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SOME PROBLEMS OF PSEUDO-DIFFERENTIAL OPERATORS THEORY

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We study the pseudo-differential equation $(Au)(x) = f(x),\ x \in D$, in the Sobolev-Slobodetskii spaces $H^s(D)$, where A is a elliptic pseudo-differential operator, D is an m-dimensional piecewise smooth manifold with boundary having singularity points. The singularity points of D are called the points breaking smoothness property for the boundary ∂D. Using the wave factorization concept for elliptic symbols, it is possible to describe solvability conditions for the equation with singularities of the "cone" as well as "wedge" types. Most of author's results on solvability were related to the planar case. Here we consider an essentially multi-dimensional situation.

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Ми вивчаємо псевдодиференціальне рівняння $(Au)(x) = f(x),\ x \in D$, в просторах Соболєва-Слободецького $H^s(D)$, де A – еліптичний псевдодиференціальний оператор, D – m-вимірний кусково-гладкий многовид з межею, що містить сингулярні точки. Сингулярними ми називаємо точки многовиду D, в яких немає гладкості межі. За допомогою поняття хвильової факторизації для еліптичних символів вдалося описати умови існування розв'язку для рівнянь з сингулярностями типу "конуса" та "ребра". Більшість результатів автора щодо розв'язності стосувалися двовимірного випадку. Тут ми розглядаємо суттєво багатовимірний випадок.

Introduction

Our main goal is to describe possible solvability conditions for the pseudo-differential equation

$$(Au)(x) = f(x), \quad x \in D,$$

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where D is a manifold with boundary, A is a pseudo-differential operator with symbol $A(x,\xi)$. Such operators are defined locally by the formula

$$u(x) \longmapsto \int\limits_{\mathbb{R}^m}\int\limits_{\mathbb{R}^m} A(x,\xi)u(y)e^{-iy\cdot\xi}\,d\xi dy \tag{1}$$

in the case of a smooth compact manifold D, since "the freezing coefficients principle" (or "the local principle") can be applied. For a manifold with smooth boundary we need a new local formula for definition of A. In the inner points of D we use the formula (1), whereas we need to introduce another formula in the boundary points:

$$u(x) \longmapsto \int\limits_{\mathbb{R}^m_+}\int\limits_{\mathbb{R}^m} A(x,\xi)u(y)e^{-iy\cdot\xi}d\xi dy.$$

For invertibility of such an operator with symbol $A(\cdot,\xi)$ that does not depend on the spatial variable x one can apply theory of the classical Riemann boundary problem for upper and lower complex half-planes with a parameter ξ'. This approach was systematically studied in [4]. But if the boundary ∂D has at least one conical point, this approach is not effective.

The conical point at boundary is a point having a neighborhood, diffeomorphic to the cone

$$C^a_+ = \{(x_1,\dots,x_m) \in \mathbb{R}^m :\ x_m > a|x'|,\ x' = (x_1,\dots,x_{m-1})\},\ a > 0,$$

hence the local definition for a pseudo-differential operator near the conical point can be given by

$$u(x) \longmapsto \int\limits_{C^a_+}\int\limits_{\mathbb{R}^m} A(x,\xi)u(y)e^{-iy\cdot\xi}d\xi dy. \tag{2}$$

1. Spaces, operators, factorization

We consider the operator (1) in the Sobolev-Slobodetskii space $H^s(\mathbb{R}^m)$ with norm

$$\|u\|_s^2 = \int\limits_{\mathbb{R}^m} |\tilde{u}(\xi)|^2(1+|\xi|)^{2s}d\xi,$$

and introduce the following class of symbols non-depending on spatial variable x:

$$c_1 \le |A(\xi)(1+|\xi|)^{-\alpha}| \le c_2, \qquad \xi \in \mathbb{R}^m, \tag{3}$$

where c_1, c_2 are some positive constants. The number $\alpha \in \mathbb{R}$ is called *the order* of the pseudo-differential operator A. It is well-known that a pseudo-differential operator with symbol $A(\xi)$ satisfying (2) is a linear bounded operator acting from $H^s(\mathbb{R}^m)$ into $H^{s-\alpha}(\mathbb{R}^m)$ [4].

We are interested in studying the invertibility of operator (2) in the corresponding Sobolev-Slobodetskii spaces. By definition, the space $H^s(C^a_+)$ consists of distributions from $H^s(\mathbb{R}^m)$ with support in C^a_+. The norm in $H^s(C^a_+)$ is induced by the $H^s(\mathbb{R}^m)$-norm. Such an operator is associated with the corresponding equation

$$(Au_+)(x) = f(x),\ x \in C^a_+, \tag{4}$$

where the right-hand side f belongs to $H_0^{s-\alpha}(C_+^a)$. Next, $H_0^s(C_+^a)$ is the space of distributions $S'(C_+^a)$, which admit a continuation on $H^s(\mathbb{R}^m)$. The norm in $H_0^s(C_+^a)$ is defined by

$$\|f\|_s^+ = \inf \|lf\|_s,$$

the infimum taken over all possible continuations l of f.

From now on, we assume that symbols $A(\xi)$ satisfy the condition (3).

Definition 1.1. We say that a symbol $A(\xi)$ admits *the wave factorization* provided

$$A(\xi) = A_{\neq}(\xi)\, A_{=}(\xi),$$

where the factors $A_{\neq}(\xi)$, $A_{=}(\xi)$ satisfy the following conditions:

- $A_{\neq}(\xi), A_{=}(\xi)$ are defined everywhere, except for points of the set $\{\xi \in \mathbb{R}^m : |\xi'|^2 = a^2\xi_m^2\}$;
- $A_{\neq}(\xi), A_{=}(\xi)$ admit analytical continuations into the radial tube domains $T(\overset{*}{C}{}_+^a)$, $T(\overset{*}{C}{}_-^a)$ respectively, and these continuations satisfy the estimates
$|A_{\neq}^{\pm 1}(\xi + i\tau)| \le c_1(1+|\xi|+|\tau|)^{\pm\varkappa}, \quad |A_{=}^{\pm 1}(\xi - i\tau)| \le c_2(1+|\xi|+|\tau|)^{\pm(\alpha-\varkappa)}, \quad \tau \in \overset{*}{C}{}_+^a$.

The number $\varkappa$ is called the *index of wave factorization.*

Here $\overset{*}{C}{}_+^a$ is the conjugate cone to C_+^a, and $\overset{*}{C}{}_-^a = -\overset{*}{C}{}_+^a$.

Example 1.2. Let

$$A = -\frac{\partial^2}{\partial x_1^2} - \cdots - \frac{\partial^2}{\partial x_m^2} + k^2, \quad k \in \mathbb{R} \setminus \{0\},$$

Then the symbol of this operator has form $A(\xi) = \xi_1^2 + \xi_2^2 + \cdots + \xi_m^2 + k^2$, by properties of the Fourier transform. The following equality is the wave factorization of the Helmholtz operator:

$$\xi_m^2 + |\xi'|^2 + k^2 = \left(\sqrt{a^2+1}\,\xi_m + \sqrt{a^2\xi_m^2 - |\xi'|^2 - k^2}\right)\left(\sqrt{a^2+1}\,\xi_m - \sqrt{a^2\xi_m^2 - |\xi'|^2 - k^2}\right),$$

where the value $\sqrt{a^2\xi_m^2 - |\xi'|^2 - k^2}$ is treated as the boundary value $\sqrt{a^2(\xi_m + i0)^2 - |\xi'|^2 - k^2}$.

Remark 1.3. Two interesting applied problems from the diffraction and elasticity theory can be solved by the wave factorization mentioned above [3, 5]. For these problems we have the two-dimensional equation (4) with symbol

$$A(\xi_1, \xi_2) = (\xi_1^2 + \xi_2^2 - k^2)^{\pm 1/2}.$$

The existence of the wave factorization permits to obtain a solution of certain analogue of the multidimensional Riemann problem as follows

$$(G_m u)(x) = \lim_{\tau \to 0+} \int\limits_{\mathbf{R^m}} \frac{u(y', y_m)dy'dy_m}{(|x'-y'|^2 - a^2(x_m - y_m + i\tau)^2)^{m/2}}. \tag{5}$$

The integral is a multidimensional analogue of the Cauchy type integral, i.e. its limit case corresponds to the boundary values. It looks as a convolution with the kernel which is the Fourier image of C^a_+-indicator. But this multiplier is a non-integrable function. Therefore we need to go into the complex plane to destroy the divergence. The definition (5) is one of possible definitions for the singular integral. Of course, it is very desirable to give this definition for real variables (as the principal value type of the Cauchy integral like in the one-dimensional case), but it is worth noting however that such definition was used in classical papers.

2. Solvability theorems

The concept of wave factorization permits to describe the full solvability cases for the equation (4). For simplicity we assume $m = 2$. From now on we also suppose that the symbol admits the wave factorization.

Theorem 2.1. *Let $\varkappa - s = \delta$, $|\delta| < 1/2$. For any right-hand side $f \in H_0^{s-\alpha}(C^a_+)$ the equation* (4) *admits a unique solution $u_+ \in H_s(C^a_+)$ with the Fourier transform of the form*

$$\tilde{u}_+ = A_{\neq}^{-1} G_2 A_{=}^{-1} \tilde{\ell f},$$

where ℓf is an arbitrary continuation of $f \in H_0^{s-\alpha}(C^a_+)$ on $H^{s-\alpha}(\mathbb{R}^2)$. In addition, the following estimate holds

$$\|u_+\|_s \leq c\|f\|^+_{s-\alpha}.$$

Theorem 2.2. *Let $\varkappa - s = n + \delta$, $n > 0$ be an integer, $|\delta| < 1/2$. Then for any right-hand side $f \in H_0^{s-\alpha}(C^a_+)$ there exists a solution $u_+ \in H_s(C^a_+)$ of the equation* (4) *with the Fourier transform*

$$\begin{aligned}\tilde{u}_+(\xi) = {} & A_{\neq}^{-1} Q G_2 Q^{-1} A_{=}^{-1} \tilde{\ell f} \\ & + A_{\neq}^{-1}\left(\sum_{k=0}^{n-1}\left(\tilde{c}_k(\xi_1 - a\xi_2)^k(\xi_1 + a\xi_2)^k + \tilde{d}_k(\xi_1 + a\xi_2)(\xi_1 - a\xi_2)^k\right)\right. \\ & \left. + \sum_{k_1+k_2=0}^{n_\delta} a_{k_1k_2}(\xi_1 - a\xi_2)^{k_1}(\xi_1 + a\xi_2)^{k_2}\right),\end{aligned}$$

where c_k, d_k are arbitrary functions from $H_{s_k}(\mathbb{R}_-)$, $H_{s_k}(\mathbb{R}_+)$ respectively, $Q(\xi)$ is an arbitrary elliptic polynomial of order n satisfying the estimate (3) *with $\alpha = n$, $s_k = s - \varkappa + k + 1/2$, $k = 0, 1, \ldots, n-1$, $a_{k_1k_2} \in \mathbf{C}$,*

$$n_\delta = \begin{cases} n-1, & if\, \delta > 0 \\ n-2, & if\, \delta \leq 0. \end{cases}$$

The latter formula describes all possible solutions of equation (4)*. Moreover, these solution satisfy the a priori estimate*

$$\|u_+\|_s \leq c\left(\|f\|^+_{s-\alpha} + \sum_{k=0}^{n-1}\left([c_k]_{s_k} + [d_k]_{s_k}\right) + \sum_{k_1+k_2=0}^{n_\delta}|a_{k_1k_2}|\right).$$

Theorem 2.3. *Let $\varkappa - s = n + \delta$, $n \in \mathbf{Z}$, $n < 0$, $|\delta| < 1/2$. The equation* (4) *admits a solution u_+ from $H_s(C_+^a)$ if and only if the following conditions hold*

$$\left(\frac{1}{a}\frac{\partial}{\partial y_1} - \frac{\partial}{\partial y_2}\right)^{\beta_1}\left(\frac{1}{a}\frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right)^{\beta_2} A_{=}^{-1}\ell f(y)\bigg|_{y=0} = 0,$$

$$\left(\frac{1}{a}\frac{\partial}{\partial y_1} - \frac{\partial}{\partial y_2}\right)^{\beta_1}\left(\frac{1}{a}\frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right)^{\beta_2} A_{=}^{-1}\ell f(y)\bigg|_{\substack{ay_1 - y_2 \le 0\\ ay_1 + y_2 = 0}} = 0,$$

$$\left(\frac{1}{a}\frac{\partial}{\partial y_1} - \frac{\partial}{\partial y_2}\right)^{\beta_1}\left(\frac{1}{a}\frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right)^{\beta_2} A_{=}^{-1}\ell f(y)\bigg|_{\substack{ay_1 - y_2 = 0\\ ay_1 + y_2 \ge 0}} = 0,$$

where $|\beta| \in \{0, 1, \dots, |n| - 2\}$. Moreover, there exists a constant c such that $\|u_+\|_s \le c\|f\|_{s-\alpha}^+$.

In particular, Theorem 2.2 helps us to state correct boundary value problems for identifying the unknown functions c_k, d_k. For simplicity we assume that $n = 1$, $a = 1$, $f \equiv 0$. In the case of the Dirichlet or Neumann boundary conditions we have two unknown functions $\tilde{c}_0(\xi_1 - \xi_2)$, $\tilde{d}_0(\xi_1 + \xi_2)$, and an application of the Mellin transform leads to the system of linear algebraic equations with the matrix

$$R(\lambda) = \begin{pmatrix} K(\lambda) & I \\ I & M(\lambda) \end{pmatrix},$$

where $K(\lambda)$, $M(\lambda)$, I are square matrices of the order 2.

The conditions $\det R(\lambda) \neq 0$, $\operatorname{Re}\lambda = 1/2$ are called the *conical Shapiro-Lopatinsky condition.*

If A is the Laplacian, then $R(\lambda)$ can be calculated explicitly [3, 5].

3. Some distributions

If we will try to consider more complicated singularities like a cusp point at the boundary, we need some additional investigation. Each singularity corresponds to a certain distribution and it is useful to know what kind of distributions we will obtain in special limit cases. All results below in this section are treated in the sense of distributions.

Let us denote by $\otimes$ the direct product of distributions. Next, the distribution $\mathrm{P}\frac{1}{\mathrm{x}}$ is introdused in V. S. Vladimirov's book [2].

Theorem 3.1. *The following equality holds*

$$\lim_{a\to\infty} \frac{a}{2\pi^2}\frac{1}{\xi_1^2 - a^2\xi_2^2} = \frac{i}{2\pi}\,\mathrm{P}\frac{1}{\xi_1} \otimes \delta(\xi_2), \tag{6}$$

where δ is the Dirac function.

The distribution (6) corresponds to a half-infinite crack with an adjoint mass.

If we find another asymptotic for distribution (6) as $a \to 0$, then we have

$$\lim_{a\to 0} \frac{a}{2\pi^2}\frac{1}{\xi_1^2 - a^2\xi_2^2} = \frac{1}{2\pi i}\delta(\xi_1) \otimes \mathrm{P}\frac{1}{\xi_2},$$

and it corresponds to half-plane case (see [4]).

Now we will speak on another asymptotics related to multi-wedge angle. The simplest variant of such angle is $\{x \in \mathbb{R}^3 : \ x_3 > a\,|x_1| + b\,|x_2|\}$, where a, b are two parameters. If these parameters tend to 0 or ∞, then we obtain new types of thin singularities.

The distribution corresponding to such angle is [3, 5]

$$K_{a,b}(\xi_1,\xi_2,\xi_3) = \frac{4iab}{(2\pi)^3}\frac{\xi_3}{\left(\xi_1^2 - a^2\xi_3^2\right)\left(\xi_2^2 - b^2\xi_3^2\right)}.$$

We consider different relations between a and b.

Theorem 3.2. $\displaystyle \lim_{b\to\infty} \frac{4iab\xi_3}{(2\pi)^3\left(\xi_1^2 - a^2\xi_3^2\right)\left(\xi_2^2 - b^2\xi_3^2\right)} = \frac{i}{2\pi}\delta(\xi_1)\otimes \mathrm{P}\frac{1}{\xi_2}\otimes\delta(\xi_3).$

Theorem 3.3. $\displaystyle \lim_{a\to\infty} \frac{4iab\xi_3}{(2\pi)^3\left(\xi_1^2 - a^2\xi_3^2\right)\left(\xi_2^2 - b^2\xi_3^2\right)} = \frac{i}{2\pi}\mathrm{P}\frac{1}{\xi_1}\otimes\delta(\xi_2)\otimes\delta(\xi_3).$

Theorem 3.4. $\displaystyle \lim_{b\to 0} \frac{4iab}{(2\pi)^3}\frac{\xi_3}{\left(\xi_1^2 - a^2\xi_3^2\right)\left(\xi_2^2 - b^2\xi_3^2\right)} = \delta(\xi_2)\otimes K_a(\xi_1,\xi_3).$

Theorem 3.5. $\displaystyle \lim_{a\to 0} \frac{4iab}{(2\pi)^3}\frac{\xi_3}{\left(\xi_1^2 - a^2\xi_3^2\right)\left(\xi_2^2 - b^2\xi_3^2\right)} = \delta(\xi_1)\otimes K_b(\xi_2,\xi_3).$

Theorem 3.6. $\displaystyle \lim_{a\to 0,\, b\to 0} \frac{4iab}{(2\pi)^3}\frac{\xi_3}{\left(\xi_1^2 - a^2\xi_3^2\right)\left(\xi_2^2 - b^2\xi_3^2\right)} = \frac{1}{2\pi i}\delta(\xi')\otimes \mathrm{P}\frac{1}{\xi_3}, \quad \xi' = (\xi_1,\xi_2).$

The last result corresponds to the half-space case $x_3 > 0$ [4].

In the case $m = 2$, $a \to +\infty$, the following formal representations are useful

$$K_a(\xi_1,\xi_2) = \sum_{n=0}^{k}\frac{(-1)^n}{n!\,a^n}\,\mathrm{P}\frac{1}{\xi_1}\otimes\delta^{(n)}(\xi_2) + R_k(\xi_1,\xi_2),$$
$$K_a(\xi_1,\xi_2) = \sum_{n=0}^{+\infty}\frac{(-1)^n}{n!\,a^n}\,\mathrm{P}\frac{1}{\xi_1}\otimes\delta^{(n)}(\xi_2).$$

4. Quasi-elliptic case

Freezing coefficients yields symbols $A(\cdot,\xi)\equiv A(\xi)$, which are homogeneous of order m in the generalized sense:

$$A(t^{\alpha_1}\xi_1,\dots,t^{\alpha_m}\xi_m) = t^m A(\xi),$$

for all $t > 0$ and $\alpha_1 + \alpha_2 + \dots + \alpha_m = m$. The heat operator

$$A:\ u \longmapsto \frac{\partial u}{\partial t} - a^2\left(\frac{\partial^2 u}{\partial x_1^2} + \dots + \frac{\partial^2 u}{\partial x_m^2}\right)$$

with the symbol $A(\xi) = i\xi_0 - a^2(\xi_1^2 + \cdots + \xi_m^2)$ has the homogeneity order $m+1$:
$\alpha_1 = \alpha_2 = \cdots = \alpha_m,\ \alpha_0 = 2.$

One can adapt definition of the wave factorization for studying such operators (equations) by the following way. According to above we separate one variable and introduce the following notation $(0 < \gamma < +\infty)$:

$$C_+^a = \{x \in \mathbb{R}^{m+1} : x = (x_0, x_1, \dots, x_m),\ x_0 > a|x'|,\ x' = (x_1, \dots, x_m)\},$$
$$\overset{*}{C}{}_+^a = \{x \in \mathbb{R}^{m+1} : ax_0 > |x'|\}.$$

Definition 4.1. By the *wave factorization* of a symbol $A(\xi)$ we understand its representation in the form

$$A(\xi) = A_{\neq}(\xi)A_{=}(\xi),$$

where the factors $A_{\neq}(\xi)$, $A_{=}(\xi)$ satisfy the following conditions:

- $A_{\neq}(\xi)$, $A_{=}(\xi)$ are defined everywhere without may be the points of the set $\{\xi \in \mathbb{R}^{m+1} : |\xi'|^2 = a^2\xi_m^2\}$;
- $A_{\neq}(\xi)$, $A_{=}(\xi)$ admit an analytical continuation into radial tube domains $T(\overset{*}{C}{}_+^a)$, $T(\overset{*}{C}{}_-^a)$ respectively, satisfying the estimates $|A_{\neq}^{\pm 1}(\xi + i\tau)| \le c_1(1 + |\xi'| + |\xi_0|^{1/\gamma} + |\tau|)^{\pm\varkappa}$, $|A_{=}^{\pm 1}(\xi - i\tau)| \le c_2(1 + |\xi'| + |\xi_0|^{1/\gamma} + |\tau|)^{\pm(\alpha-\varkappa)}$ for every $\tau \in \overset{*}{C}{}_+^a$.

The number $\varkappa$ is called the *index of wave factorization*.

Furthermore, if we consider the equation (4) in the Sobolev–Slobodetskii space with the norm

$$\|u\|_{s,\gamma}^2 = \int\limits_{\mathbb{R}^{m+1}} |\tilde{u}(\xi)|^2(1 + |\xi'| + |\xi_0|^{1/\gamma})^{2s} d\xi,$$

we can obtain the following simple result ($m = 1$).

Theorem 4.2. *Let $\varkappa - s = \delta$, $|\delta/\gamma| < 1/2$. Then for any right-hand side $f \in H_0^{s-\alpha}(C_+^a)$ the equation* (4) *has a unique solution $u_+ \in H_s(C_+^a)$ for which the Fourier transform is given by $\tilde{u}_+ = A_{\neq}^{-1}G_2A_{=}^{-1}\widetilde{\ell f}$, where ℓf is an arbitrary continuation of $f \in H_0^{s-\alpha}(C_+^a)$ on $H_{s-\alpha}(\mathbb{R}^2)$. Moreover, this solution satisfies the a priori estimate $\|u_+\|_{s,\gamma} \le c\|f\|_{s-\alpha}^+$.*

5. Future extensions

The author is going to study in the nearest future the following cases.

(i) The essentially multi-dimensional case with the distribution

$$\sum a_k(P)\delta^{(k)}(P),$$

on the boundary. Here P is the surface of cone (cf. [1]).

(ii) The asymptotical case (thin singularities) [7].

(iii) The discrete case, for which there are some interesting results related to the Calderon-Zygmund operators [6].

(iv) The non-elliptic case, e.g. parabolic equations.

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