УДК 517.5

A. Talhaoui

THE CAUCHY-RIEMANN EQUATIONS FOR A CLASS OF $(0,1)$-FORMS IN l^2

A. Talhaoui. *The Cauchy-Riemann equations for a class of* $(0,1)$*-forms in* l^2, Mat. Stud. **46** (2016), 171–177.

We study the local exactness of $\overline{\partial}$ operator in the unit ball of l^2 for a particular class of $(0,1)$-forms ω of the type $\omega(z) = \sum_i z_i\omega^i(z)d\overline{z_i}$, $z = (z_i)$ in l^2. We suppose each function $\omega^i(z)$ of class C^∞ in the closed unit ball of l^2 of the form $\omega^i(z) = \sum_k \omega^i_k(z^k)$, where $\mathbb{N} = \bigcup I_k$ is a partition of $\mathbb{N}$, $(\mathrm{card} I_k) < +\infty$, and z^k is the projection of z on $\mathbb{C}^{I_k}$. We establish sufficient conditions for exactness of ω related to the expansion in Fourier series of the functions ω^i_k.

1. Introduction. The study of local exactness of infinitely differentiable $(0,1)$-forms was the object of important work, in particular those of L. Lempert. This author gets local exactness in the space l^1 and on any space of Banach when the forms are real analytical ([1], [2]).

In Hilbert spaces few results are known, however we must mention an important result due to G.Coeuré: he gives an example of $(0,1)$-form ω of class $\mathcal{C}^1$ in the unit ball of an infinite dimensional separable Hilbert space such that the equation $\overline{\partial}f = \omega$ does not admit any local solution around 0, (see Mazet [3]). No other example is known with ω of the class $\mathcal{C}^p(1 < p \leq \infty)$.

In this paper, we study the local exactness of $\overline{\partial}$ in the Hilbert space l^2, for a particular class of (0,1)-forms of the type

$$\omega(z) = \sum_i z_i\omega^i(z)d\overline{z}_i, \qquad z = (z_i) \text{ in } l^2$$

under the following assumptions (H_1):

i) Each function ω^i is indefinitely differentiable on the closed unit ball of l^2 denoted $\overline{B}$, and of the form

$$\omega^i(z) = \sum_k \omega^i_k(z^k) \tag{1}$$

where the series (1) is supposed to be absolutely convergent, and where $\mathbb{N} = \bigcup I_k$ is a partition of $\mathbb{N}$, with z^k standing for the projection of z on $\mathbb{C}^{I_k}$, and ω^i_k being a function of class $\mathcal{C}^\infty$ on the closed unit ball of $\mathbb{C}^{I_k}$ provided with the norm of l^2.

ii) For all k, card I_k noted $|k|$ is finite.

The used method is based on the expansion in Fourier series of the indefinitely differentiable functions f on the closed unit ball of $\mathbb{C}^N$. In ([5], Theorem 2.1) we show that such

2010 *Mathematics Subject Classification:* 32F20, 46G20.
Keywords: $\overline{\partial}$ operator; Hilbert space; infinite dimension.
doi:10.15330/ms.46.2.171-177

functions admit necessarily a Fourier series expansion of the form

$$f(z) = \sum_{(\alpha,\beta)\in(\mathbb{N}\times\mathbb{N})^N} z^\alpha \overline{z}^\beta f_{\alpha,\beta}(|z|^2)\ , \text{ with } z^\alpha = z_1^{\alpha_1}\dots z_N^{\alpha_N}$$

and $|z|^2 = (|z_1|^2, \dots, |z_N|^2)$. This allows us to study the local exactness of $\overline{\partial}$ for a restricted class of forms ω which respond moreover to the additional assumption (H_2):

$$\omega_k^i = \sum_{\alpha\in\mathbb{N}^{|k|}} (z^k)^\alpha \omega_{\alpha,k}^i(|z^k|^2), \text{ for all } i \text{ and } k.$$

In [5] the following results was proved.

Theorem A. *Let ω be a closed $(0,1)$-form of the class C^∞ on $\overline{B}$ of the type $\omega(z) = \sum z_i\omega^i(z)d\overline{z_i}$ and verifying the assumptions (H_1) and (H_2). If there exists a positive integer M such that the coefficients $\omega_{\alpha,k}^i$ are null for all $|\alpha| > M$, all k and all i in I_k, then the series F_k and F converge and define indefinitely differentiable functions on $\overline{B}$.*

Theorem B. *Let ω be a closed $(0,1)$-form of class C^∞ on $\overline{B}$ according to the type*

$$\omega(z) = \sum z_i\omega^i(z)d\overline{z}_i$$

and verifying the assumptions (H_1) and (H_2). We assume furthermore that the sequence $(|k|)$ is bounded and that the derivatives $D^p\omega^i$ are uniformly bounded in i on the unit ball of l^2 for $0 \le p \le 2$. Then there exists a real number $r > 0$ and a function F of class C^∞ on the ball with radius r such that

$$\overline{\partial}F = \omega \text{ and } |F(z)| \le C\|z\|^2 \sup_{i,\ 0\le p\le 2} \|D^p\omega^i\|_\infty \text{ for } \|z\| < r$$

where C is a constant and D designates the differentiation operator.

Here we study the local exactness of $\overline{\partial}$ when the sequence $(|k|)$ is not bounded.

If $z = (z_i)$ is a finite or infinite sequence of numbers, we denote by $\#z$ the number of nonzero entries z_i. For every integers $0 \le n \le N$, let $\mathbb{N}_n^N$ be the set of all multiindicies $\alpha \in \mathbb{N}^N$ such that $\#\alpha = n$. In section 3, we establish the following result which generalize Theorem B.

Theorem 1. *Let ω be a closed $(0,1)$-form of the class C^∞ on the closed unit ball of l^2 according to the type $\omega(z) = \sum z_i\omega^i(z)d\overline{z}_i$ and verifying the assumptions (H_1) and (H_2). Let (n_k) be a sequence of integers such that $1 \le n_k \le |k|$ for all k, and $\liminf_{k\to+\infty} \frac{n_k}{|k|} > 0$. We assume furthermore that for every k, the coefficients $\omega_{\alpha,k}^i$ are null if $\alpha \in \mathbb{N}_n^{|k|}$ for all $n < n_k$, and all i in I_k, and that the derivatives $D^p\omega^i$ are uniformly bounded in i on the unit ball of l^2 for $0 \le p \le 2$. Then there exist a real number $r > 0$ and a function F of the class C^∞ on the ball of radius r such that*

$$\overline{\partial}F = \omega \text{ and } |F(z)| \le C \sup_{i,\ 0\le p\le 2} \|D^p\omega^i\|_\infty \text{ for } \|z\| < r,$$

where C is a constant and D designates the differentiation operator.

2. Preliminaries.

2.1. Notations. In this work our main concern will be the Hilbert space l^2, and so, unless indicated otherwise, $\| \ \|$ will denote the l^2-norm on l^2 or on $\mathbb{C}^N$: if $z = (z_i) \in l^2$ or $\mathbb{C}^N$, $\|z\| = \sum |z_i|^2$. $B(r)$ and $B_N(r)$ will denote the ball $\|z\| < r$ in l^2 and $\mathbb{C}^N$ respectively. When $r = 1$, we simply write B and B_N for $B(1)$ and $B_N(1)$, respectively. We shall make extensive use of muti-indices. A multi-index $\alpha = (\alpha_i)_{i=1}^{\infty}$ for us is a sequence of integers $\alpha_i \geq 0$ with $\alpha_i = 0$ for i sufficiently large. The length of α is $|\alpha| = \sum_{i=1}^{\infty} \alpha_i$. We let $\alpha! = \prod_{i=1}^{\infty} \alpha_i!$, where the usual convention $0! = 1$ is observed. For a sequence of complex numbers $z = (z_i)_{i=1}^{\infty}$, we put $z^\alpha = \prod_{i=1}^{\infty} z_i^{\alpha_i}$, where 0^0 is defined to be 1.

If z and w are in $\mathbb{C}^N$, the following notations will be used in the sequel: $z_i' = (z_1, z_2, \ldots, z_i)$; $z_i'' = (z_i, z_{i+1}, \ldots, z_N)$ $(i = 1, 2, \ldots, N)$. When α is a multi-index of $\mathbb{N}^N$, we simply write $z^{\alpha_i'}$ for $(z_i')^{\alpha_i'}$. $|z|^2 = (|z_1|^2, \ldots, |z_N|^2)$, $zw = (z_1 w_1, \ldots, z_N w_N)$. If x is a vector of $\mathbb{R}_+^N$, then $\sqrt{x} = (\sqrt{x_1}, \ldots, \sqrt{x_N})$.

If f is in $\mathcal{C}^\infty(\overline{B_N})$, in the sense of Frechet, then for each $p \in \mathbb{N}$, we put

$$\|D^p f\|_\infty = \sup_{z \in \overline{B_N}} \|D^p f(z)\|,$$

where $\|D^p f(z)\|$ denotes the norm of the pth differential operator.

2.2. A series in infinitely many variables. If $z = (z_i)_{i=1}^{\infty}$ is in the unit ball of l^2, we put

$$P_n(z) = \sum_{\#\alpha \geq n} \frac{|\alpha|^{\frac{|\alpha|}{2}}}{\alpha^{\frac{\alpha}{2}}} z^\alpha.$$

Lemma 1. *Given $1 \leq n \leq N$, and $\epsilon \in \left]0, \frac{1}{2}\right[$, there is a real number $\rho > 0$ and a constant $C > 0$ such that if $z \in B_N(\rho)$, then*

$$|P_n(z)| \leq C \left(e^{1/2\epsilon} \rho\right)^n \mathrm{C}_N^n$$

C depends only on ρ and ϵ but not on N.

Proof. Let us consider in $\mathbb{C}$ the entire function $g(z) = \sum_{\alpha \geq 1} \frac{z^\alpha}{\alpha^{\alpha/2}}$. For every $\epsilon > 0$, we have

$$|g(z)| \leq \sum_{\alpha \geq 1} \frac{|z|^\alpha}{\epsilon^{\alpha/2} \sqrt{\alpha!}} \epsilon^{\alpha/2}.$$

Using the Cauchy-Schwarz inequality, we obtain

$$|g(z)| \leq \left(\sum_{\alpha \geq 1} \frac{|z|^{2\alpha}}{\epsilon^\alpha \alpha!} \right)^{1/2} \left(\sum_{\alpha \geq 1} \epsilon^\alpha \right)^{1/2}.$$

Let $1 \leq n \leq N, q \in \mathbb{N}^*$, and let $z \in \mathbb{C}^N$, we have

$$\sum_{\substack{\alpha \in \mathbb{N}_n^{|k|} \\ |\alpha| = q}} \frac{z^\alpha}{\alpha^{\alpha/2}} = \sum_{1 \leq i_1 < \ldots < i_n \leq N} \sum_{\substack{\alpha_{i_1}, \ldots, \alpha_{i_n} > 0 \\ |\alpha| = q}} \frac{z_{i_1}^{\alpha_{i_1}}}{\alpha_{i_1}^{\alpha_{i_1}/2}} \cdots \frac{z_{i_n}^{\alpha_{i_n}}}{\alpha_{i_n}^{\alpha_{i_n}/2}}. \tag{2}$$

For any $1 \leq i_1 < \ldots < i_n \leq N$, we observe that the second sum in the right hand of (2) is the homogeneous component of degree q of the product $g(z_{i_1}) \ldots g(z_{i_n})$. It follows, when $z \in B_N(\sqrt{q})$, the majorization

$$\left| \sum_{\substack{\alpha_{i_1},\ldots,\alpha_{i_n}>0 \\ |\alpha|=q}} \frac{z_{i_1}^{\alpha_{i_1}}}{\alpha_{i_1}^{\alpha_{i_1}/2}} \cdots \frac{z_{i_n}^{\alpha_{i_n}}}{\alpha_{i_n}^{\alpha_{i_n}/2}} \right| \leq \exp\left(\frac{q}{2\epsilon}\right) \left(\sqrt{\frac{\epsilon}{1-\epsilon}}\right)^n .$$

By homothety on the ball of radius ρ, we get

$$\left| \sum_{\substack{\alpha_{i_1},\ldots,\alpha_{i_n}>0 \\ |\alpha|=q}} \frac{z_{i_1}^{\alpha_{i_1}}}{\alpha_{i_1}^{\alpha_{i_1}/2}} \cdots \frac{z_{i_n}^{\alpha_{i_n}}}{\alpha_{i_n}^{\alpha_{i_n}/2}} \right| \leq \left(\frac{\rho}{\sqrt{q}}\right)^q \exp\left(\frac{q}{2\epsilon}\right) \left(\sqrt{\frac{\epsilon}{1-\epsilon}}\right)^n$$

and therefore, if ρ is sufficiently small, we get

$$|P_n(z)| \leq \sum_{q \geq n} \left| \sum_{\substack{\alpha \in \mathbb{N}_n^{|k|} \\ |\alpha|=q}} \frac{q^{q/2} z^\alpha}{\alpha^{\alpha/2}} \right| \leq \frac{\left(e^{1/2\epsilon}\rho\right)^n}{1-\rho e^{1/2\epsilon}} \mathrm{C}_N^n .$$

□

2.3. Fourier series expansion.

Theorem 2. *If f is in $\mathcal{C}^\infty\left(\overline{B_N}\right)$, then it admits the Fourier series expansion*

$$f(z) = \sum_{\alpha \in \mathbb{Z}^N} z^\alpha f_\alpha(|z|^2), \tag{3}$$

where $z^\alpha = z_1^{\alpha_1} z_2^{\alpha_2} \ldots z_N^{\alpha_N}$ with $z_i^{\alpha_i} = (\overline{z_i})^{|\alpha_i|}$ if $\alpha_i < 0$.

The series (3) is normally convergent with its derivatives on $\overline{B_N}$; the coefficients f_α are $\mathcal{C}^\infty$ on $\left\{x \in \mathbb{R}_+^N ; x_1 + \ldots + x_N < 1\right\}$, that satisfy

$$\forall \alpha \in \mathbb{Z}^N \ ; \ z^\alpha f_\alpha(|z|^2) = \int_{[0,2\pi]^N} f(ze^{i\theta}) e^{-i(\alpha\cdot\theta)} \frac{d\theta}{(2\pi)^N},$$

where $(\alpha \cdot \theta) = \sum_{i=1}^N \alpha_i \theta_i$, $e^{i\theta} = (e^{i\theta_1}, \ldots, e^{i\theta_N})$, and $\frac{d\theta}{(2\pi)^N} = \frac{d\theta_1}{2\pi} \cdots \frac{d\theta_N}{2\pi}$.

For the proof see ([5], Theorem 2.1).

3. Exactness of a class of (0,1)-forms. We study the local exactness of $\overline{\partial}$ in the Hilbert space l^2 for a particular class of $(0,1)$-forms of the type

$$\omega(z) = \sum_i z_i \omega^i(z) d\overline{z_i}, \qquad z = (z_i) \text{ in } \ l^2$$

under the following assumptions (H_1):

i) Each function ω^i is indefinitely differentiable on the closed unit ball of l^2, and it takes the form

$$\omega^i(z) = \sum_k \omega_k^i(z^k) \tag{4}$$

where the series (4) is supposed to be absolutely convergent, and where $\mathbb{N} = \bigcup I_k$ is a partition of $\mathbb{N}$, with z^k standing for the projection of z on $\mathbb{C}^{I_k}$, and ω_k^i being a function of class $\mathcal{C}^\infty$ on the closed unit ball of $\mathbb{C}^{I_k}$ provided with the norm of l^2.
ii) For all k, card I_k noted $|k|$ is finite.

According to Theorem 2, for all i and k the function ω_k^i admits a Fourier series expansion in the form $\omega_k^i(z^k) = \sum_{\alpha\in\mathbb{Z}^{|k|}}(z^k)^\alpha\omega_{\alpha,k}^i(|z^k|^2)$, where the coefficients $\omega_{\alpha,k}^i$ are functions of class $\mathcal{C}^\infty$, on the closed unit ball of $\mathbb{C}^{I_k}$.

In what follows, we make the assumption (H_2):

$$\omega_k^i(z^k) = \sum_{\alpha\in\mathbb{N}^{|k|}}(z^k)^\alpha\omega_{\alpha,k}^i(|z^k|^2) \quad \text{for all } i \text{ and } k \text{ in } \mathbb{N}.$$

Following ([5], Theorem 3.2), ω is $\overline{\partial}$-closed if and only if the form $\Phi_{\alpha,k} = \sum_{i\in I_k}\omega_{\alpha,k}^i dt_i$ is d-closed in the closed unit ball of $\mathbb{R}_+^{|k|}$ for each α and k. So we are led to integrate the $(0,1)$-form $\tilde{\omega} = \sum_k \tilde{\omega}_k$ such that

$$\tilde{\omega}_k(z^k) = \sum_{i\in I_k} z_i\left[\sum_{\alpha\in\mathbb{N}^{|k|}}(z^k)^\alpha\frac{\partial\Omega_{\alpha,k}}{\partial t_i}(|z^k|^2)\right]d\overline{z_i},$$

where $\Omega_{\alpha,k}$ is an anti-derivative of the form $\Phi_{\alpha,k}$.

Each $\tilde{\omega}_k$ is a $\overline{\partial}$-closed $(0,1)$-form of class $\mathcal{C}^\infty$ on the closed unit ball of $\mathbb{C}^{I_k}$.

Let $F_k(z^k) = \sum_{\alpha\in\mathbb{N}^{|k|}}(z^k)^\alpha\Omega_{\alpha,k}(|z^k|^2)$. Then $F = \sum_{k=1}^\infty F_k$ is a formal solution of the equation $\overline{\partial}F = \tilde{\omega}$ and according to ([3], Appendix 3, Lemma 5) the problem is reduced to the existence of a real number $r \in \left]0,1\right[$, and for every α and k an anti-derivatives $\Omega_{\alpha,k}$ such that the series F_k converge and satisfies an estimate independent of k on the ball of radius r of $\mathbb{C}^{I_k}$. We give a positive response for two particular cases.

i) The polynomial case.

Theorem 3. *Let ω be a closed $(0,1)$-form of class C^∞ on $\overline{B}$ of the type $\omega(z) = \sum z_i\omega^i(z)d\overline{z_i}$ and verifying the assumptions (H_1) and (H_2). If there exists a positive integer M such that the coefficients $\omega_{\alpha,k}^i$ are null for all $|\alpha| > M$, all k and all i in I_k, then the series F_k and F converge and define indefinitely differentiable functions on $\overline{B}$.*

For the proof see ([5], Theorem 4.1).

ii)Non-polynomial case. Let (n_k) be a sequence of integers such that $1 \le n_k \le |k|$ for all k, and $\liminf \frac{n_k}{|k|} > 0$. If we suppose that for every k, the coefficients $\omega_{\alpha,k}^i$ are null if $\alpha \in \mathbb{N}_n^{|k|}$ for all $n < n_k$, and all i in I_k, then we shall prove that there exists $r > 0$ such that for k sufficiently large the series F_k and F converge and define indefinitely differentiable functions on $B(r)$.

The proof of Theorem 1 is a direct consequence of the forthcoming proposition.

Proposition 1. *Let ω be a closed $(0,1)$-form of the class C^∞ on the closed unit ball of l^2 according to the type $\omega(z) = \sum z_i\omega^i(z)d\overline{z}_i$ and verifying the assumptions (H_1) and (H_2). Let (n_k) be a sequence of integers such that $1 \le n_k \le |k|$ for all k, and $\liminf_{k\to+\infty}\frac{n_k}{|k|} > 0$. We suppose moreover that for each k, the coefficients $\omega_{\alpha,k}^i$ are null if $\alpha \in \mathbb{N}_n^{|k|}$ for all $n < n_k$, and* *all i in I_k, and that the derivatives $D^p\omega^i$ are uniformly bounded in i on the unit ball of l^2 for $0 \le p \le 2$. Then there exists $r > 0$ and $\lambda > 0$ such that for k sufficiently large and $z \in B(r)$, the series F_k converge and defines a $\overline{\partial}$-antiderivates of $\tilde{\omega}_k$ of class C^∞ on $\overline{B_{|k|}}(r)$ for which*

$$|F_k(z^k)| \le C\left(\|z^k\|^2 + (2r^\lambda)^{|k|}\right) \sup_{\substack{i \\ 0\le p\le 2}} \|D^p\omega^i\|_\infty \tag{5}$$

where C is a constant independent of k.

Proof. We recall that $\Phi_{\alpha,k}$ designates the closed form in $\mathbb{R}_+^{|k|}$ defined by $\Phi_{\alpha,k} = \sum_{i\in I_k} \omega_{\alpha,k}^i dt_i$. Its anti-derivative is given by $\Omega_{\alpha,k}(|z^k|^2) = \int_\gamma \Phi_{\alpha,k}$, and the path γ defined below joins the point $|z^k|^2$ to a fixed point of the closed unit ball of $\mathbb{R}_+^{|k|}$.

We also recall that we can take the function F_k for a $\overline{\partial}$-antiderivates of $\tilde{\omega}_k$ conditioned by its series convergence.

Given $0 < r < 1$ and $z \in \overline{B(r)}$, the path γ is the union in $\mathbb{R}_+^{|k|}$ of the adjacent segments $[M^m, M^{m+1}]$ $(m = 0, \dots, |k|)$, defined by $M^0 = |z^k|^2$, and for $m \in \{1, 2, \dots, |k| + 1\}$,

$$M_i^m = \begin{cases} \frac{\alpha_i}{|\alpha|} & \text{if } \ i < m \\ \left(\frac{|z_i|}{r}\right)^2 & \text{if } \ i \ge m \end{cases}$$

Let

$$F_k^m(z^k) = \sum_{n\ge n_k} \sum_{\alpha\in\mathbb{N}_n^{|k|}} (z^k)^\alpha \int_{M^m}^{M^{m+1}} \Phi_\alpha, \quad m = 0, \dots, |k|.$$

Since $F_k = \sum_{m=0}^{|k|} F_k^m$, it will be enough to prove that, for all $m = 0, \dots, |k|$, the series F_k^m converges and satisfies an estimate independent of k.

Let us start with the case $m = 0$.

$$\begin{aligned} F_k^0(z^k) &= \sum_{n\ge n_k} \sum_{\alpha\in\mathbb{N}_n^{|k|}} \int_1^{1/r^2} (z^k)^\alpha \sum_{i\in I_k} \omega_\alpha^i(u|z^k|^2) \cdot |z_i|^2 du \\ &= \sum_{q\ge n_k} \int_1^{1/r^2} \int_0^{2\pi} \frac{1}{(\sqrt{u})^{q+1}} \langle \tilde{\omega}(\sqrt{u} z^k e^{i\theta}), z^k \rangle e^{-iq\theta} \frac{d\theta}{2\pi} du. \end{aligned}$$

By making two integrations by parts relatively to θ in each term of the above sum, we obtain

$$|F_k^0(z^k)| \le C\|z^k\|^2 \sup_{p\le 2} \|D^p\tilde{\omega}\|_\infty, \tag{6}$$

where C is a constant independent of k. Now, let us consider the series F_k^m for $m \ge 1$. We can write

$$F_k^m(z^k) = \sum_{\substack{\alpha\in\mathbb{N}_n^{|k|} \\ n\ge n_k}} \int_0^{2\pi} \int_{M^m}^{M^{m+1}} \frac{|\alpha|^{\frac{|\alpha|}{2}} (z_k)^{\alpha'_m} (\alpha''_m)^{\frac{1}{2}\alpha''_m} r^{|\alpha''_{m+1}|}}{\alpha^{\frac{\alpha}{2}} (t_m|\alpha|)^{\frac{1}{2}\alpha_m} |\alpha|^{\frac{1}{2}|\alpha''_{m+1}|}} \omega^m\left(\sqrt{t} e^{i\theta}\right) e^{-i|\alpha|\theta} \frac{d\theta}{2\pi} dt_m.$$

An easy computation shows that

$$|z_m^{\alpha_m}| \int_{M^m}^{M^{m+1}} \frac{dt_m}{(\sqrt{t_m})^{\alpha_m}} \le C r^{\alpha_m},$$

where C is a constant.

If we choose r sufficiently small, then an application of Lemma 1 shows that

$$|F_k^m(z^k)| \leq C 2^{|k|} \sup_i \|\omega^i\|_\infty r^{n_k}.$$

for every $z \in \overline{B(r)}$ and $m \geq 1$.

Since $\liminf\limits_{k \to +\infty} \frac{n_k}{|k|} > 0$, there exists a real number $\lambda > 0$ such that $n_k \geq \lambda |k|$ for k sufficiently large, hence we are led to the majorization

$$\sum_{m=1}^{|k|} |F_k^m(z^k)| \leq C(2r^\lambda)^{|k|} \sup_i \|\omega^i\|_\infty \tag{7}$$

for all $z \in \overline{B(r)}$, where r is choosen sufficiently small, and C is a constant independent of k. Now, (6) and (7) implies the required estimate (5). □

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National Polythecnic School of Oran, Algeria
talhaoui_abd@yahoo.fr

Received 1.10.2016