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# MEASUREMENT OF PHASE SHIFT BETWEEN HARMONIC SIGNALS USING BINARY SAMPLING 

Ihor Buchma<br>Lviv Polytechnic National University, 12, Bandera St., Lviv, 79013, Ukraine

The article analyzes the measurement error of the phase shift between the harmonic signals using binary sampling. We consider four methods of determining the phase shift. Mathematical models of errors caused by inequality signal amplitudes have been designed. Their graphical dependence has been presented. The method that provides the smallest additive error has been determined. The structural circuit of the device that implements the mentioned method has been shown.

Key words: measuring phase shift, errors, harmonic signals, low frequency, binary sampling, inequality of the amplitudes, threshold sensitivity.

INTRODUCTION. The cyber-physical systems become more common [1]. An important role belongs to eddy current measurement and computational tools of the first level $[2,10,11]$. These tools of cyber-physical system are based on measuring the level of the quadrature component of the secondary magnetic field consolidated to the primary magnetic field or small phase shift between the primary magnetic field intensity and the total intensity (primary and secondary) harmonic low frequency magnetic fields [2]. When measuring small phase shifts in low frequency cyber-physical system, the preference is given to algorithmic sum-difference methods based on the use of digital measurement and computational tools and the ability to provide a low threshold [10].

ANALYSIS OF THE PROBLEM. Measuring a small phase shift between the harmonic signals in the range of low frequencies and infra-low frequencies requires using methods to reduce the influence of additive flicker noise. One of the most effective methods for reducing the influence of low-frequency flicker noise is the use of the periodic comparison method, in which the angular frequency comparison $\Omega$ of signal is much higher than the comparable circular frequency of signals $\omega$ [3-5]. This method of comparison, particularly when measuring the phase shift is called a binary sampling [6].

Here we would like to note that the mentioned method would reduce the flicker noise effect only if the flicker noise, acting on comparable signals, are correlated. The correlation of the flicker noise occurs, particularly in cases of including sensors in four arms measuring bridge or semi measuring bridge [7].

So, sum-difference methods for measuring the phase shift with binary sample can be assigned to the most sensitive ones, because under certain conditions they allow to reduce the influence of the flicker noise which other sum-difference methods do not provide. Therefore, in range of infra-low frequencies where the flicker noise
has a significant impact on measurement accuracy, the use of binary sampling can reduce the threshold.

FORMULATION OF THE PROBLEM. However, in scientific works there is no theoretical analysis of errors related with the use of binary sampling. In particular, the method of binary sampling, as other sum-difference methods, requires the equality of amplitudes of compared signals [6], which is hard to provide.

Therefore, the goal of this article is the theoretical analysis of the measurement errors of the phase shift caused by the inequality of comparable signals amplitude when using binary sampling.

PRESENTING MAIN MATERIAL. Deducing of basic dependencies to determine the phase shift

Let us consider the harmonic signals with unequal amplitudes and initial phases

$$
\mathrm{u}_{1}=\mathrm{U}_{1} \sin \omega \mathrm{t}
$$

and

$$
\mathrm{u}_{2}=\mathrm{U}_{2} \sin \left(\omega \mathrm{t}+\varphi_{x}\right)
$$

where $U_{1}$ and $U_{2}$ - are respectively the amplitudes of the first and the second signals;
$\omega$ - is the circular frequency of signals; $\varphi_{x}$ - is the phase shift between the signals.
After binary sampling of these signals with the circular frequency $\Omega>\omega$ by using the automatic switch with two inputs and one output, controlled by voltage

$$
\mathrm{u}_{\mathrm{K}}(\mathrm{t})=\mathrm{U}_{\mathrm{K}} \operatorname{sign} \sin (\Omega \mathrm{t}+\varphi)
$$

where $\varphi$ - is the initial phase of a switching signal.
On the output of the switch we get the amplitude-modulated signal

$$
\begin{align*}
\mathrm{u}_{\mathrm{k}}(\mathrm{t})= & \frac{\mathrm{U}_{2}+\mathrm{U}_{1}}{2} \cdot\left[1+\frac{\mathrm{U}_{2}-\mathrm{U}_{1}}{\mathrm{U}_{2}+\mathrm{U}_{1}} \cdot \operatorname{sign} \sin (\Omega \mathrm{t}+\varphi)\right] \times \\
& \times \sin \left\{\omega \mathrm{t}+\frac{\varphi_{\mathrm{x}}}{2}[1+\operatorname{sign} \sin (\Omega \mathrm{t}+\varphi \rrbracket\}\right. \tag{1}
\end{align*}
$$

shown in Fig. 1.


Fig.1. Binary-sampled signal at the output of the switch
It is accepted that the switch is ideal, i.e. its transmission coefficient is equal to one. This means that the resistance of the opened and closed switch inputs is respectively equal to zero and infinity.

Based on the Fig. 1, switch output signal can be presented by another expression

$$
\begin{aligned}
u_{V_{K}}(t) & =\frac{1}{2} \sqrt{U_{1}^{2}+U_{2}^{2}+2 U_{1} U_{2} \cos \varphi_{\mathrm{x}}} \sin \left\{\omega \mathrm{t}+\frac{\varphi_{\mathrm{x}}}{2}+\right. \\
+ & \left.\operatorname{arctg} \frac{-U_{1} \sin \frac{\varphi_{\mathrm{x}}}{2}+\mathrm{U}_{2} \sin \frac{\varphi_{\mathrm{x}}}{2}}{\mathrm{U}_{1} \cos \frac{\varphi_{\mathrm{x}}}{2}+\mathrm{U}_{2} \cos \frac{\varphi_{\mathrm{x}}}{2}}\right\}+ \\
+ & \frac{1}{2} \sqrt{\mathrm{U}_{1}^{2}+U_{2}^{2}-2 \mathrm{U}_{1} U_{2} \cos \varphi_{\mathrm{x}}} \sin \{\omega \mathrm{t}- \\
- & \left.\operatorname{arctg} \frac{U_{2} \sin \varphi_{\mathrm{x}}}{\mathrm{U}_{1}-U_{2} \cos \varphi_{\mathrm{x}}}\right\} \operatorname{sign} \sin (\Omega \mathrm{t}+\varphi) .
\end{aligned}
$$

Accepting that $\mathrm{U}_{1}=\mathrm{U}$ and $\mathrm{U}_{2}=\mathrm{U}+\Delta \mathrm{U}=\mathrm{U}\left(1+\frac{\Delta \mathrm{U}}{\mathrm{U}}\right)=\mathrm{U}(1+\delta \mathrm{U})$, after a simple transformation, the last expression can be written in the following way

$$
\mathrm{u}_{\mathrm{VK}}(\mathrm{t})=\frac{\mathrm{U}}{2} \sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}} \sin \left\{\omega \mathrm{t}+\frac{\varphi_{\mathrm{x}}}{2}+\operatorname{arctg}\left[\frac{\delta \mathrm{U}}{2+\delta \mathrm{U}} \mathrm{t} \frac{\varphi_{\mathrm{x}}}{2}\right\}+\right.
$$

$$
\begin{align*}
& +\frac{\mathrm{U}}{2} \sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}} \sin \left\{\omega \mathrm{t}+\frac{\varphi_{\mathrm{x}}}{2}+\frac{\pi}{2}-\right. \\
& \left.\quad-\operatorname{arctg}\left[\frac{\delta \mathrm{U}}{2+\delta \mathrm{U}} \mathrm{~g}\left(\frac{\pi}{2}-\frac{\varphi_{\mathrm{x}}}{2}\right)\right]\right\} \operatorname{sign} \sin (\Omega \mathrm{t}+\varphi) \tag{2}
\end{align*}
$$

Now let us consider the expression (2), assuming that the amplitudes of the input signals are equal, i.e. $\delta \mathrm{U}=0$. Then from (2) we get

$$
\begin{gather*}
\mathrm{u}_{\mathrm{VK}}(\mathrm{t})=\mathrm{U} \cos \frac{\varphi_{\mathrm{x}}}{2} \sin \left(\omega \mathrm{t}+\frac{\varphi_{\mathrm{x}}}{2}\right)+\mathrm{U} \sin \frac{\varphi_{\mathrm{x}}}{2} \sin (\omega \mathrm{t}+ \\
+  \tag{3}\\
\left.+\frac{\varphi_{\mathrm{x}}}{2}+\frac{\pi}{2}\right) \operatorname{sign} \sin (\Omega \mathrm{t}+\varphi)
\end{gather*}
$$

Now, based on the expression (3), we find the dependence, which can determine the phase shift $\varphi_{x}$ between the switch input signals.

## Deducing of the first dependence

For the first dependence we select the first summand of (3) using a low-pass filter and by measuring its amplitude $\mathrm{U}_{\text {mol }}$ we are able to write

$$
\mathrm{U}_{\mathrm{m} \omega \mathrm{l}}=\mathrm{K}_{F N C H} \mathrm{U} \cos \frac{\varphi_{\mathrm{x}}}{2}
$$

where $\mathrm{K}_{\mathrm{FNCH}}-$ is the low-pass filter transmission coefficient.
From here we get the first expression, in which, when $U_{\text {mol }}$ and $\mathrm{K}_{\mathrm{FNCH}}$, we can determine the phase shift.

$$
\begin{equation*}
\varphi_{\mathrm{x}}=2 \arccos \frac{\mathrm{U}_{\mathrm{mol}}}{\mathrm{~K}_{\mathrm{FNCH}} \mathrm{U}} \tag{4}
\end{equation*}
$$

## Deducing of the second dependence

For deducing the second dependence, we select the second summand of (3) using a high-pass filter. Multiplying it to the function $\operatorname{sign} \sin (\Omega t+\varphi)$, we get the harmonic signal

$$
\mathrm{K}_{\mathrm{FVCH}} \mathrm{U} \sin \frac{\varphi_{\mathrm{x}}}{2} \sin \left(\omega \mathrm{t}+\frac{\varphi_{\mathrm{x}}}{2}+\frac{\pi}{2}\right)
$$

with the amplitude

$$
\mathrm{U}_{\mathrm{m} \omega 2}=\mathrm{K}_{\mathrm{FVCH}} \mathrm{U} \sin \frac{\varphi_{\mathrm{x}}}{2}
$$

where $\mathrm{K}_{\mathrm{FVCH}}$ - is the high-pass filter transmission coefficient.
Now, by measuring the amplitude, based on the last expression, we can determine the phase shift by the formula

$$
\begin{equation*}
\varphi_{\mathrm{x}}=2 \arcsin \frac{\mathrm{U}_{\mathrm{m} 02}}{\mathrm{~K}_{\mathrm{FVCH}} \mathrm{U}} \tag{5}
\end{equation*}
$$

The expressions (4) and (5) only partially coincide with the expressions obtained in [10], because there is filters influence.

Deducing of the third dependence
The third dependence is obtained from the ratio of the first and second summand amplitudes of (3), i.e.

$$
\frac{\mathrm{U}_{\mathrm{m} \omega 1}}{\mathrm{U}_{\mathrm{m} \omega 2}}=\frac{\mathrm{K}_{\mathrm{FNCH}}}{\mathrm{~K}_{\mathrm{FVCH}}} \operatorname{ctg} \frac{\varphi_{\mathrm{x}}}{2} .
$$

From the last expression we get the third phase shift dependence.

$$
\begin{equation*}
\varphi_{\mathrm{x}}=2 \operatorname{arcctg} \frac{\mathrm{~K}_{\mathrm{FVCH}} \mathrm{U}_{\mathrm{m} \omega 1}}{\mathrm{~K}_{\mathrm{FNCH}} \mathrm{U}_{\mathrm{m} \omega 2}} \tag{6}
\end{equation*}
$$

## Deducing of the fourth dependence

Similarly, for the inverse ratio of amplitudes we get the fourth phase shift dependence

$$
\begin{equation*}
\varphi_{\mathrm{x}}=2 \operatorname{arctg} \frac{\mathrm{~K}_{\mathrm{FNCH}} \mathrm{U}_{\mathrm{m} \omega 2}}{\mathrm{~K}_{\mathrm{FVCH}} \mathrm{U}_{\mathrm{m} \omega 1}} \tag{7}
\end{equation*}
$$

The expressions (6) and (7), in their turn, are partially similar to the expression obtained in [10] without binary sampling, but they also do not take into account the impact of the filters.

From the expressions (4), (5), (6), (7) it is evident that the error in measuring the phase shift will be determined by the measurement error of the amplitudes $\mathrm{U}, \mathrm{U}_{\text {mol }}$, $\mathrm{U}_{\mathrm{m} \omega 2}$ and $K_{F N C H}, K_{F V C H}$. But we are interested in the error caused by the inequality of the signal amplitudes.

## Assessing the impact of the amplitude inequality on measurement error

Let us estimate the errors caused by the amplitude inequality $\delta U$ of comparable signals. To do this, let us select the first summand of (2) using a low-pass filter and by measuring its amplitude, we get

$$
\begin{equation*}
\mathrm{U}_{\mathrm{mol}}^{\prime}=\mathrm{K}_{\mathrm{FNCH}} \frac{\mathrm{U}}{2} \sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}} \tag{8}
\end{equation*}
$$

Similarly, selecting the second summand of expression (2) with a high-pass filter, multiplying it by the function $\operatorname{sign} \sin (\Omega t+\varphi)$ and by measuring its amplitude, we get

$$
\begin{equation*}
\mathrm{U}_{\mathrm{m} \omega 2}^{\prime}=\mathrm{K}_{\mathrm{FVCH}} \frac{\mathrm{U}}{2} \sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}} \tag{9}
\end{equation*}
$$

## Error analysis for the first dependence

Similarly to the way how the expression (4) is obtained, we select from the expression (2) using a low-pass filter the first summand by measuring its amplitude and the amplitude of the first input signal and considering the expression (8), we get

$$
\begin{aligned}
\varphi_{\mathrm{x}}^{\prime}= & 2 \arccos \frac{\mathrm{~K}_{\mathrm{FNCH}} \frac{\mathrm{U}}{2} \sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{\mathrm{U}}= \\
& =2 \arccos \frac{\mathrm{~K}_{\mathrm{FNCH}} \sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{2}
\end{aligned}
$$

To evaluate the error from the influence of the amplitudes inequality of the comparable signal $\delta U$, we accept that $\mathrm{K}_{\mathrm{FVCH}}=1$. Thus, the absolute error of the phase shift measurement will look like

$$
\Delta \varphi=\varphi_{x}^{\prime}-\varphi_{x}=2 \arccos \frac{\sqrt{4 \cos ^{2} \frac{\varphi_{x}}{2}(1+\delta U)+(\delta U)^{2}}}{2}-\varphi_{x}
$$

Based on the last expression we will present the dependence for a relative error

$$
\begin{equation*}
\delta \varphi=\frac{\Delta \varphi}{\varphi_{\mathrm{x}}}=\left[\frac{2}{\varphi_{\mathrm{x}}} \arccos \frac{\sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{2}-1\right] \cdot 100 \% \tag{10}
\end{equation*}
$$

Based on (10), the dependence of the relative amplitude measurement error of the phase shift $\delta \varphi$ from the phase shift $\varphi_{\mathrm{x}}$ in the range of change from 0 to 1 rad . for various values of $\delta U$ is presented in Fig. 2.


Fig.2. Charts of relative errors of the phase shift measurement $\varphi_{x}$, based on the dependence (10) for different values of the amplitudes inequality $\delta \mathrm{U}$

Fig. 2 shows that reducing the phase shift measurement sensitivity threshold requires high precision in alignment of the comparable signals amplitude. Therefore, this measurement method is not suitable for measuring small phase shifts.

## Error analysis for the second dependence

Now similarly to how the expression (5) is obtained, we select from the expression (2) using a high-pass filter the second summand, measuring its amplitude $U_{m \omega 2}^{\prime}$ and having already the result of the measurement of the amplitude of the first input signal U and considering (9), we obtain

$$
\begin{aligned}
& \varphi_{\mathrm{x}}^{\prime}=2 \arcsin \frac{\mathrm{~K}_{\mathrm{FVCH}} \frac{\mathrm{U}}{2} \sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{\mathrm{U}}= \\
&=2 \arcsin \frac{\mathrm{~K}_{\mathrm{FVCH}} \sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{2}
\end{aligned}
$$

To evaluate the error from the influence of the amplitudes inequality of comparable signal $\delta \mathrm{U}$, we accept that $\mathrm{K}_{\mathrm{FVCH}}=1$. Then accordingly, the absolute and relative errors of the phase shift measurement will be

$$
\begin{gather*}
\Delta \varphi=\varphi_{\mathrm{x}}^{\prime}-\varphi_{\mathrm{x}}=2 \arcsin \frac{\sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{2}-\varphi_{\mathrm{x}} \\
\delta \varphi=\frac{\Delta \varphi}{\varphi_{\mathrm{x}}}=\left[\frac{2}{\varphi_{\mathrm{x}}} \arcsin \frac{\sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{2}-1\right] \cdot 100 \% \tag{11}
\end{gather*}
$$

Based on (11), the dependence of the relative amplitude measurement error of the phase shift $\delta \varphi$ from the phase shift $\varphi_{\mathrm{x}}$ in the range of change from 0 to 0.2 rad . for various values of $\delta \mathrm{U}$ is presented in Fig. 3.


Fig.3. Dependence of the relative phase shift measurement error for different values of amplitudes of the comparable signal inequality

As shown in Fig. 3, in this case you can get a much lower threshold. Therefore, this approach to measuring the phase shift can be applied for measuring small phase shifts.

Error analysis for the third dependence
To evaluate the error of the third method based on the (8) and (9), we write the expression for the phase shift similarly to (6)

$$
\varphi_{\mathrm{x}}^{\prime}=2 \operatorname{arcctg} \frac{\mathrm{~K}_{\mathrm{FNCH}} \sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{\mathrm{~K}_{\mathrm{FVCH}} \sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}} .
$$

Now we write the expression for the absolute error

$$
\Delta \varphi=\varphi^{\prime}{ }_{x}-\varphi_{x}=2 \operatorname{arcctg} \frac{\mathrm{~K}_{\mathrm{FNCH}} \sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{\mathrm{~K}_{\mathrm{FVCH}} \sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}-\varphi_{\mathrm{x}}
$$

Based on this expression, we will get the dependence for the relative error

$$
\begin{equation*}
\delta \varphi=\frac{\Delta \varphi}{\varphi_{\mathrm{x}}} \cdot 100 \%=\left[\frac{2 \operatorname{arcctg} \frac{\mathrm{~K}_{\mathrm{FNCH}} \sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{\mathrm{~K}_{\mathrm{FVCH}} \sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}}{\varphi_{\mathrm{x}}}-1\right] \cdot 100 \% \tag{12}
\end{equation*}
$$

To select the amplitude error, we will accept that $\mathrm{K}_{\mathrm{FNCH}}=\mathrm{K}_{\mathrm{FVCH}}$. Based on (12), the dependence of the relative amplitude measurement error of the phase shift $\delta \varphi$ from the phase shift $\varphi_{\mathrm{x}}$ in the range of change from 0 to 0.2 rad . for various values of $\delta \mathrm{U}$ is presented in Fig. 4.

This case also causes great values of the sensitivity threshold, making it impossible to use when measuring small phase shifts.


Fig.4. Charts of relative errors of the phase shift measurement $\varphi_{x}$, based on the dependence (12) for different values of the amplitudes inequality $\delta \mathrm{U}$

## Error analysis for the fourth dependence

Now we consider the amplitude error of the fourth method described by the expression (7). Here for the phase shift, we get this dependence

$$
\varphi_{x}^{\prime}=2 \operatorname{arctg} \frac{\mathrm{~K}_{\mathrm{FVCH}} \sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{\mathrm{~K}_{\mathrm{FNCH}} \sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}
$$

Let us write the expression for the absolute error

$$
\Delta \varphi=\varphi^{\prime}{ }_{x}-\varphi_{\mathrm{x}}=2 \operatorname{arcctg} \frac{\mathrm{~K}_{\mathrm{FVCH}} \sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{\mathrm{~K}_{\mathrm{FNCH}} \sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}-\varphi_{\mathrm{x}},
$$

and also for the relative error

$$
\begin{equation*}
\delta \varphi=\frac{\Delta \varphi}{\varphi_{\mathrm{x}}} \cdot 100 \%=\left[\frac{2 \operatorname{arctg} \frac{\mathrm{~K}_{\mathrm{FVCH}} \sqrt{4 \sin ^{2} \frac{\varphi_{\mathrm{x}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}{\mathrm{~K}_{\mathrm{FNCH}} \sqrt{4 \cos ^{2} \frac{\varphi_{\mathrm{X}}}{2}(1+\delta \mathrm{U})+(\delta \mathrm{U})^{2}}}}{\varphi_{\mathrm{x}}}-1\right] \cdot 100 \% \tag{13}
\end{equation*}
$$

In the case of $\mathrm{K}_{\mathrm{FVCH}}=\mathrm{K}_{\mathrm{FNCH}}$ based on (13), the dependence of the relative amplitude measurement error of the phase shift $\delta \varphi$ from the phase shift $\varphi_{x}$ in the range of change from 0 to 0.2 rad . for various values of $\delta \mathrm{U}$ is presented in Fig. 5.


Fig.5. Dependencies (13) of relative measurement errors of the phase shift $\varphi_{\mathrm{x}}$ by the third method for different values of the amplitudes of inequality $\delta U$

As shown in Fig. 5, the errors for the third and fourth methods are the same. Therefore, this option is also not suitable for measuring small phase shifts as it leads to high values of sensitivity threshold.

## Analysis of results

Using the expressions (10), (11), (12) and (13), we can calculate the sensitivity thresholds $\Delta \varphi_{\text {threshold }}$ determining the phase $\operatorname{shift} \varphi_{\mathrm{x}}$ for different values of the amplitudes of inequality $\delta \mathrm{U}$ of comparable signals. The values of sensitivity thresholds are summarized in Table 1.

Table 1
Values of sensitivity thresholds $\Delta \varphi_{\text {threshold }}$ for different values of the amplitudes inequality $\delta \mathrm{U}$ of the comparable signals for different dependencies for determining the phase shift (4-7)

| Formula number for determin- <br> ing the phase shift | Amplitudes inequality $\delta \mathrm{U}$ | Sensitivity threshold $\Delta \varphi$ threshold, rad |
| :---: | :---: | :---: |
| (4) | 0.001 | 0.06 |
|  | 0.01 | 0.2 |
|  | 0.05 | 0.44 |
| $(5)$ | 0.001 | 0.001 |
|  | 0.01 | 0.006 |
|  | 0.05 | 0.029 |
| $(6,7)$ | 0.001 | 0.006 |
|  | 0.01 | 0.028 |
|  | 0.05 | 0.055 |

According to Table 1, we construct schemes of dependences for threshold phase shifts $\Delta \varphi_{\text {threshold }}$ from the amplitudes inequality $\delta U$ for all expressions (4-7) (Fig. 6).


Figure 6. Dependence of phase shifts threshold from the amplitudes inequality for expressions (4-7) that determine the phase shift $\varphi_{\mathrm{x}}$

In Fig. 6, the dependencies are numbered, meaning the formulas number that is used to calculate the phase shift. Fig. 6 shows that the lowest threshold can be achieved if the phase shift is determined by the formula (5). The highest threshold happens if the phase shift is determined by the formula (4). The sensitivity threshold in case of determining the phase shift using formulas $(6,7)$ takes an intermediate value.

From the information above it follows that before measuring the phase shift to reduce the sensitivity threshold we should apply the amplitudes equalization of the compared signal (Fig. 7). It is hoped that the relative amplitudes inequality, which can be practically achieved, will be no worse than $\delta \mathrm{U}=0,001$. From Table 1 we find that in this case the sensitivity threshold of measuring of the phase shift caused by the amplitudes inequality will be $\Delta \varphi_{\text {threshold }}=0.001 \mathrm{rad}$.

This corresponds to about 3.5 arc minutes.

## The method of reducing error

The additive error can be reduced if the amplitude equalization is carried out with greater accuracy. This scheme is implemented in Fig. 7.

In Fig. 7 the following notation are used: AES - amplitude equalization scheme; DF1, DF2 - respectively the first and the second differentiator; CM1, CM2 - respectively the first and the second comparator; MM1, MM2 - respectively the first and the second monostable multivibrator; SVG - switching voltage generator; S - automatic switch; SAselective amplifier; SD - synchronous detector; A - amplifier; MC - microcomputer, Input
signals $u_{I N I}(t)$ and $u_{I N 2}(t)$ come to the amplitude equalization scheme AES, which does not change the phase shift between the signals. At frequencies in few Hz , this requirement can be met. The AES output signals are sent to the appropriate inputs of the automatic switch A, controlled by impulse voltage from the output of the switching voltage generator SVG. Switch output signal (Fig. 1), which is described by the expression (2) goes to the selective amplifier SA, that provides the second member of the expression (3), which is a balance-modulated signal. After the synchronous detection from the last signal sinusoidal signal envelope are selected, the amplitude of which must be measured. To make this, the envelope is first boosted by the amplifier A with the gain K and then sent to the first analog input of the microcomputer MC. Simultaneously, the envelope goes to the differentiator, which shifts the phase by $\pi / 2$, and then on the comparator CM2, which converts shifted in phase sine wave to a signal with rectangular form, from the front of which the monostable multivibrator MM2 generates short pulses that coincide with the moment in which the instantaneous value of the output amplifier A reach the amplitude value. Output pulses from MM2 go to the first digital input of microcomputer MC. If MC detecting on the first digital input high level, using an ADC does a count of instantaneous signal value in the first analog input and saves it into memory. This count corresponds to the $\mathrm{KU}_{\mathrm{m} \omega} 2$, where K - is the gain of the amplifier A.


Fig. 7. Block scheme of the phase shift measurement using binary sampling

At the same time one of the AES output signals enters the second analog input of the microcomputer MC and simultaneously to the differentiator DF1, which shifts the phase in $\pi / 2$, and then goes to the comparator CM1, which converts the rectangular signal, from the front of which the monostable multivibrator MM1 generates short pulses that coincide with the moment in which the instantaneous value of the sine wave on the second analog input of microcomputer MC reach the amplitude value $U$. And at this moment, if on the second digital input a high level is detected, coming from the monostable multivibrator MM1, the microcomputer MC using the ADC does a count of the analog signal, operating on the first analog input, and writes it into RAM.

Then, using the formula, the microcomputer MC calculates the phase shift

$$
\varphi_{\mathrm{x}}=2 \arcsin \frac{\mathrm{~K} \cdot \mathrm{U}_{\mathrm{m} \omega 2}}{\mathrm{k} \cdot \mathrm{~K}_{\mathrm{FVCH}} \cdot \mathrm{U}},
$$

where K - I the gain of the amplifier A ; k - is a constant, which is in memory and its value is equal to the gain of the amplifier $A$.

## CONCLUSIONS

The additive error caused by inequality amplitude occurs when measuring the phase shift between the harmonic signals when applying the sumo-difference method.

This paper analyzes the error for this case, when applying the binary sampling to signals. In the analysis rangeof the binary-sampled signal we have detected four dependencies determining the algorithms, which can calculate the phase shift by measuring the amplitude of a comparable signal and the amplitude of spectral components of a binary-sampled signal.

We have found the algorithm with the smallest error in which we measure the amplitude of the high frequency components of the signal after sampling and binary amplitude compared with other signals. We have shown the graphs of errors for different values of the amplitudes of inequality.

The block diagram of the phasemeter of small phase shifts has been suggested based on the results that implements the algorithm for measuring the phase shift with the least errors.

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# ВИМІРЮВАННЯ ФАЗОВОГО ЗСУВУ МІЖ ГАРМОНІЧНИМИ СИГНАЛАМИ З ВИКОРИСТАННЯМ БІНАРНОЇ ДИСКРЕТИЗАЦІЇ 

Ігор Бучма<br>Національний університет "Львівська політехніка" вул. С. Бандери, 12, Львів, 79013, Україна<br>e-mail: ibuchma1@gmail.com

В статті проаналізовано похибки вимірювання фазового зсуву між гармонічними сигналами з використанням бінарної дискретизаиії. Розглянуто чотири методи визначення фазового зсуву. Створено математичні моделі похибок, зумовлених нерівністю амплітуд сигналів. Приведено їх графічні залежності. Виокремлено метод, що забезпечує найменші адитивні похибки. Наведено структурну схему пристрою, що реалізує згаданий метод.

Ключові слова: вимірювання фазового зсуву, похибки, гармонічні сигнали, низькі частоти, бінарна дискретизаиія, нерівність амплітуд, порогова чутливість.

