y \in X$ continuously.

A topological semilattice is called *Lawson* [7] if in each its point it possesses a local base consisting of subsemilattices.

An upper semilattice is *complete* if each it non-empty subset has the least upper bound. It is well-known that any compact topological upper semilattice is complete and contains agreatest element [6]. A compact Hausdorff topological upper semilattice X is Lawson if and only if the mapping $\sup : \exp X \to X$ that assigns the least upper bound to each non-empty closed subset $A \subset X$ is continuous w.r.t. the Vietoris topology.

We call $(X, \oplus, \circledast)$ a (left idempotent) $(I, \max, *)$-*semimodule* if X is a set with operations $\oplus : X \times X \to X$, $\circledast : I \times X \to X$ such that for all $x, y, z \in X$, $\alpha, \beta \in I$ the following holds:

1. $x \oplus y = y \oplus x$;
2. $(x \oplus y) \oplus z = x \oplus (y \oplus z)$;
3. there is a unique $\bar{0} \in X$ such that $x \oplus \bar{0} = x$ for all x;
4. $\alpha \circledast (x \oplus y) = (\alpha \circledast x) \oplus (\alpha \circledast y)$, $\max\{\alpha, \beta\} \circledast x = (\alpha \circledast x) \oplus (\beta \circledast x)$;
5. $(\alpha * \beta) \circledast x = \alpha \circledast (\beta \circledast x)$;
6. $1 \circledast x = x$;
7. $0 \circledast x = \bar{0}$.

In the sequel we use a shorter term "I-semimodule" for $(I, \max, *)$-semimodule.

A triple $(X, \oplus, \circledast)$ is called *a compact Hausdorff Lawson I-semimodule* if $(X, \oplus, \circledast)$ is an I-semimodule and a compact Hausdorff topology is fixed on X that makes it a compact Lawson upper semilattice with $\oplus$ being pairwise supremum (hence the partial order is defined as $x \leq y \Leftrightarrow x \oplus y = y$), and the multiplication $\circledast$ is continuous.

For all points $x_1, x_2, \ldots, x_n \in X$ and coefficients $\alpha_1, \alpha_2, \ldots, \alpha_n \in I$ such that $\max\{\alpha_1, \alpha_2, \ldots, \alpha_n\} = 1$ we define *the I-convex combination* of a finite number of elements $\alpha_1 \circledast x_1 \oplus \alpha_2 \circledast x_2 \oplus \ldots \oplus \alpha_n \circledast x_n$, which from now on is denoted simply as $\alpha_1 x_1 \oplus \alpha_2 x_2 \oplus \ldots \oplus \alpha_n x_n$. It can be calculated stepwise using pairwise convex combinations of the form $x \oplus \alpha y$, which in fact are values of a mapping $X \times I \times X \to X$.

If the mentioned pairwise I-convex combination is continuous, then $(X, \oplus, \circledast)$ is called an *I-convex compactum* [5]. Hence an I-convex compactum is a compact Hausdorff space X with a Lawson continuous pairwise I-convex combination $(x, \alpha, y) \mapsto x \oplus \alpha y$, $X \times I \times X \to X$, which (for $\alpha = 1$) makes X a compact Hausdorff Lawson upper semilattice.

In compact Hausdorff Lawson I-semimodules we can define an I-convex combination of an infinite number of elements using finite combinations as follows:

$$\bigoplus_{i \in \mathcal{I}} \alpha_i x_i = \inf\{\sup_{i \in \mathcal{I}_1} \alpha_i \circledast \sup_{i \in \mathcal{I}_1} x_i \oplus \ldots \oplus \sup_{i \in \mathcal{I}_n} \alpha_i \circledast \sup_{i \in \mathcal{I}_n} x_i \mid n \in \mathbb{N}, \mathcal{I} = \mathcal{I}_1 \cup \mathcal{I}_2 \cup \ldots \cup \mathcal{I}_n\}.$$

Observe that the above I-convex combination does not depend on $\alpha_i x_i$ such that the respective α_i are equal to zero. Theorem [5, 5.9.2] implies an important property of the mapping that sends each collection of elements with coefficients to their I-convex combination.

Lemma 1. *Let $(X, \oplus, \circledast)$ be an I-convex compactum and $\exp_1(X \times I) \subset \exp(X \times I)$ the subspace of all closed subsets of $X \times I$ that contain at least one pair of the form $(x, 1)$. Then the mapping $h : \exp_1(X \times I) \to X$ defined for $\mathcal{A} \underset{\text{cl}}{\subset} X \times I$ by the formula*

$$h(\mathcal{A}) = \bigoplus_{i \in \mathcal{I}} \{\alpha_i x_i | (x_i, \alpha_i) \in \mathcal{A}\}$$

is continuous.

2 SOME MAPPINGS IN METRIC I-CONVEX COMPACTA

We need some auxilliary statements. Let $S \subset X$ be a non-empty closed I-convex subset of a metric I-convex compactum $(X, \oplus, \circledast)$, i.e. S contains all I-convex combinations of its elements. Then S is known [5] to be an I-convex compactum as well. For the product topology on $X \times \mathbb{R}$ the metric $\rho((x_1, a_1), (x_2, a_2)) = \max\{d(x_1, x_2), |a_1 - a_2|\}$ is admissible.

For an element $x \in X$ consider the set $\mathfrak{F}_x = \{(x', a) | x \in S, d(x, x') \leq a \leq \operatorname{diam} X\}$.

Proposition 1. *The set $\mathfrak{F}_x \subset S \times [0, \operatorname{diam} X]$ is closed and the mapping $f : X \to \exp(S \times [0, \operatorname{diam} X])$ that assigns $\mathfrak{F}_x$ to each $x \in X$ is continuous.*

The proof relies on the two following lemmas.

Lemma 2. *Let (X, d) be a metric compactum, then for all $x \in X$ the set $\mathcal{F}_x = \{(x', a) | x' \in X, d(x, x') \leq a \leq \operatorname{diam} X\}$ is non-empty and closed in $X \times [0, \operatorname{diam} X]$.*

Proof. Obviously $(x', \operatorname{diam} X) \in \mathfrak{F}_x$ for all $x' \in X$, hence the set in question is non-empty. We show that the complement $X \times [0, \operatorname{diam} X] \setminus \mathcal{F}_x$ is open. Let a point (x', a) belong to the complement, i.e. $d(x, x') > a$. Put $\varepsilon = \dfrac{d(x, x') - a}{2}$. Then $\varepsilon > 0$ and for any point (y, b) in the ε-neighborhood of (x', a), which is a ball $B_\varepsilon(x') \times (a - \varepsilon, a + \varepsilon)$, the inequalities $d(y, x) \geq d(x', x) - d(x', y) > (a + 2\varepsilon) - \varepsilon = a + \varepsilon > b$ are valid. Hence the ε-neighborhood of the point (x', a) is contained in the set $X \times [0, \operatorname{diam} X] \setminus \mathcal{F}_x$. □

Therefore the set $\mathfrak{F}_x = \mathcal{F}_x \cap (S \times I)$ is non-empty and closed in $S \times [0, \operatorname{diam} X]$ as well.

Lemma 3. *Let (X, d) be a metric compactum and S its non-empty closed subset, then the mapping f from X to the space $\exp(S \times [0, \operatorname{diam} X])$ of all non-empty closed subsets with the Hausdorff metric that sends each $x \in X$ to the set $\mathfrak{F}_x$, is non-expanding.*

Proof. Let $x, y \in X$, $x \neq y$, hence $r = d(x, y) > 0$. If $(x', a) \in \mathfrak{F}_x$, i.e. $d(x, x') \leq a$, put $b = \min\{a + r, \operatorname{diam} X\}$. Thus $|b - a| = \rho((x', a), (x', b)) \leq r$ and $d(y, x') \leq d(x, y) + d(x, x') = d(x, x') + r$. Taking into account $d(y, x') \leq \operatorname{diam} X$ we deduce $d(y, x') \leq b$, hence $(x', b) \in \mathfrak{F}_y$.

Thus for each point $(x', a) \in \mathfrak{F}_x$ there is a point $(x', b) \in \mathfrak{F}_y$ at a distance $\leq r$, and vice versa. Thus the Hausdorff distance ρ_H between $\mathfrak{F}_x$ and $\mathfrak{F}_y$ does not exceed $r = d(x, y)$, i.e. f is non-expanding. This completes the proof. □

Assign to all $x \in X$ and $\varepsilon > 0$ the set $\mathfrak{G}_x \subset S \times I$ of the form

$$\mathfrak{G}_x = \left\{ (x', \alpha) | x' \in S,\ \alpha \in I,\ \alpha \le \max \left\{ 0, 1 - \frac{d(x,x') - d(x,S)}{\varepsilon} \right\} \right\}.$$

Observe that a point (x', α), with $\alpha > 0$, can belong to $\mathfrak{G}_x$ only if $x' \in S$, $d(x,x') < d(x,S) + \varepsilon$.

Proposition 2. *The following statements hold:*

(1) the set $\mathfrak{G}_x$ is closed in $S \times I$;

(2) the mapping $g : X \times (0, +\infty) \to \exp(S \times I)$ that assigns $\mathfrak{G}_x$ to each element $x \in X$ and $\varepsilon > 0$ is continuous;

(3) for all $x \in X$, $\varepsilon > 0$ the equality $\max\{\alpha \in I \mid (x', \alpha) \in \mathfrak{G}_x$ for some $x' \in S\} = 1$ is valid.

Proof. The set $\mathfrak{G}_x \subset S \times I$ is the image of the set $\mathfrak{F}_x \subset S \times [0, \operatorname{diam} X]$, namely $\mathfrak{G}_x = (1_X \times \theta_{x,\varepsilon})(\mathfrak{F}_x)$, where $\theta_{x,\varepsilon} : [0, \operatorname{diam} X] \to I$ is defined by the formula $\theta_{x,\varepsilon}(a) = \max\{1 - \dfrac{a - d(x,S)}{\varepsilon}, 0\}$. Hence $\mathfrak{G}_x$ is closed as the image of a closed set under a continuous mapping of compacta (1). Moreover $\mathfrak{F}_x$ and $\theta_{x,\varepsilon}$ depend on x and ε continuously, therefore the same holds for $\mathfrak{G}_x$ (2). Compactness of $S \subset X$ implies existence of $x' \in S$ such that $d(x,x') = d(x,S)$, hence $(x', 1) \in \mathfrak{G}_x$ (3). ☐

Proposition 3. *The mapping $\Phi : X \times (0, +\infty) \to S$ defined as*

$$\Phi(x, \varepsilon) = \bigoplus_{i \in \mathcal{I}} \{\alpha_i x_i | (x_i, \alpha_i) \in \mathfrak{G}_x\}$$

is continuous.

Proof. Continuity of Φ is a corollary of Proposition 2 and Lemma 1 because Φ is the composition of the continuous mappings g and h (cf. Lemma 1). ☐

3 CONSTRUCTION OF ALMOST OPTIMAL APPROXIMATIONS OF CAPACITIES

Consider the space $\underline{M}X$ of subnormalized capacities. For reader's convenience we present and prove properties of $\underline{M}X$ [5] in the following statement.

Proposition 4. *The triple $(\underline{M}X, \vee, \wedge)$ is a $(I, \max, \min)$-convex compactum, if the operations $\vee : \underline{M}X \times \underline{M}X \to \underline{M}X$ and $\wedge : I \times \underline{M}X \to \underline{M}X$ are defined by the formulae:*

$$c_1 \vee c_2(F) = \max\{c_1(F), c_2(F)\},\ \alpha \wedge c(F) = \min\{\alpha, c(F)\}$$

for $c_1, c_2 \in \underline{M}X$, $\alpha \in I$, $F \underset{\mathrm{cl}}{\subset} X$.

Proof. It is almost obvious that the defined above functions $c_1 \vee c_2 : \exp X \to I$, $\alpha \wedge c : \exp X \cup \{\varnothing\} \to I$ are capacities on X. Put $\oplus = \vee$, $\circledast = \wedge$ and set the zero element $\bar{0} \in \underline{M}X$ to the "zero capacity" with the values $\bar{0}(F) = 0$ for all $F \underset{\mathrm{cl}}{\subset} X$. It is easy to observe that axioms (1)—(7) from the definition of semimodule hold. Thus $(\underline{M}X, \vee, \wedge)$ is a (left idempotent) $(I, \max, \min)$-semimodule. Recall (see [2]) that the subbase of all sets of the form $O_-(F, a)$ and $O_+(U, a)$, for $A \underset{\mathrm{cl}}{\subset} X$, $U \underset{\mathrm{op}}{\subset} X$, $a \in I$, determines a compact Hausdorff topology τ on $\underline{M}X$.

It a partial order at $\underline{M}X$ is defined as

$$c_1 \leq c_2 \Leftrightarrow c_1 \vee c_2 = c_2 \Leftrightarrow c_1(F) \leq c_2(F), \text{ for all } F \underset{\text{cl}}{\subset} X,$$

then the pairwise suprema are calculated argumentwise: $c_1 \vee c_2(F) = \max\{c_1(F), c_2(F)\}$, and $\underline{M}X$ is an upper semilattice with the least element $\bar{0}$. It was proved in [5] that $(\underline{M}X, \leq)$ is a topological (i.e. the pairwise supremum $c_1 \vee c_2$ depends on c_1 and c_2 continuously w.r.t. the topology τ) upper Lawson semilattice (because subbase elements $O_-(F, a)$ and $O_+(U, a)$ are subsemilattices), and τ is the Lawson topology.

The function $c_1 \vee \alpha c_2 : \exp X \cup \{\varnothing\} \to I$ defined by the formula

$$c_1 \vee \alpha c_2(F) = c_1 \vee (\alpha \wedge c_2)(F) = \max\{c_1(F), \min\{\alpha, c_2(F)\}\}$$

is a subnormalized capacity on X, and the mapping $\underline{M}X \times I \times \underline{M}X \to \underline{M}X$ that assigns $c_1 \vee \alpha c_2$ to (c_1, α, c_2) is continuous. Hence $\underline{M}X$ is a compact Hausdorff space with a Lawson continuous pairwise I-convex combination which makes it a compact Hausdorff Lawson upper semilattice, i.e. $(\underline{M}X, \vee, \wedge)$ is an I-convex compactum. □

If a compact topology on X is determined with an admissible metric d, then $(\underline{M}X, \hat{d})$ is a metric compactum and the defined above metric $\hat{d}$ on $\underline{M}X$ is admissible, i.e. $(\underline{M}X, \vee, \wedge)$ is a metric I-convex compactum. The following property of $\hat{d}$ is crucial.

Lemma 4. *Let (X, d) be a metric compactum, $c_0, c_i \in \underline{M}X$ for $i \in \mathcal{I}$ are capacities such that $\hat{d}(c_0, c_i) \leq \varepsilon$ for some $\varepsilon \geq 0$ and all i. Then for arbitrary coeficients $\alpha_i \in I$ such that $\sup_{i \in \mathcal{I}} \alpha_i = 1$ the inequality $\hat{d}(c_0, \bigvee_{i \in \mathcal{I}} \alpha_i c_i) \leq \varepsilon$ is valid.*

For a finite number of c_i the inequality is straightforward, and by continuity we extend it to infinite combinations.

Remark. *Since $MX \subset \underline{M}X$ is a closed subsemimodule,* everything *said above on $\underline{M}X$ applies also to MX.*

Therefore the above statements can be used to approximate a capacity $c \in \underline{M}X$ (or $c \in MX$) with capacities from a closed I-convex subspace $S \subset \underline{M}X$ (resp. $S \subset MX$). The convexity means that S contains all I-convex combinations of the form $\bigvee_{i \in \mathcal{I}} (\alpha_i \wedge c_i)$, where $c_i \in S$, $\alpha_i \in I$, $\max\{\alpha_i | i \in \mathcal{I}\} = 1$. For simplicity consider a more general case of $\underline{M}X$.

For a capacity $c \in \underline{M}X$ and a number $\varepsilon > 0$ construct the set

$$\mathfrak{G}_c = \left\{ (c', \alpha) | c' \in S, \alpha \in I, \ \alpha \leq \max\left\{ 0, 1 - \frac{\hat{d}(c, c') - \hat{d}(c, S)}{\varepsilon} \right\} \right\},$$

which is closed in $S \times I$ due to Proposition 2.

Define a capacity $\tilde{c}_\varepsilon$ with the formula $\tilde{c}_\varepsilon = \bigvee_{i \in \mathcal{I}} \{\alpha_i \wedge c_i | (c_i, \alpha_i) \in \mathfrak{G}_c\}$. Equivalently $\tilde{c}_\varepsilon$ can be defined as

$$\tilde{c}_\varepsilon(F) = \sup\left\{ (1 - \frac{\hat{d}(c, c') - \hat{d}(c, S)}{\varepsilon}) \wedge c'(F) | c' \in S, \hat{d}(c, c') \leq \hat{d}(c, S) + \varepsilon \right\} \tag{1}$$

for all $F \underset{\text{cl}}{\subset} X$. Although $\tilde{c}_\varepsilon$ is not the closest to $c \in \underline{M}X$ in the subspace S, it is "almost the closest" in the sense of the following theorem.

Theorem 1. *For a capacity $c \in \underline{M}X$, a number $\varepsilon > 0$ and a closed I-convex subspace $S \subset \underline{M}X$ the capacity $\tilde{c}_\varepsilon$ belongs to S and satisfies the inequality $\hat{d}(c, \tilde{c}_\varepsilon) \leq \hat{d}(c, S) + \varepsilon$. The mapping $\Phi : \underline{M}X \times (0, \operatorname{diam} \underline{M}X] \to S$ defined as $\Phi(c, \varepsilon) = \tilde{c}_\varepsilon$ is continuous.*

Proof. Continuity of Φ and $\tilde{c}_\varepsilon \in S$ follow from Proposition 3. By the equality (1) the capacity $\tilde{c}_\varepsilon$ is an I-convex combination of capacities $c' \in S$ such that $\hat{d}(c, c') \leq \hat{d}(c, S) + \varepsilon$, hence by Lemma 4 the inequality $\hat{d}(c, \tilde{c}_\varepsilon) \leq \hat{d}(c, S) + \varepsilon$ is valid as well. □

Remark. *Obviously an analogous theorem is valid for MX.*

It is easy to verify that the subspaces $M_\cap X$ and MX_0 are closed and I-convex subsets of the semimodule $(MX, \vee, \wedge)$ ($M_\cup X$ is I-convex if the I-convex combination on $(MX, \vee, \wedge)$ is defined in a dual manner, cf. [5]). Methods of calculating of the distances $\hat{d}(c, M_\cap X)$, $\hat{d}(c, M_\cup X)$, $\hat{d}(c, MX_0)$ were presented in [4]. Thus we can use the latter theorem to construct approximations of an arbitrary subnormalized capacity c on X with $\cup$-capacities, $\cap$-capacities or capacities on $X_0 \subset X$ that are ε-closed to optimal and depend on c, ε continuously.

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Представлено метод "майже оптимального" неперервного наближення ємностей на метричному компакті мірами можливості, мірами необхідності чи ємностями на замкненому підпросторі.

Ключові слова і фрази: ємність, метричний компакт, наближення.