UDC 519.21

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STABILITY OF LINEAR SYSTEMS OF DIFFERENTIAL EQUATIONS WITH RANDOM JUMP LINEAR SOLUTIONS IN HILBERT SPACE

The conditions of stability in the middle and in the mean square solutions of stochastic differential equations with random perturbations in Hilbert spaces are obtained.

Key words: *Hilbert space, stability, generators operator Markov process.*

Introduction. The statement about the stability or instability of the first approach are formulated as follows: if the solution of linear equations, built on the original stable or unstable, then the solution of the original equation will also be under stable or unstable. This assertion is true under certain assumptions that are reasonable and at work. Analysis of the behavior of solutions of stochastic differential equations of Ito-Skorokhod devoted a number of works, including [1–6].

Formulation of the problem. Consider a Markov process $\xi(t)$, which can be in states $\theta_1, \theta_2, ..., \theta_n, ...$ with probabilities

$$p_k(t) = \mathrm{P}\{\xi(t) = \theta_k\} \quad (k = 1, 2, ...), \tag{1}$$

which satisfy the system of differential equations

$$\frac{dp_k(t)}{dt} = \sum_{k=1}^{\infty} \alpha_{ks} p_s(t) . \tag{2}$$

This constant coefficients α_{ks} $(k = 1, 2, ...)$ satisfy the relation

$$\alpha_{ks} < 0 , \ \sum_{s=1}^{\infty} \alpha_{ks} = 0 , \ \alpha_{ks} \geq 0 \ \ (k \neq s) \ \ (k, s = 1, 2, ...) .$$

The Markov process $\xi(t)$ abruptly changes its value at random times t_j $(j = 0, 1, 2, ...)$.

Along with Markov processes consider the system of linear differential equations

$$\frac{dX(t)}{dt} = AX(t) , \ t \neq t_j \ \ (j = 0, 1, 2, ...) , \tag{3}$$

which time jump t_j changes its value under the law

$$X(t_j + 0) = C_{ks} X(t_j - 0) , \tag{4}$$

the transition of a random process from a state θ_s to a state θ_k $(k \neq s)$, C_{ks} — constant matrix.

Solution of system (3) is a random process. The paper investigates the stability of the average and the stability of the mean square solutions of linear differential equations (3). Thus the derived equations to determine the necessary and sufficient conditions for asymptotic stability.

Difference approximation of differential equations. Turn of differential equations (2) and (3) the difference, putting $t_n = nh \ \ (n = 0,1,2,...)$, $h > 0$.

The system of differential equations (2) is approximated by a system of difference equations

$$p_k(t_{n+1}) = p_k(t_n) + h\sum_{s=1}^{\infty} \alpha_{ks} p_s(t_n) \ \ (k = 1,2,...) . \tag{5}$$

Let the conditions $1 + h\alpha_{kk} \geq 0 \ \ (k = 1,2,...)$ are true. The system of differential equations (5) describes Markov chain of infinite, can be in states $\theta_1, \theta_2, ..., \theta_n, ...$ with probabilities $p_k(t) = \mathrm{P}\{\xi(t) = \theta_k\} \ \ (k = 1,2,...)$.

The system of differential equations (3) is approximated by a system of difference equations

$$X_{n+1} = X_n + hA(\xi_{n+1}, \xi_n) X_n , \tag{6}$$

where

$$A(\theta_k, \theta_s) = A \ \ (s = 1,2,...) ; \ A(\theta_k, \theta_s) = h^{-1}(C_{ks} - E) \ \ (k \neq s) . \tag{7}$$

Moment equation for a system of difference equations (6). Let $f(t_n, X, \xi)$ — density distribution of discrete- continuous random process X, that can be represented as

$$f(t_n, X, \xi) = \sum_{k=1}^{\infty} f_k(t_n, X) \delta(\xi - \theta_k) , \tag{8}$$

where $\delta(\theta)$ — the Dirac delta function. Partial density distribution $f_k(t_n, X)$ satisfying the conditions

$$\begin{gathered} f_k(t_{n+1}, X) = \\ = (1 + h\alpha_{kk}) f_k(t_{n1}(E + hA(\theta_k, \theta_s))^{-1} X) \,|\det(E + hA(\theta_k, \theta_s))^{-1}| + \\ + \sum_{s=1, s\neq q}^{\infty} ha_{ks} f_s(t_{n_1}, (E + hA(Q_k, Q_s))^{-1} X) \,|\det(E + hA(Q_k, Q_s))^{-1}, \\ (k = 1,2,...) \end{gathered} \tag{10}$$

Expand the terms of the equation (10) the degree of the parameter h, in anticipation of differentiation $f_k(t, X) \ \ (k = 1,2,...)$ for all arguments. Thus we arrive at a system of differential equations with partial derivatives

$$\frac{\partial f_k(t,X)}{\partial t} = -\frac{Df_k(t,X)}{DX} A_k X - f_k(t,X) S_p A_k + \\ + \sum_{s=1}^{\infty} \alpha_{ks} f_s(t, C_{ks}^{-1} X) \,|\det C_{ks}^{-1}, \ (k = 1,2,...) \tag{11}$$

Denote the mathematical expectation vector X and matrix XX' symbols

$$M(t) = \langle \tilde{O} \rangle = \int_{E_m} Xf(t,X)dX = \sum_{k=1}^{\infty} \int_{E_m} Xf_k(t,X)dX; \\ D(t) = \langle \tilde{O}\tilde{O}^* \rangle = \int_{E_m} XX^* f(t,X)dX = \sum_{k=1}^{\infty} \int_{E_m} XX^* f_k(t,X)dX \tag{12}$$

Vector $M(\mathrm{t})$ and matrix $D(\mathrm{t})$ are defined by equations

$$M(t) = \sum_{k=1}^{\infty} M_k(t), \ M_k(t) = \int_{E_m} Xf_k(t,X)dX, \ (k = 1,2,...)$$

$$D(t) = \sum_{k=1}^{\infty} D_k(t), \ D_k(t) = \int_{E_m} XX^* f_k(t,X)dX, \ (k = 1,2,...).$$

Vectors $M_k(\mathrm{t})$ $(k = 1,2,...)$ satisfying the system of differential equations

$$\frac{dM_k(\mathrm{t})}{dt} = A_k M_k + \sum_{s=1}^{q} \alpha_{ks} C_{ks} M_k \quad (k = 1,2,...) \tag{13}$$

is derived from the system (11) by Matrix multiplication XX' and than by integration over the entire space E_m.

Further assume that the partial density distribution $f_k(0,X)$, $(k = 1,2,...)$ have finite moments of first and second order. From the system of equations (13) and (14) we obtain the following theorem.

Theorem. To trivial solutions of linear differential equations (3) with random jumps solution of the form (4), caused by the Markov process $\xi(t)$ (1) be asymptotically stable on average, it is necessary and sufficient that the trivial solution of system (13) was asymptotically stable.

Conclusions. We built the moment equations, which follows from the stability of the initial stability of stochastic equations in a Hilbert space.

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Отримані умови стійкості в середньому та в середньому квадратичному розв'язків стохастичних диференціальних рівнянь з випадковими збуреннями у гільбертових просторах.

Ключові слова: *Гільбертовий простір, стійкість, твірний оператор марківського процесу.*

Отримано: 16.03.2013